Fading regularization inverse methods for the identification of boundary conditions in thin plate theory

Mohamed Aziz Boukraa[1,2], **Franck Delvare**[2]

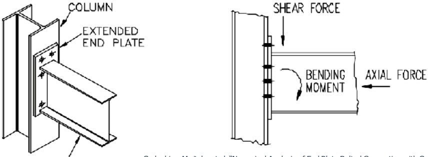
[1]*Laboratoire de mathématiques Blaise Pascal*
Université Clermont Auvergne

[2]*Laboratoire de Mathématiques Nicolas Oresme*
Université de Caen Normandie

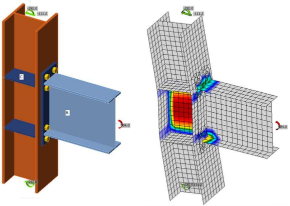
CFM2022, Nantes, 29/08-02/09/2022

Motivations : Identifications of boundary conditions in structural mechanics

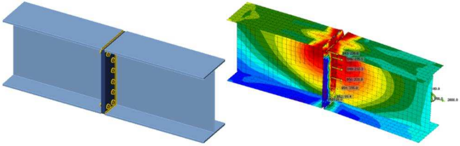
Motivations : Identifications of boundary conditions in structural mechanics



Sadeghian, Mojtaba et al. "Numerical Analysis of End Plate Bolted Connection with Corrugated Beam." *World Academy of Science, Engineering and Technology, International Journal of Civil, Environmental, Structural, Construction and Architectural Engineering* 9 (2015): 1496-1500.



https://www.ideastatica.com/support-center/extended-end-plate-moment-connections-aisc



https://www.ideastatica.com/support-center/bolted-plate-to-plate-en

https://www.energierecrute.com

Cauchy problem associated with the biharmonic equation

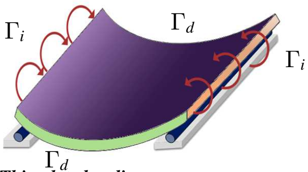
$$\Delta^2 u = 0 \quad \forall x \in \Omega$$

ou

$$\begin{cases} \Delta u = v & \forall x \in \Omega \\ \Delta v = 0 & \forall x \in \Omega \end{cases}$$

$\partial\Omega = \Gamma_d \cup \Gamma_i$ et $\Gamma_d \cap \Gamma_i = \emptyset$

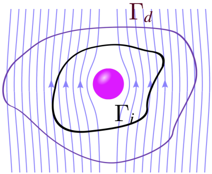
où $u_{,n} = \frac{\partial u}{\partial n}$ et $v_{,n} = \frac{\partial v}{\partial n}$



Thin plate bending

u : the deflection of the plate

v : the bending moment



Stokes flow

u : the stream function

v : the vorticity of the fluid

Cauchy problem associated with the biharmonic equation

$$\Delta^2 u = 0 \quad \forall x \in \Omega$$

ou

$$\begin{cases} \Delta u = v & \forall x \in \Omega \\ \Delta v = 0 & \forall x \in \Omega \end{cases}$$

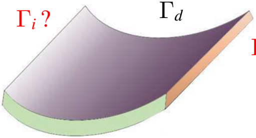
with

$$\begin{cases} u = \varphi_d & \forall x \in \Gamma_d \\ u_{,n} = \psi_d & \forall x \in \Gamma_d \\ v = \mu_d & \forall x \in \Gamma_d \\ v_{,n} = \phi_d & \forall x \in \Gamma_d \end{cases}$$

$\partial\Omega = \Gamma_d \cup \Gamma_i$ et $\Gamma_d \cap \Gamma_i = \emptyset$

où $u_{,n} = \frac{\partial u}{\partial n}$ et $v_{,n} = \frac{\partial v}{\partial n}$

No boundary condition is given on Γ_i



→ ill-posed problem in the sens of Hadamard

the stability of the solution cannot be guaranteed

→ It's an inverse problem !

→ Cannot be solved by the usual methods

Examples of regularization methods

Based on a reformulation of the Cauchy problem :

- The method based on minimization of an energy-like error Functional (*Andrieux et al.* (2005-2006))

  Transform the problem into two well-posed problem with mixed boundary conditions and minimize the gap between the two field solutions.

- Steklov-Poincaré algorithm (*Belgacem et al.* (2005))

  Transform the problem into a Steklov-Poincaré problem, two direct problems with Dirichlet and Neumann boundary data respectively.

- ...

Based on the regularization of the continuous problem :

- Quasi-reversibility method (*Lattès et al. (1967)*)

  Second order ill-posed Cauchy problem $\rightsquigarrow$ Fourth order well-posed problem

- Tikhonov methods (*Tikhonov et al. (1986)*)

  Regularization by adding a control term (well-posed problem).

- Fading regularization method (*Cimetière et al. (2000,2001), Delvare (2000)*)

  Iterative regularization by adding a control term that tend to 0 (well-posed problems).

- ...

Equivalent formulation of the problem

For $\Phi_d = (\varphi_d, \psi_d, \mu_d, \phi_d)$ a quadruplet of compatible data on Γ_d, (i.e. $\Phi_d \in H(\Gamma_d)$), the biharmonic Cauchy problem is equivalent to :

$$\begin{cases} \mathbf{U} = (u, u_{,n}, v, v_{,n}) \in H(\Gamma) \text{ such as :} \\ \mathbf{U} = \Phi_d \qquad \text{on } \Gamma_d \end{cases}$$

with

$$H(\Gamma) = \{\Phi = (\varphi, \psi, \mu, \phi) \in X(\Gamma) \text{ such as } \exists u \in \mathscr{H}_0^2 \\ \text{with } v = \Delta u \text{ and } (u, u', v, v') = (\varphi, \psi, \mu, \phi)\},$$

such as

$$X(\Gamma) = H^{3/2}(\Gamma) \times H^{1/2}(\Gamma) \times H^{-1/2}(\Gamma) \times H^{-3/2}(\Gamma)$$

and

$$\mathscr{H}_0^2 = \{u \in H^2(\Omega) \quad / \quad \Delta^2 u = 0\}.$$

The fading regularization method

Cimetière et al. (2000,2001), Delvare (2000)

Basic idea : Seeking among all solutions of the equilibrium equation in Ω, the one that fits the best the boundary conditions available on Γ_d, with :

- independence to a regularization parameter,
- stability towards noisy data,

$$\mathbf{U}^{k+1} = \underset{\mathbf{V}\in H(\Gamma)}{\text{Argmin}} \left\{ \|\mathbf{V} - \Phi_d\|^2_{\Gamma_d} + c\|\mathbf{V} - \mathbf{U}^k\|^2_{\Gamma} \right\}$$

✓ A sequence of well-posed optimization problems,
✓ Best agreement to the data (data relaxation),
✓ Independence of the solution with respect to c,
✓ Convergent algorithm.

→ At iteration k, there exists a unique minimum characterized by the optimality equation :

$$\langle \mathbf{U}^{k+1} - \Phi_d, \mathbf{V} \rangle_{\Gamma_d} + c\langle \mathbf{U}^{k+1} - \mathbf{U}^k, \mathbf{V} \rangle_{\Gamma} = 0 \quad \forall \mathbf{V} \in H(\Gamma)$$

Convergence of the continuous formulation

Theorem

Let Φ_d be the compatible Cauchy data associated with the compatible solution $\mathbf{U}_e \in \mathcal{H}(\Gamma)$. Then, the sequence $(\mathbf{U}^k)_{k\in\mathbb{N}}$ generated by the iterative algorithm verifies :

$$\begin{aligned} \mathbf{U}^k &\to \Phi_d \quad \textit{in} \quad H(\Gamma_d) \quad \textit{strongly} \\ \mathbf{U}^k &\rightharpoonup \mathbf{U}_e \quad \textit{in} \quad H(\Gamma) \quad \textit{weakly} \end{aligned}$$

Convergence of the continuous formulation

Theorem

Let Φ_d be the compatible Cauchy data associated with the compatible solution $\mathbf{U}_e \in \mathcal{H}(\Gamma)$. Then, the sequence $(\mathbf{U}^k)_{k\in\mathbb{N}}$ generated by the iterative algorithm verifies :

$$\begin{aligned} \mathbf{U}^k &\to \Phi_d \quad \text{in} \quad H(\Gamma_d) \quad \text{strongly} \\ \mathbf{U}^k &\rightharpoonup \mathbf{U}_e \quad \text{in} \quad H(\Gamma) \quad \text{weakly} \end{aligned}$$

Lemma

For all $n \in \mathbb{N}$, the sequence $(\mathbf{U}^k)_k$ generated by the iterative algorithm verifies :

$$\|\mathbf{U}^{n+1} - \mathbf{U}_e\|_\Gamma^2 + \sum_{k=0}^{n} \|\mathbf{U}^{k+1} - \mathbf{U}^k\|_\Gamma^2 + \frac{2}{c}\sum_{k=0}^{n} \|\mathbf{U}^{k+1} - \Phi_d\|_{\Gamma_d}^2 = \|\mathbf{U}^0 - \mathbf{U}_e\|_\Gamma^2$$

where $\mathbf{U}_e$ is the compatible solution of the Cauchy problem.

Convergence of the continuous formulation

Lemma

For all $n \in \mathbb{N}$, the sequence $(\mathbf{U}^k)_k$ generated by the iterative algorithm verifies :

$$\|\mathbf{U}^{n+1} - \mathbf{U}_e\|_\Gamma^2 + \sum_{k=0}^{n} \|\mathbf{U}^{k+1} - \mathbf{U}^k\|_\Gamma^2 + \frac{2}{c} \sum_{k=0}^{n} \|\mathbf{U}^{k+1} - \Phi_d\|_{\Gamma_d}^2 = \|\mathbf{U}^0 - \mathbf{U}_e\|_\Gamma^2$$

where $\mathbf{U}_e$ is the compatible solution of the Cauchy problem.

- The strong convergence
  - The series $\sum_{k=0}^{n} \|\mathbf{U}^{k+1} - \Phi_d\|_{\Gamma_d}^2$ is bounded,

$\rightarrow$ $\|\mathbf{U}^k - \Phi_d\|_{\Gamma_d}^2$ tends to 0,

$\rightarrow$ $\mathbf{U}^k \underset{k \to +\infty}{\longrightarrow} \Phi_d$ on Γ_d.

Convergence of the continuous formulation

Lemma

For all $n \in \mathbb{N}$, the sequence $(\mathbf{U}^k)_k$ generated by the iterative algorithm verifies :

$$\|\mathbf{U}^{n+1} - \mathbf{U}_e\|_\Gamma^2 + \sum_{k=0}^{n} \|\mathbf{U}^{k+1} - \mathbf{U}^k\|_\Gamma^2 + \frac{2}{c}\sum_{k=0}^{n} \|\mathbf{U}^{k+1} - \Phi_d\|_{\Gamma_d}^2 = \|\mathbf{U}^0 - \mathbf{U}_e\|_\Gamma^2$$

where $\mathbf{U}_e$ is the compatible solution of the Cauchy problem.

- The weak convergence
  - Existence of a sub-sequence of $(\mathbf{U}^k)_k$ that is weakly convergent to $\mathbf{U}_e$
    - $(\|\mathbf{U}^k - \mathbf{U}_e\|_\Gamma^2)_k$ is bounded, hence $(\mathbf{U}^k)_k$ is bounded in $H(\Gamma)$

    $\rightarrow$ there exists a sub-sequence $(\mathbf{U}^\mu)_\mu$ of $(\mathbf{U}^k)_k$ such as :

    $$\mathbf{U}^\mu \rightharpoonup \mathbf{U}_L \text{ in } H(\Gamma)$$

    - $\lim_{\mu\to+\infty} \|\mathbf{U}^\mu - \Phi_d\|_{\Gamma_d}^2 = 0$, hence $\lim_{\mu\to+\infty} \mathbf{U}^\mu = \Phi_d$
    - by uniquness of the limit on Γ_d : $\mathbf{U}_L|_{\Gamma_d} = \Phi_d$
    - by uniquness of the harmonic extension (**Holmgren's theorem**) :

    $$\mathbf{U}_L = \mathbf{U}_e \text{ on } \Gamma.$$

Convergence of the continuous formulation

Lemma

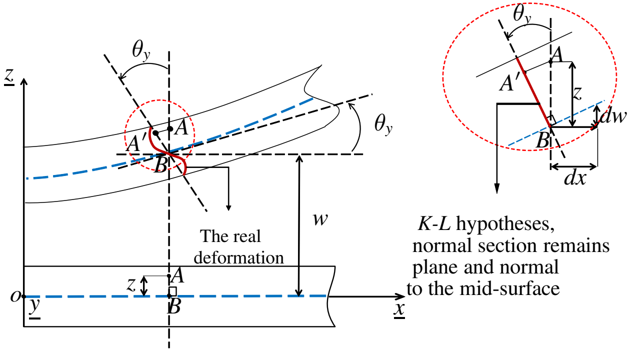
For all $n \in \mathbb{N}$, the sequence $(\mathbf{U}^k)_k$ generated by the iterative algorithm verifies :

$$\|\mathbf{U}^{n+1} - \mathbf{U}_e\|_\Gamma^2 + \sum_{k=0}^{n} \|\mathbf{U}^{k+1} - \mathbf{U}^k\|_\Gamma^2 + \frac{2}{c} \sum_{k=0}^{n} \|\mathbf{U}^{k+1} - \Phi_d\|_{\Gamma_d}^2 = \|\mathbf{U}^0 - \mathbf{U}_e\|_\Gamma^2$$

where $\mathbf{U}_e$ is the compatible solution of the Cauchy problem.

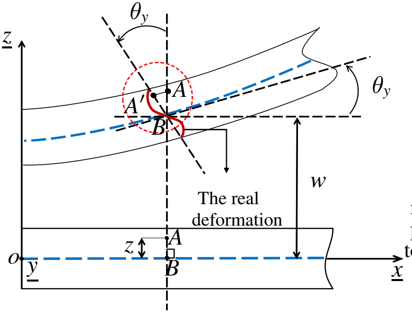
- The weak convergence
  - Existence of a sub-sequence of $(\mathbf{U}^k)_k$ that is weakly convergent to $\mathbf{U}_e$
  - Weak convergence of all the sequence $(\mathbf{U}^k)_k$ to $\mathbf{U}_e$ on Γ
    - $\rightarrow$ Proof by contradiction.

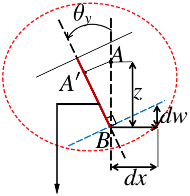
Kirchhoff-Love hypotheses



- *"Sections normal to the middle plane remain plane during deformation"*
- *"Sections normal to the middle plane remain normal to the middle plane during deformation"*

Kirchhoff-Love hypotheses

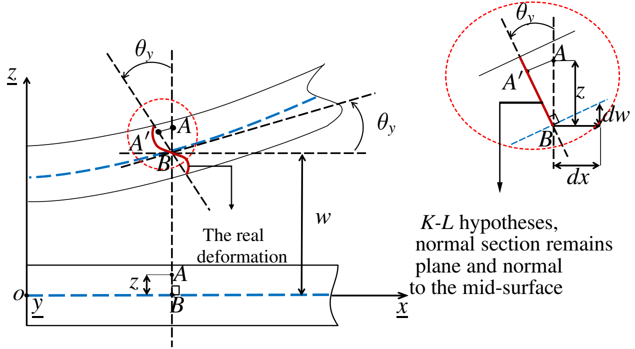




K-L hypotheses, normal section remains plane and normal to the mid-surface

$$\begin{cases} \theta_x = \dfrac{\partial w}{\partial y} \\ \theta_y = -\dfrac{\partial w}{\partial x} \end{cases}$$

Kirchhoff-Love hypotheses



K-L hypotheses, normal section remains plane and normal to the mid-surface

- Variational formulation :

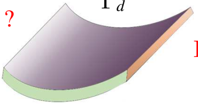
$$\int_\Omega \underbrace{\Big((\mathbf{L}\nabla)^t \mathbf{D} (\mathbf{L}\nabla) w\Big)}_{D\Delta^2 w} \delta w \; dxdy = \int_\Omega q(x,y)\delta\omega \quad dxdy$$

$$+ \int_\Gamma \left[\mathcal{M}_n \frac{\partial \delta w}{\partial n} - \mathcal{V}_n \delta w \right] \quad ds + \sum_i \delta w_i R_i$$

where $(\mathbf{L}\nabla) = \begin{bmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial y^2} & 2\frac{\partial^2}{\partial x \partial y} \end{bmatrix}^t$ et $\mathbf{D}$ is the flexural rigidity of the plate.

Cauchy problem in thin plate theory

- Cauchy problem associated with the biharmonic equation with mechanical boundary conditions that relate to the thin plate bending problem



$$\begin{cases} \Delta^2 w = 0 & \text{in } \Omega \\ w = \varphi_d & \text{on } \Gamma_d \\ w_{,n} = \psi_d & \text{on } \Gamma_d \\ \mathcal{M}_n = \mathcal{M}_d & \text{on } \Gamma_d \\ \mathcal{V}_n = \mathcal{V}_d & \text{on } \Gamma_d \end{cases}$$

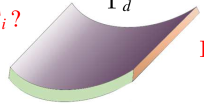
- The boundary conditions of the *Kirchhoff* thin plate theory amount to identifying the quantities w, $\frac{\partial w}{\partial n}$ and the forces :

$$\mathcal{M}_n = -D\left[\Delta w + (1-\nu)\left(2n_x n_y \frac{\partial^2 w}{\partial x \partial y} - n_y^2 \frac{\partial^2 w}{\partial x^2} - n_x^2 \frac{\partial^2 w}{\partial y^2}\right)\right]$$

$$\mathcal{V}_n = -D\left[\frac{\partial \Delta w}{\partial n} + (1-\nu)\frac{\partial}{\partial s}\left[n_x n_y\left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2}\right) + (n_x^2 - n_y^2)\frac{\partial^2 w}{\partial x \partial y}\right]\right]$$

Cauchy problem in thin plate theory

- Cauchy problem associated with the biharmonic equation with mechanical boundary conditions that relate to the thin plate bending problem



$$\begin{cases} \Delta^2 w = 0 & \text{in } \Omega \\ w = \varphi_d & \text{on } \Gamma_d \\ w_{,n} = \psi_d & \text{on } \Gamma_d \\ \mathcal{M}_n = \mathcal{M}_d & \text{on } \Gamma_d \\ \mathcal{V}_n = \mathcal{V}_d & \text{on } \Gamma_d \end{cases}$$

- The regularization functional becomes :

$$\begin{aligned} J_c^{k+1}(W) = {} & \|w_{|\Gamma_d} - \phi_d\|^2_{H^{3/2}(\Gamma_d)} + \|w_{,n|\Gamma_d} - \mu_d\|^2_{H^{1/2}(\Gamma_d)} + \|\mathcal{M}_{n|\Gamma_d} - \mathcal{M}_d\|^2_{H^{-1/2}(\Gamma_d)} \\ & + \|\mathcal{V}_{n|\Gamma_d} - \mathcal{V}_d\|^2_{H^{-3/2}(\Gamma_d)} + c\Big(\|w - w^k\|_{H^{3/2}(\Gamma)} + \|w_{,n} - w_{,n}{}^k\|_{H^{1/2}(\Gamma)} \\ & + \|\mathcal{M}_n - \mathcal{M}_n^k\|_{H^{-1/2}(\Gamma)} + \|\mathcal{V}_n - \mathcal{V}_n^k\|_{H^{-3/2}(\Gamma)}\Big), \\ & \forall W = (w, w_{,n}, \mathcal{M}_n, \mathcal{V}_n) \in \mathbf{H}(\Gamma). \end{aligned}$$

where $c > 0$ and $\mathbf{H}(\Gamma)$ is the space of the compatible quadruplets.

Plate finite elements

Thin plates
Kirchhoff-Love theory

$\mathcal{C}^1$ regularity for the displacement w

Example of non-conforming elements :

- **-Adini 1960**
- **-Cheung 1968**
- **-Bazeley 1966**
- **-Melosh 1963**

Poor performance due to $\mathcal{C}^1$ regularity issue when rotation field interpolation comes from displacement field

Thick plates
Reissner-Mindlin theory

$\mathcal{C}^0$ regularity for w and independent interpolation for the rotation field

Example of thick plate elements :

- **-MITC4**
- **-DSQ**
- **-DKMQ**

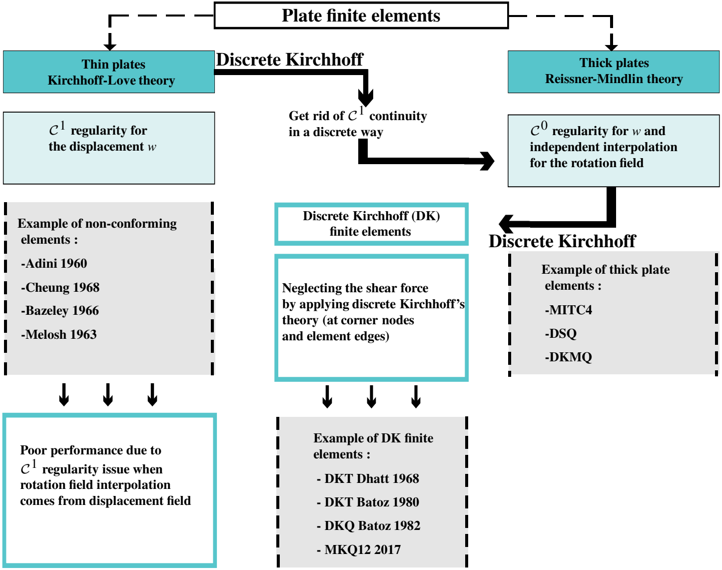
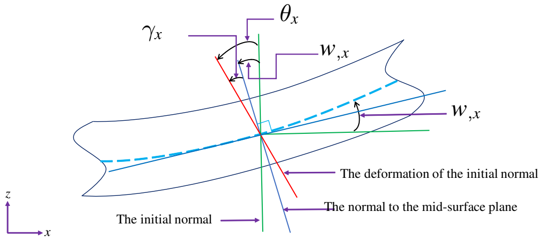


Plate finite elements
Thin plates
Kirchhoff-Love theory
Discrete Kirchhoff
Thick plates
Reissner-Mindlin theory
$\mathcal{C}^1$ regularity for
the displacement w
Get rid of $\mathcal{C}^1$ continuity
in a discrete way
$\mathcal{C}^0$ regularity for w and
independent interpolation
for the rotation field
Example of non-conforming
elements :
-Adini 1960
-Cheung 1968
-Bazeley 1966
-Melosh 1963
Discrete Kirchhoff (DK)
finite elements
Discrete Kirchhoff
Example of thick plate
elements :
-MITC4
-DSQ
-DKMQ
Neglecting the shear force
by applying discrete Kirchhoff's
theory (at corner nodes
and element edges)
Poor performance due to
$\mathcal{C}^1$ regularity issue when
rotation field interpolation
comes from displacement field
Example of DK finite
elements :
- DKT Dhatt 1968
- DKT Batoz 1980
- DKQ Batoz 1982
- MKQ12 2017

DK (Discrete Kirchhoff) finite elements

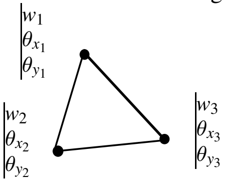
- **Thick plate finite element** : Including shear deformation $\theta_s = \gamma_s + \frac{\partial w}{\partial s}$

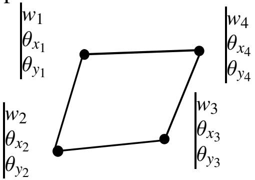


- Independent discretization of the displacement and the rotation field :

  $w = \sum_i N_i w_i \quad \theta_x = \sum_i N_i \theta_{x_i} \quad \theta_y = \sum_i N_i \theta_{y_i}$

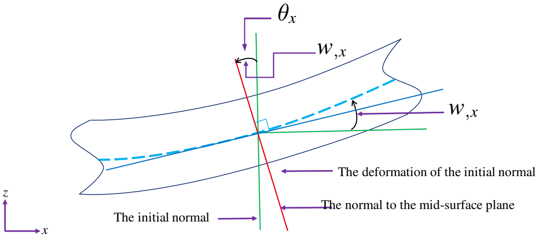
- Finite element with 3 degrees of freedom per node : :





DK (Discrete Kirchhoff) finite elements

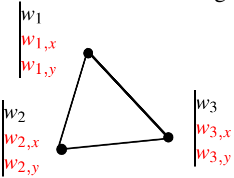
- **DK finite element** : *Kirchhoff* hypotheses $\gamma_s = 0 \Rightarrow \theta_s = \frac{\partial w}{\partial s}$

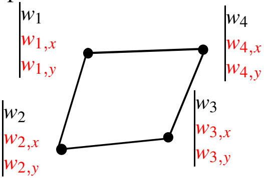


- Independent discretization of the displacement and the rotation field :

  $w = \sum_i N_i w_i \quad \theta_x = \sum_i N_i \theta_{x_i} \quad \theta_y = \sum_i N_i \theta_{y_i}$ such that $\theta_{s_i} = \frac{\partial w}{\partial s}|_i$

- Finite element with 3 degrees of freedom per node :





Numerical implementation of the iterative algorithm

- Interpolation of the displacement vector :

$$\underline{w}^e = \underline{\mathbf{N}}\, \underline{d}^e, \quad \underline{d}^{e_i} = \begin{bmatrix} w_i \\ \theta_{x_i} \\ \theta_{y_i} \end{bmatrix} = \begin{bmatrix} w_i \\ w_{,y_i} \\ -w_{,x_i} \end{bmatrix}.$$

- Interpolation the strain vector :

$$(\mathbf{L}\nabla)\underline{w}^e = \begin{bmatrix} \theta_{x,x} \\ \theta_{y,y} \\ \theta_{x,y} + \theta_{y,x} \end{bmatrix} = \underline{B}^e\ \underline{d}^e$$

- Finite element formulation :

$$\left(\int_\Omega \underline{B}^t \underline{D}\, \underline{B}\, d\Omega \right) \underline{d} = \int_\Gamma \left[-\underline{\mathbf{N}}^t_{,n} \mathcal{M}_n + \underline{\mathbf{N}}^t \mathcal{V}_n \right] ds$$

$$\mathbf{K}\underline{d} = \underbrace{\left[-\int_\Gamma \mathbf{N}^t_{,n} ds \quad \int_\Gamma \mathbf{N}^t ds \right]}_{\equiv \mathbf{F}} \underbrace{\begin{bmatrix} \underline{\mathcal{M}}_n \\ \underline{\mathcal{V}}_n \end{bmatrix}}_{\equiv \underline{b}}$$

$$\mathcal{E}(\underline{\mathbf{V}}) := \mathbf{K}\underline{d} - \mathbf{F}\ \underline{b} = 0, \text{tel que } \underline{\mathbf{V}} = (\underline{d}, \underline{\mathcal{M}}_n, \underline{\mathcal{V}}_n)$$

Numerical implementation of the iterative algorithm

- The fading regularization algorithm :

$$\left\| \begin{array}{l} \underline{\mathbf{V}}^{k+1} = \underset{\underline{\mathbf{V}} \in \mathbb{R}^{5N}}{\text{Argmin}}\, J_c^{k+1}(\underline{\mathbf{V}}) \\ \text{with } \underline{\mathbf{V}} = (\underline{d}, \underline{\mathcal{M}}_n, \underline{\mathcal{V}}_n) = (\underline{W}, \underline{\theta}_{,x}, \underline{\theta}_{,y}, \underline{\mathcal{M}}_n, \underline{\mathcal{V}}_n) \\ \text{under the equality constraints } \mathcal{E}(\underline{\mathbf{V}}) = 0 \end{array} \right.$$

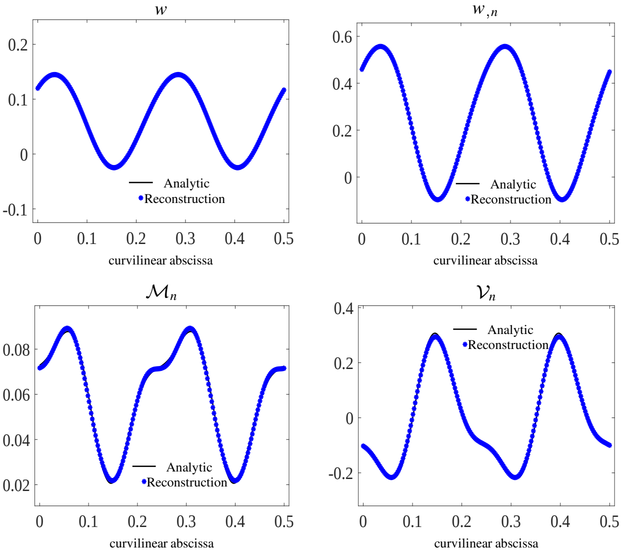
- The functional to be optimized :

$$\begin{aligned} J_c^{k+1}(\underline{\mathbf{V}}) = {} & \|\underline{W}_{|\Gamma_d} - \underline{\phi}_d\|^2_{L^2(\Gamma_d)} + \|\underline{n}_y \underline{\theta}_{,x} + \underline{n}_x \underline{\theta}_{,y}{}_{|\Gamma_d} - \mu_d\|^2_{L^2(\Gamma_d)} \\ & + \|\underline{\mathcal{M}}_{n|\Gamma_d} - \underline{\mathcal{M}}_d\|^2_{L^2(\Gamma_d)} + \|\underline{\mathcal{V}}_{n|\Gamma_d} - \underline{\mathcal{V}}_d\|^2_{L^2(\Gamma_d)} + c\Big(\|\underline{W} - \underline{W}^k\|^2_{L^2(\Gamma)} \\ & + \|\underline{\theta}_{,x} - \underline{\theta}_{,x}{}^k\|^2_{L^2(\Gamma)} + \|\underline{\theta}_{,y} - \underline{\theta}_{,y}{}^k\|^2_{L^2(\Gamma)} + \|\underline{\mathcal{M}}_n - \underline{\mathcal{M}}_n^k\|^2_{L^2(\Gamma)} + \|\underline{\mathcal{V}}_n - \underline{\mathcal{V}}_n^k\|^2_{L^2(\Gamma)}\Big) \end{aligned}$$

- Resolution of the linear system :

$$\begin{bmatrix} \nabla J_c^{k+1} & \nabla \mathcal{E}^T \\ \mathcal{E} & 0 \end{bmatrix} \begin{bmatrix} \underline{\mathbf{V}}^{k+1} \\ \underline{\eta}^{k+1} \end{bmatrix} = \begin{bmatrix} \mathcal{S}^k \\ \underline{0} \end{bmatrix}.$$

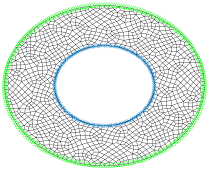
Reconstructions on the inner boundary of an annular domain (compatible data)



Analytical solution

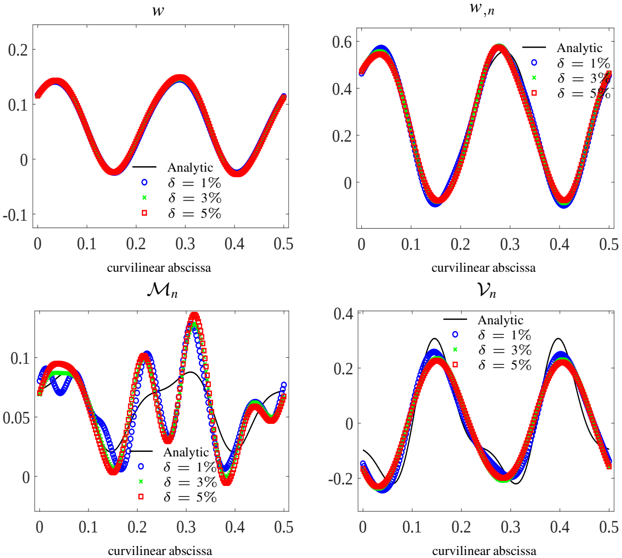
$$u_{an}(\mathbf{x}) = \frac{1}{2}x_1(\sin x_1 \cosh x_2 - \cos x_1 \sinh x_2)$$

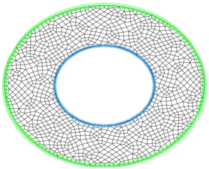
$$\mathbf{x} = (x_1, x_2) \in \Omega$$



$\triangle$ Γ_i : unknowns
$*$ Γ_d : data

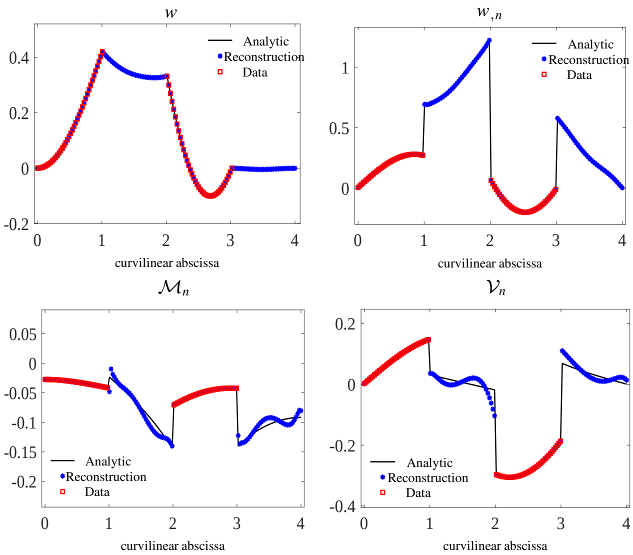
Reconstructions on the inner boundary of an annular domain (noisy data)

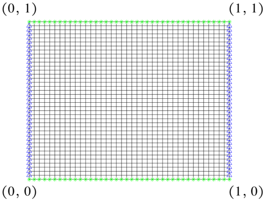




$\triangle$ Γ_i : unknowns
$*$ Γ_d : data

Reconstructions on the boundary of a square domain (compatible data located on two opposite sides)



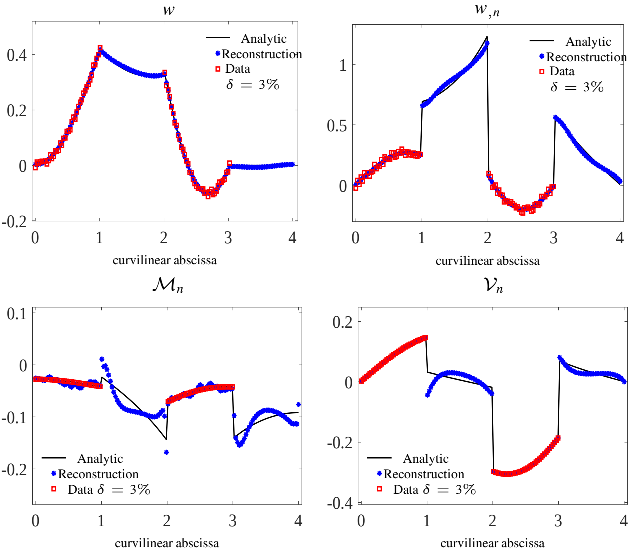


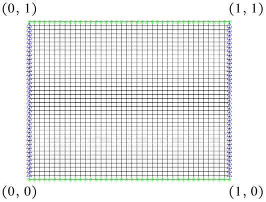
$\Omega = 40 \times 40$

△ Γ_i : unknowns

$*$ Γ_d : data

Reconstructions on the boundary of a square domain (noisy data located on two opposite sides)



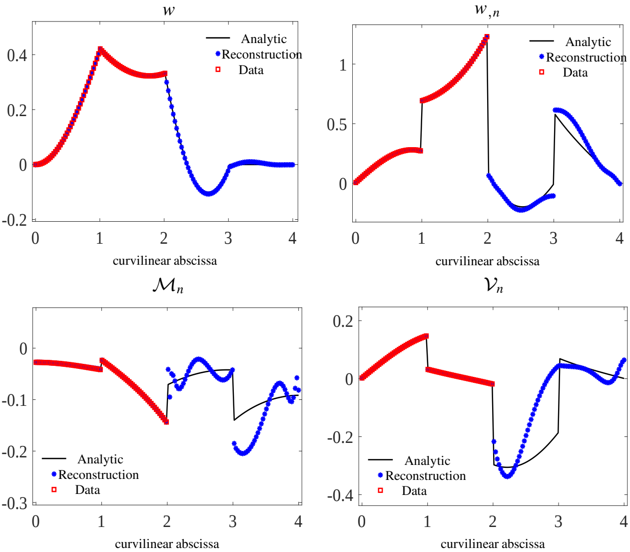


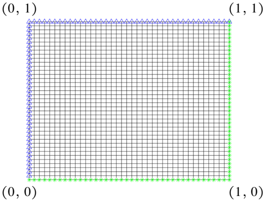
$\Omega = 40 \times 40$

$\triangle$ Γ_i : unknowns

$*$ Γ_d : data

Reconstructions on the boundary of a square domain (compatible data located on two adjacent sides)



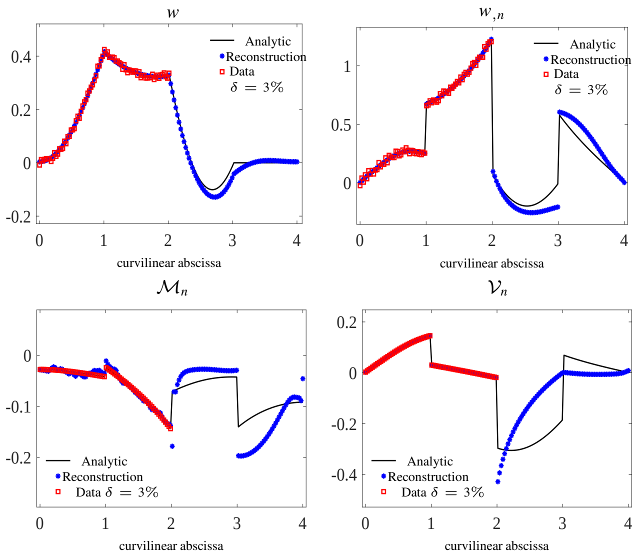


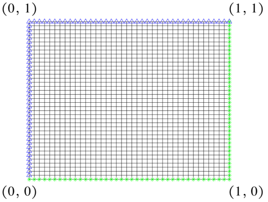
$\Omega = 40 \times 40$

$\triangle$ Γ_i : unknowns

$*$ Γ_d : data

Reconstructions on the boundary of a square domain (noisy data located on two adjacent sides)





$\Omega = 40 \times 40$

$\triangle$ Γ_i : unknowns

$*$ Γ_d : data

Perspectives

- From a numerical point of view
  - Use of other types of plate finite elements that ensure $\mathcal{C}^1$ continuity (idea : adding the cross derivative as nodal parameter (Bogner or Bazeley elements))
  - Use of other numerical methods (such as the method of fundamental solutions) for the Cauchy problem in thin plate theory
  - ...
- Perspectives related to mechanics
  - Data completion problems in thin plate theory (identification of fields and/or boundary conditions, identification of defects, etc...)
  - Use of experimental and real data
  - ...

Thank you

Feel free to add me to your contacts!

profile/Mohamed-Boukraa

in/mohamed-aziz-boukraa-231438b6/

mohamedazizboukraa@gmail.com

Inria researcher (Postdoc position)
IDEFIX group, Inria, ENSTA Paris, Institut Polytechnique de Paris