Use of polar invariants for uncertainty quantification and optimization of the aeroelastic response of composite structures

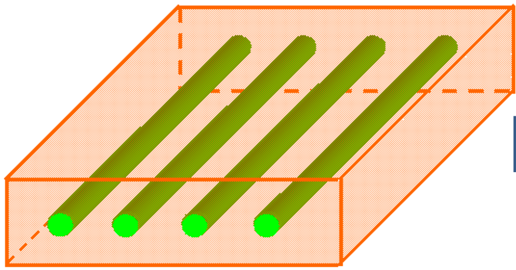
Angela Vincenti

Institut d'Alembert, Sorbonne Université

GDR-GDM, Jussieu, 23 Novembre 2023

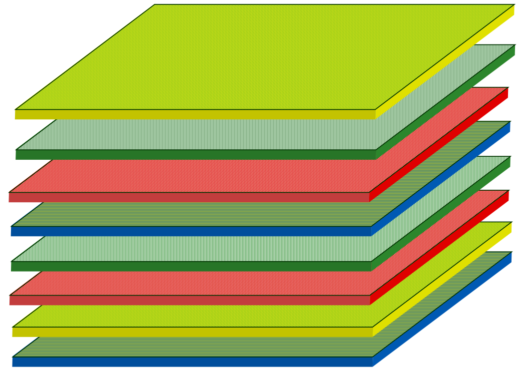
Optimization of composite materials and structures

MICROSCALE



Orthotropic lamina
(fibers + matrix)

MESOSCALE



Laminated plate

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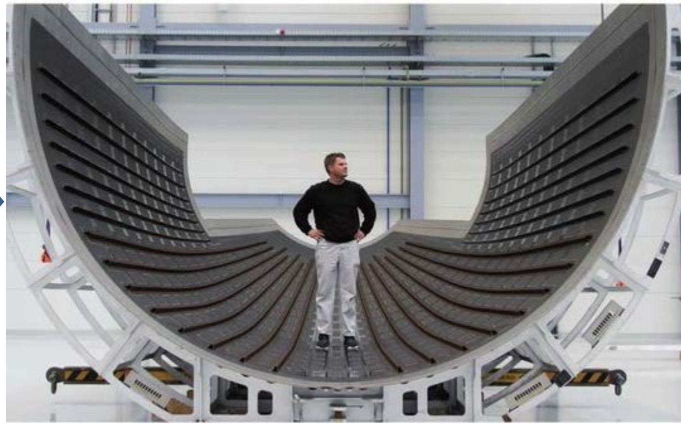

Complex composite structures
Airbus 350 [Hilken, SAE Int J Aerosp 2017]

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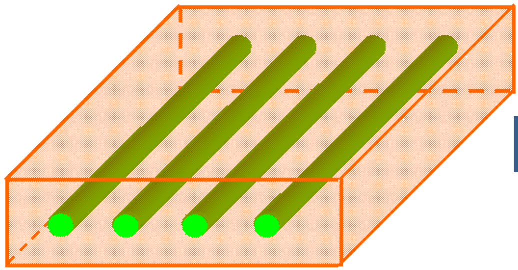

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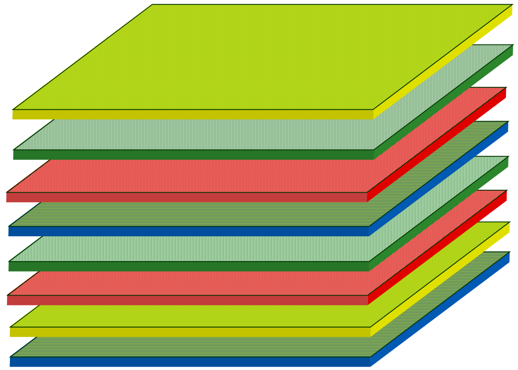

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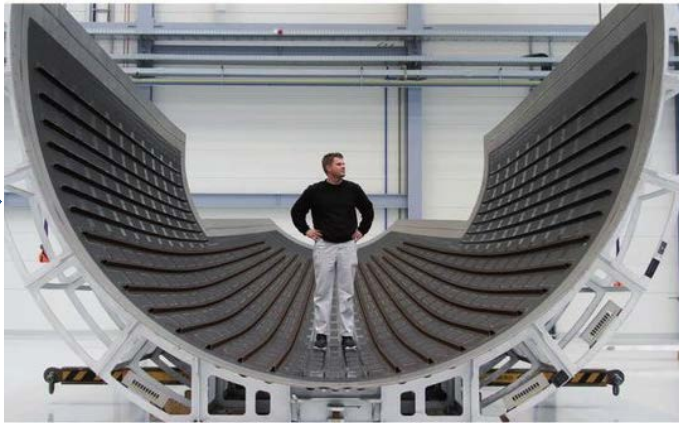

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- Large strength- and stiffness-to-weight ratio
- Heterogeneity generally hinders fracture propagation
- **Multiple combinations of reinforcement architecture : free design scheme**

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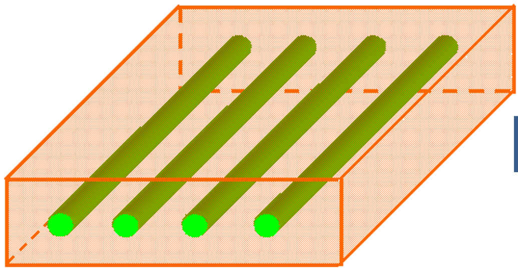

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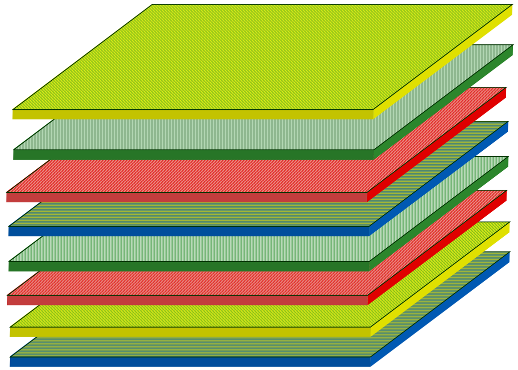

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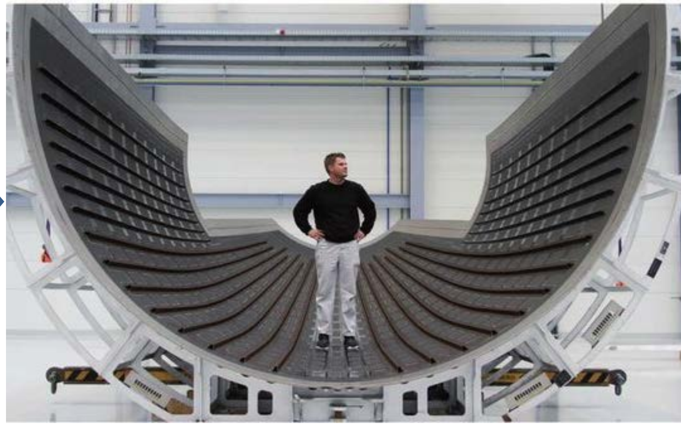

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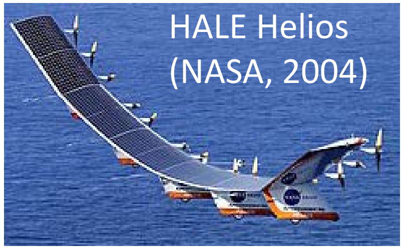
Applications to lightweight structures



Boeing 787 Dreamliner

Substance
class C catamaran
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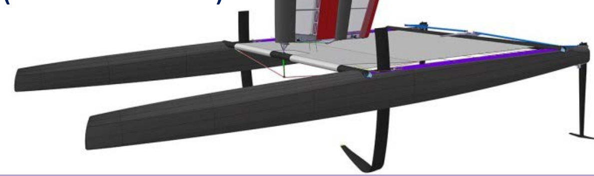

Optimization of composite materials and structures

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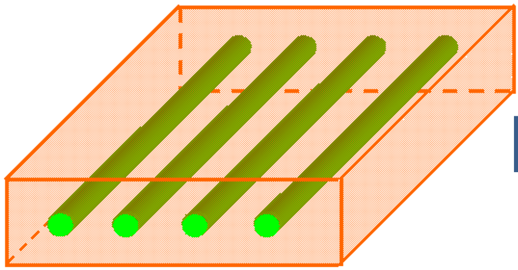

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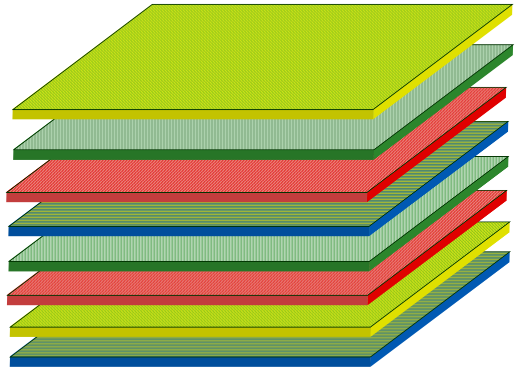

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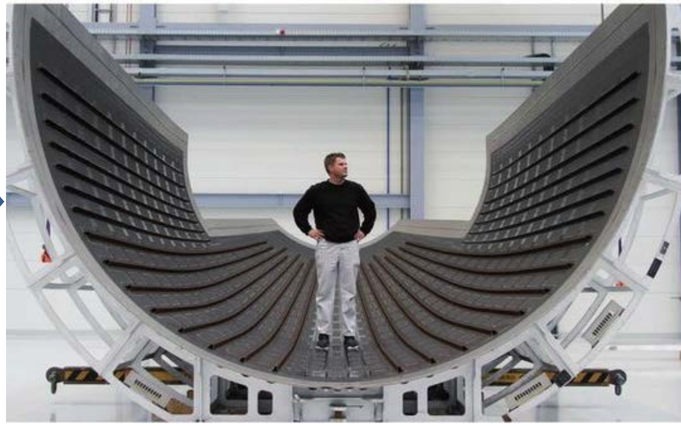

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$E_1, E_2, G_{12}, \nu_{12}$

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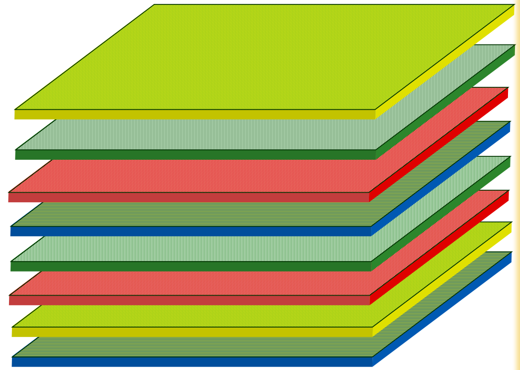

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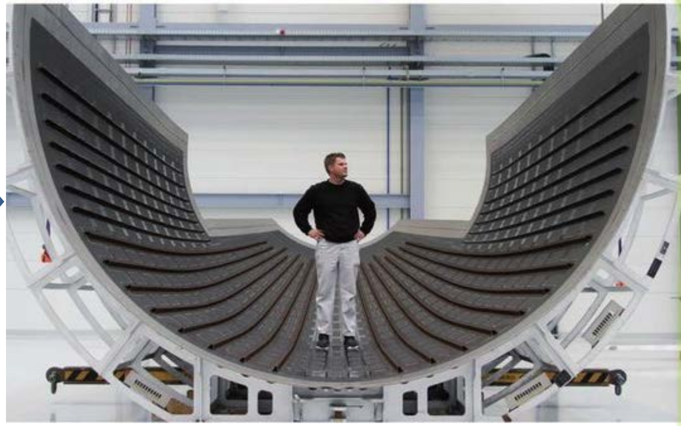

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Optimization of composite materials and structures

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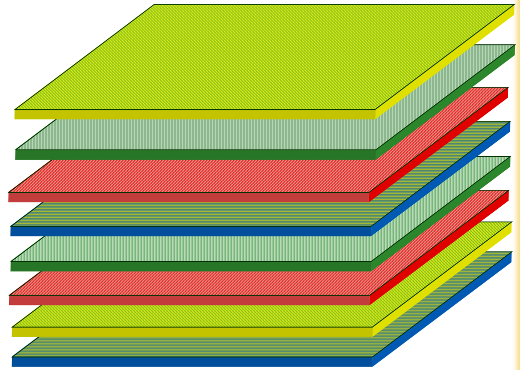

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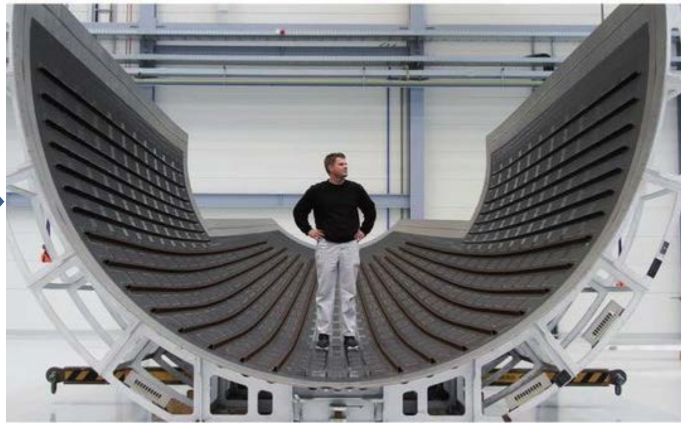

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Scientific challenges :

- Assembly of **layers oriented at angles δ_k** (k = 1, ..., N) **: high number of variables**
- **Complex behaviour : anisotropy**, different in-plane and out-of-plane response, **couplings** (elastic, thermo-elastic, etc.)
- Highly **non-linear and non-convex relations** in terms of orientation angles δ_k

Optimization of composite materials and structures

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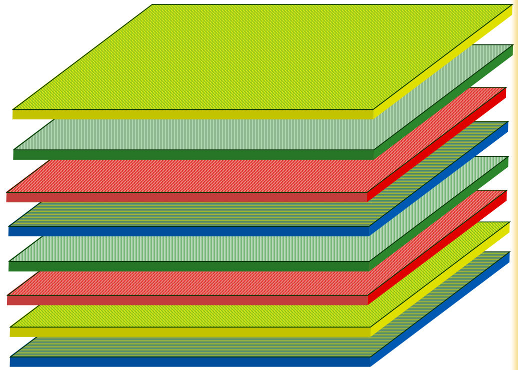

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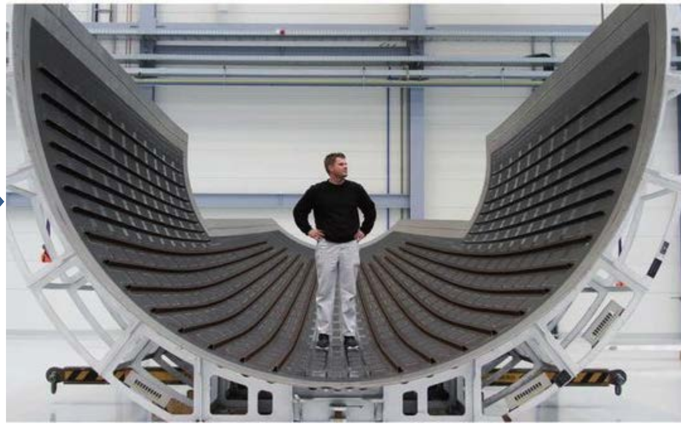

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Optimization of composite materials and structures

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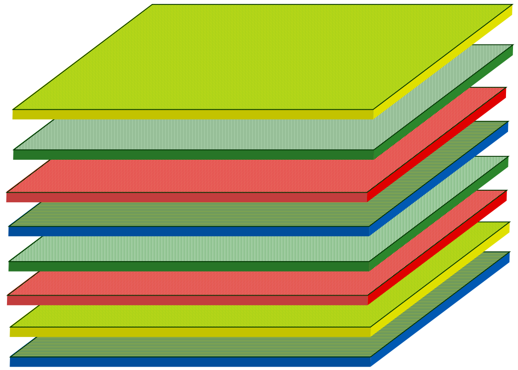

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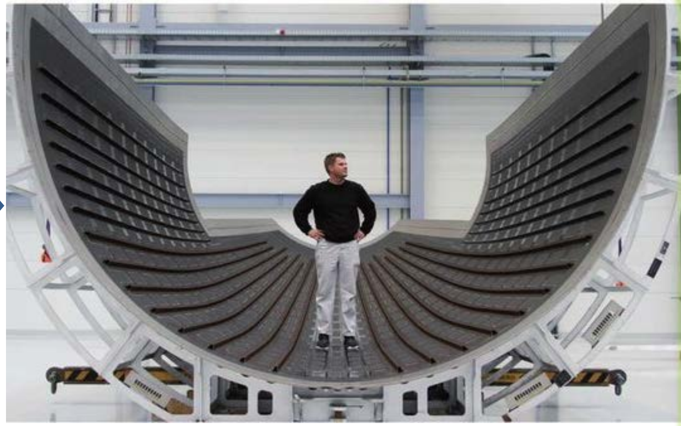

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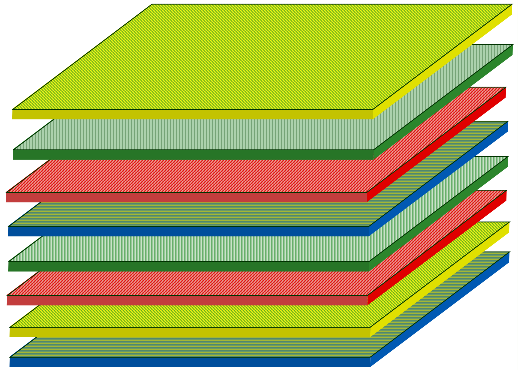

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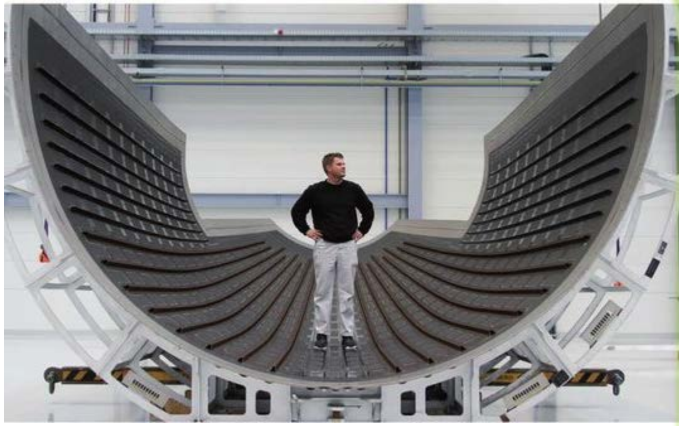

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- The **number of constitutive modules** can itself be **a design variable**
- **Variable stiffness :** change of elastic properties throughout the structure
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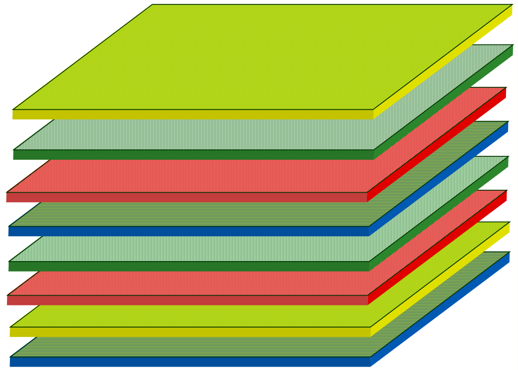

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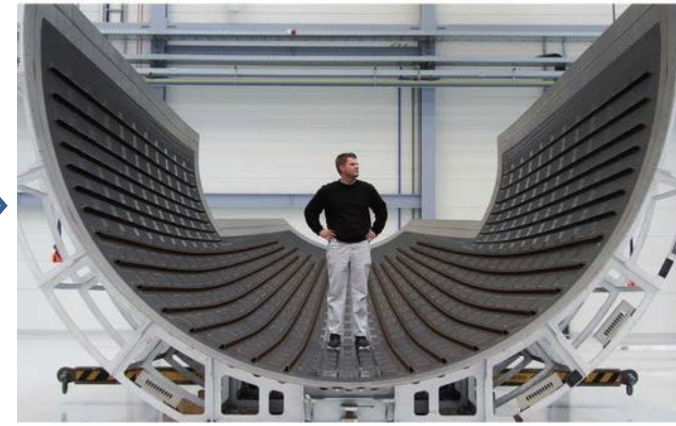

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i. A unified formulation for the optimal design of composite laminated plates

ii. Extension to multi-physical couplings (thermo-elastic, piezo-elastic, ...)

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ii. Aeroelastic optimization of composite plate wings

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Common thread : the use of the polar method in 2D elasticity

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The polar representation of plane tensors

Second-order symmetric tensors : strain $\boldsymbol{\varepsilon}$, stress $\boldsymbol{\sigma}$, forces and moments $\mathbf{N}$ and $\mathbf{M}$, etc.

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\begin{aligned}
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Polar **angles** $\boldsymbol{\Phi_0}$ and $\boldsymbol{\Phi_1}$

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$\boldsymbol{\Phi_0}-\boldsymbol{\Phi_1}$

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Invariants $\boldsymbol{T}$ and $\boldsymbol{R}$ → Spherical and deviatoric

Polar **angle** $\boldsymbol{\Phi}$ → First principal axis

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Invariants $\boldsymbol{T_0}$ and $\boldsymbol{T_1}$ → Isotropic components

$\boldsymbol{R_0}$ and $\boldsymbol{R_1}$, $\boldsymbol{\Phi_0} - \boldsymbol{\Phi_1}$ → Anisotropic components (moduli)

Polar **angles** $\boldsymbol{\Phi_0}$ and $\boldsymbol{\Phi_1}$ → Anisotropic phase angles

(Verchery, *EuroMech* 1979; Vannucci, *Meccanica* 2005)

Polar conditions for 2D elastic symmetries

Elastic symmetry	Polar condition	Remaining polar parameters	Principal axis
Orthotropy	$\Phi_0 - \Phi_1 = K\frac{\pi}{4},\ K = 0, 1$	$T_0,\ T_1,\ R_K = (-1)^K R_0,\ R_1$	Φ_1
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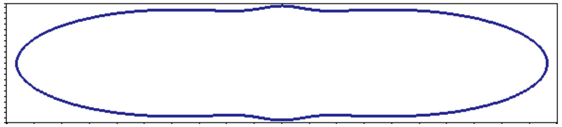
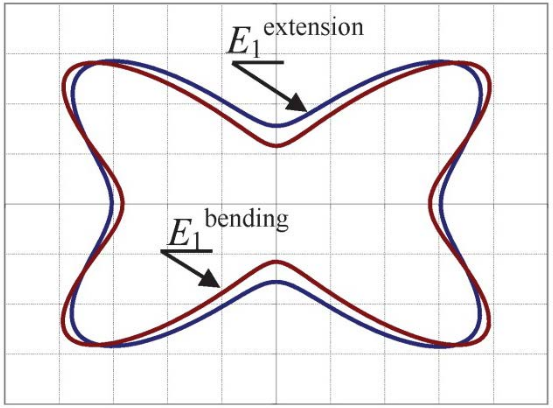
Orthotropy





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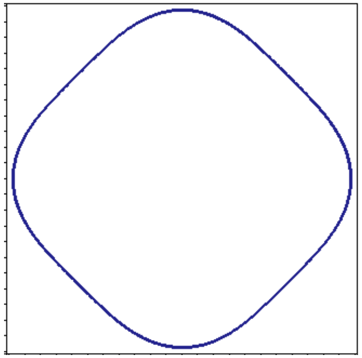
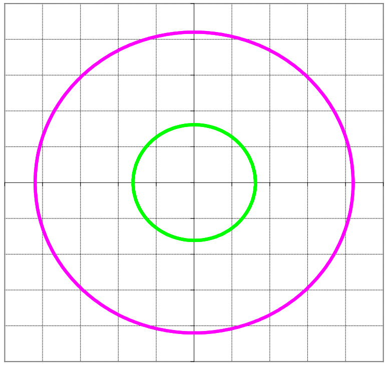


Isotropy



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Main advantages of the polar representation :

- Polar invariants are **intrinsic tensor's descriptors**
- **Simple transformation** of polar angles
- **Physical interpretation** of polar components
- **Explicit conditions for elastic symmetries** on polar invariants
- **Reduction of the number of polars parameters** for symmetric tensors
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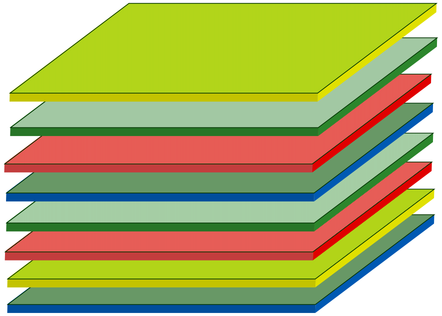
↳ **Polar representation of the behaviour of composite laminated plates**

(Verchery, *EuroMech* 1979; Vannucci, *Meccanica* 2005)

Polar representation of the Classical Laminated Plate Theory



$$\left\{ \begin{array}{c} \mathbf{N} \\ \mathbf{M} \end{array} \right\} = \left[\begin{array}{cc} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{array} \right] \left\{ \begin{array}{c} \varepsilon \\ \chi \end{array} \right\}$$

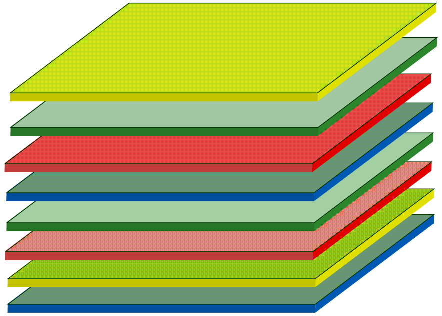
The **CLPT** expresses the **homogenised behaviour of a laminated plate** (n layers, stacking sequence $[\delta_k]$; general case of a **hybrid laminate**) :

$$\mathbf{A} = \sum_{k=1}^{n} \mathbf{Q}_k(\delta_k)(z_k - z_{k-1})$$

$$\mathbf{B} = \frac{1}{2}\sum_{k=1}^{n} \mathbf{Q}_k(\delta_k)(z_k^2 - z_{k-1}^2)$$

$$\mathbf{D} = \frac{1}{3}\sum_{k=1}^{n} \mathbf{Q}_k(\delta_k)(z_k^3 - z_{k-1}^3)$$

Polar representation of the Classical Laminated Plate Theory



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The **polar expressions of the Classical Laminated Plate Theory** (n layers, stacking sequence $[\delta_k]$; general case of a **hybrid laminate**) :

$$T_0^A, T_0^B, T_0^D = \frac{1}{m}\sum_{k=1}^{n} T_{0k}\left(z_k^m - z_{k-1}^m\right)$$

$$T_1^A, T_1^B, T_1^D = \frac{1}{m}\sum_{k=1}^{n} T_{1k}\left(z_k^m - z_{k-1}^m\right)$$

$$R_0^A e^{4i\Phi_0^A}, R_0^B e^{4i\Phi_0^B}, R_0^D e^{4i\Phi_0^D} = \frac{1}{m}\sum_{k=1}^{n} R_{0k} e^{4i(\Phi_{0k}+\delta_k)}\left(z_k^m - z_{k-1}^m\right)$$

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with m = 1, 2 , 3 for in-plane (A), coupling (B) and bending (D)

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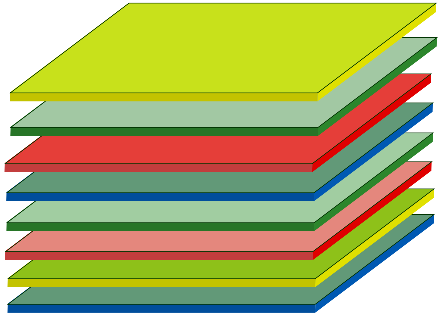

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \chi \end{Bmatrix}$$

The **polar expressions of the Classical Laminated Plate Theory** (**common case of identical layers**, n layers, stacking sequence $[\delta_k]$) :

Isotropic polar components :

$$T_0^A = hT_0\ ,\quad T_0^B = 0\ ,\quad T_0^D = \frac{h^3}{12}T_0$$

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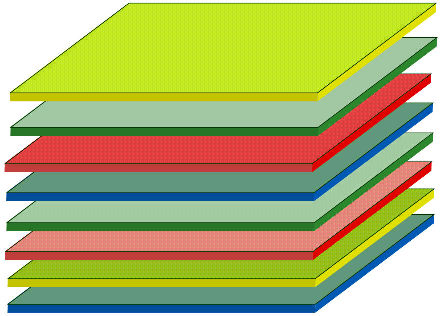

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lamination parameters

(Tsai & Pagano, 1968 ; Tsai, 1980)

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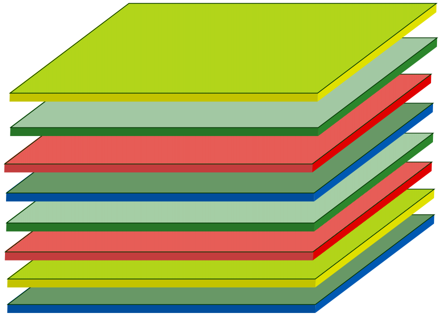

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Remark : *lamination parameters* are issued from the CLPT (stiffness response, laminates made of identical layers) whilst the **polar representation is more general**

Polar invariants and optimal design of composite laminates

1. Optimization of composite laminated plates

i. A unified formulation for the optimization of composite laminated plates

(Vincenti et al, *JOGO* 2010 ; Vincenti et al, *MAMS* 2012)

ii. Extension to multi-physical couplings (thermo-elastic, piezo-elastic, ...)

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"Class 0" formulation **: design** of laminates w.r.t. **elastic symmetries**

"Class 1" formulation **: design of elastic symmetries with constraints** on mechanical properties

"Class 2" formulation **: optimization of laminates including conditions on elastic symmetries**

PhD thesis of R. Ahmadian

Collaboration with P. Vannucci (Université de Versailles)

"Class 0" : elastic symmetries of laminates

Context :

- **Empirical rules for elastic symmetries** (symmetric stacking sequences, angle-ply, cross-ply) : **restriction of the design space** and **sub-optimal solutions**
  (Liu et al, *CMAME* 2000 ; Soremekun et al, *Comput & Struct* 2001 ; Herencia et al, *AIAA J* 2007 ; ...)
- **Proof of existence of an infinite number of solutions for uncoupling and quasi-homogeneity of laminates** (Vannucci & Verchery, *IJSS* 2001; Vincenti et al, *Compos. Part A* 2001)
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The design of elastic symmetries as an identification problem :

(P) Find $\boldsymbol{\delta} \in \boldsymbol{\Delta}_{\mathrm{adm}}$ such that : $\min_{\boldsymbol{\delta}} I\left(\mathbf{P}(\boldsymbol{\delta})\right)$

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✓ **All combinations of elastic symmetries :** in-plane, bending, uncoupling, q-homogeneity

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Examples of "Class 0" problem : unconstrained optimization

1. Uncoupled orthotropic laminate **with coincident axes** in membrane and bending **plus thermoelastic symmetries** (direction of zero expansion):

(P) Find $\boldsymbol{\delta} \in \boldsymbol{\Delta}_{\text{adm}}$ such that : $\min\limits_{\boldsymbol{\delta}} I\left(\mathbf{P}(\boldsymbol{\delta})\right)$

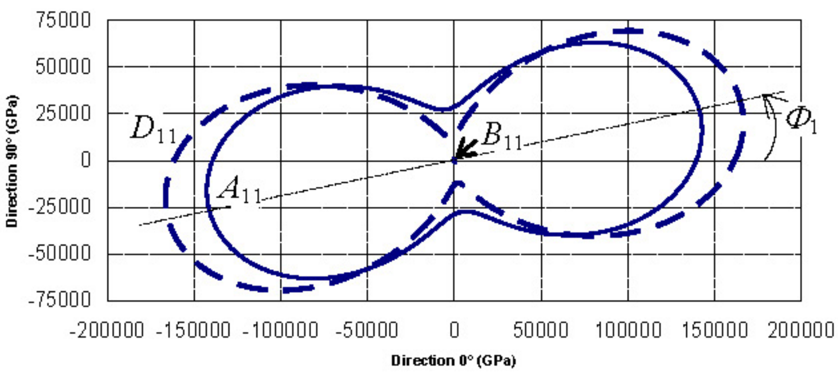
where :

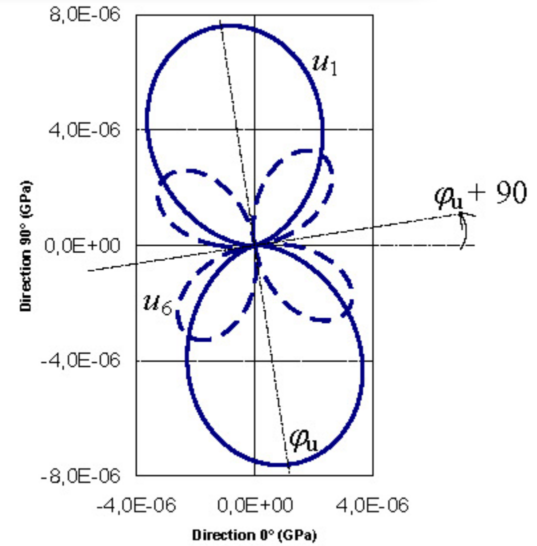
- $I\left(\mathbf{P}(\boldsymbol{\delta})\right)$ expresses the combination of objectives on elastic and thermo-elastic symmetries
- $\boldsymbol{\Delta}_{\text{adm}}$: continuous orientations, from -90° to 90°

Study case : 12-ply laminate

Solution :

[0/23.0/11.3/11.2/...
-33.7/ 85.8/30.9/...
-13.4/ -0.1/22.1/...
21.6/ -0.9]





Examples of "Class 0" problem : unconstrained optimization

2. Uncoupled quasi-homogeneous square-symmetric laminate **plus thermoelastic symmetries** (isotropic thermal expansion)**:**

(P) Find $\boldsymbol{\delta} \in \boldsymbol{\Delta}_{\mathrm{adm}}$ such that : $\min_{\boldsymbol{\delta}} I\left(\mathbf{P}(\boldsymbol{\delta})\right)$

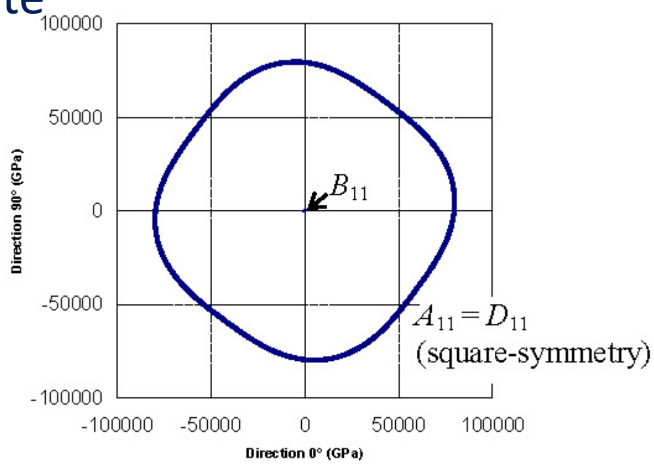
where :

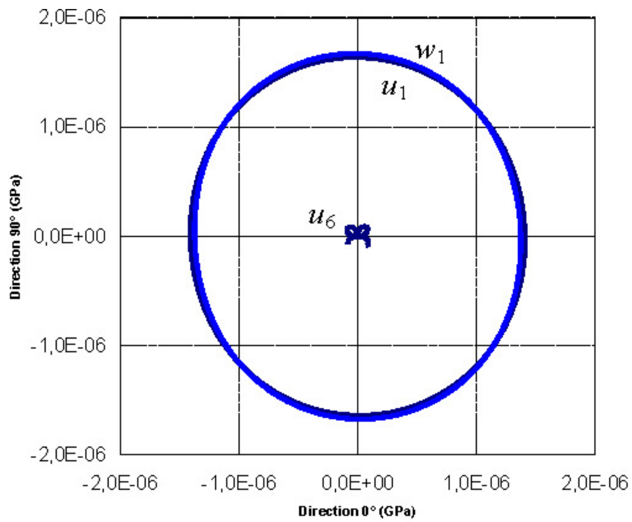
- $I\left(\mathbf{P}(\boldsymbol{\delta})\right)$ expresses the combination of objectives on elastic and thermo-elastic symmetries
- $\boldsymbol{\Delta}_{\mathrm{adm}}$: discrete set of orientations, from -90° to 90°, step 5°

Study case : 24-ply laminate

Solution :

[0/25/-45/-85/-85/75/...
-30/40/45/90/-15/-75/...
20/-45/0/30/10/-70/...
75/-20/-85/-5/-65/50]





"Class 2" : optimization of laminates with elastic symmetries

Optimization of laminates with elastic symmetries :

(P) Find $(\boldsymbol{\delta}, \mathbf{x})$ such that :

$$\min_{\boldsymbol{\delta},\mathbf{x}} f\left(\boldsymbol{\delta}, \mathbf{x}\right)$$

$$\text{subject to :} \quad \begin{cases} g_i\left(\boldsymbol{\delta}, \mathbf{x}\right) \leq 0 \quad i = 1, ..., r \ , \\ h_j\left(\boldsymbol{\delta}, \mathbf{x}\right) = 0 \quad j = 1, ..., m \ , \\ I\left(\mathbf{P}(\boldsymbol{\delta})\right) < \epsilon \\ \mathbf{x}_L \leq \mathbf{x} \leq \mathbf{x}_U \ , \\ \boldsymbol{\delta} \in \boldsymbol{\Delta}_{\text{adm}} \end{cases}$$

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(Vincenti et al, *MAMS* 2012)

- ✓ **Elastic symmetries among design constraints**
- ✓ Variables can be layers' angles $\boldsymbol{\delta}$, but also other parameters $\mathbf{x}$ (e.g. geometry)
- ✓ As for"Class 0/1" problems, functions are **highly non linear and non convex**
- ✓ **Optimization of simple laminated structures**

Example of "Class 2" problem : constrained optimization

Max buckling load with **uncoupling, in-plane and bending orthotropy with coincident axis, constraints on Young's moduli:**

(P) Find ($\boldsymbol{\delta}$) such that :

$$\max_{\boldsymbol{\delta}} \left(\min_{m,n} \lambda_{\text{crit}} \left(\boldsymbol{\delta} \right) \right)$$

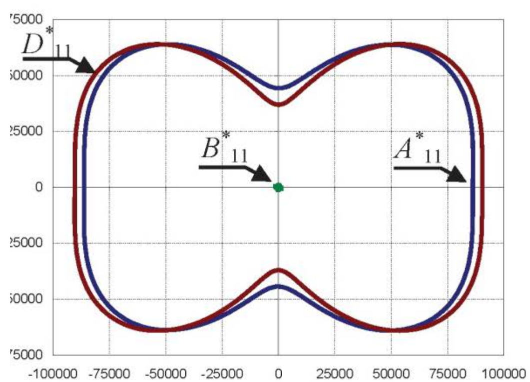
$$\text{subject to :} \quad \begin{cases} I\left(\mathbf{P}(\boldsymbol{\delta})\right) < \epsilon \\ {E_x}^A\left(\boldsymbol{\delta}\right) \geq \ 60\text{GPa} \ , \\ {E_y}^A\left(\boldsymbol{\delta}\right) \geq \ 30\text{GPa} \ , \\ \boldsymbol{\delta} \in \boldsymbol{\Delta} = \{-89, -88, ..., 90\} \end{cases}$$

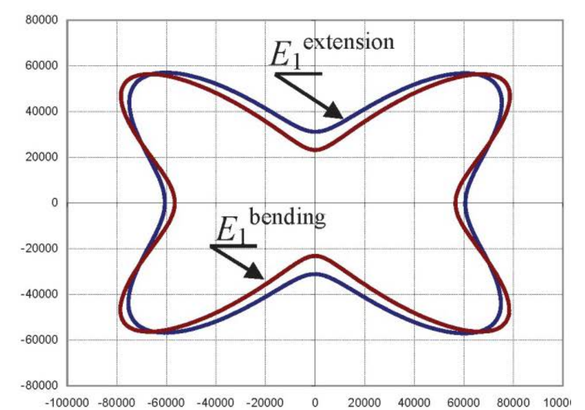
(Vincenti et al: *JOGO* 2010 ; *MAMS* 2012)

Study case : 16-ply simply-supported rectangular plate under bi-axial compression

Solution :

[-24/39/-47/37/32/-47/-6/-47/ /55/59/18/-38/-38/19/-40/42]





Optimal design of laminates using the polar formalism

1. Optimization of composite laminated plates (Vincenti & Vannucci, *ECCM* 2006 ; Vannucci & Vincenti, *Compos Struct* 2007 ; Vincenti et al, *JOGO* 2010 ; Vincenti et al., *MAMS* 2012)

Final remarks :

- ✓ **Design of elastic symmetries** formulated as an **unconstrained optimization** problem **or as a constraint** of a structural optimization problem
- ✓ Control of **multi-physical couplings** (thermo-, hygro- or piezo-elastic stability) as an **unconstrained optimization** problem

→ **towards global optimization of laminates** (cfr [O'Donnell & Weaver, *AIAA J* 2020])

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How to tackle optimization of complex composite structures ?
Two-level approach based on the polar formalism

Two-level approach : separating the macro- and meso-scale

Step 1 (macro-) : structural optimization

$$\min_{\mathbf{A},\mathbf{B},\mathbf{D},\mathbf{x}} f\left(\mathbf{A},\mathbf{B},\mathbf{D},\mathbf{x}\right)$$

subject to :

$$\begin{cases} g_i\left(\mathbf{A},\mathbf{B},\mathbf{D},\mathbf{x}\right) \leq 0 \quad (i=1,...,r) \\ h_j\left(\mathbf{A},\mathbf{B},\mathbf{D},\mathbf{x}\right) = 0 \quad (j=1,...,m) \\ \mathbf{x}_L \leq \mathbf{x} \leq \mathbf{x}_U \\ \mathbf{A},\mathbf{B},\mathbf{D} \in \mathcal{D}_{\mathrm{adm}} \end{cases}$$

Step 2 (meso-) : search of optimal stacks

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$\mathbf{A}_{\text{opt}}$
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Find the stacking sequence $\boldsymbol{\delta}$ such that :

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- Laminate's **homogenised elastic moduli : fixed number of variables** and problem **regularisation**
- Hypothesis on **elastic symmetries : reduced number** of elastic parameters
- **Representation** of $\mathbf{A}$ and $\mathbf{D}$?
  And of the admissible domain $\mathcal{D}_{\text{adm}}$?

$\mathbf{A}_{\text{opt}}$, $\mathbf{B}_{\text{opt}}$, $\mathbf{D}_{\text{opt}}$ →

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- Existing polar formulation for the optimal design of elastic symmetries
- **Extension of the polar formulation to the case of prescribed elastic moduli**

Two-level approach : separating the macro- and meso-scale

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$$\begin{cases} g_i(\mathbf{A},\mathbf{B},\mathbf{D},\mathbf{x}) \leq 0 \quad (i=1,\dots,r) \\ h_j(\mathbf{A},\mathbf{B},\mathbf{D},\mathbf{x}) = 0 \quad (j=1,\dots,m) \\ \mathbf{x}_L \leq \mathbf{x} \leq \mathbf{x}_U \\ \mathbf{A},\mathbf{B},\mathbf{D} \in \mathcal{D}_{\text{adm}} \end{cases}$$

- Laminate's **homogenised elastic moduli : fixed number of variables** and problem **regularisation**
- Hypothesis on **elastic symmetries : reduced number** of elastic parameters
- **Representation** of $\mathbf{A}$ and $\mathbf{D}$? And of the admissible domain $\mathcal{D}_{\text{adm}}$?

$\mathbf{A}_{\text{opt}}$
$\mathbf{B}_{\text{opt}}$
$\mathbf{D}_{\text{opt}}$

Step 2 (meso-) : search of optimal stacks

Find the stacking sequence $\boldsymbol{\delta}$ such that :

$$\mathbf{A}(\boldsymbol{\delta}) = \mathbf{A}_{\text{opt}}$$
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- Existing polar formulation for the optimal design of elastic symmetries
- **Extension of the polar formulation to the case of prescribed elastic moduli**

Use of the polar formalism

Polar representation of an orthotropic laminate

- **Uncoupled laminates** (**B** = **O**) **:** remaining tensors **A** and **D** (in-plane and bending)
- **Laminates made of identical layers :** isotropic components T_0 and T_1 are fixed
- **Orthotropy :** two polar invariants $R_K = (-1)^K R_0$ and R_1, plus one polar angle Φ_1

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Thermodynamic conditions :

$$T_0 - R_K > 0$$

$$T_1 \left(T_0 + R_K\right) - 2R_1^2 > 0$$

[Vincenti & Desmorat, *JElas* 2010]

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[Vannucci, *JElas* 2012]

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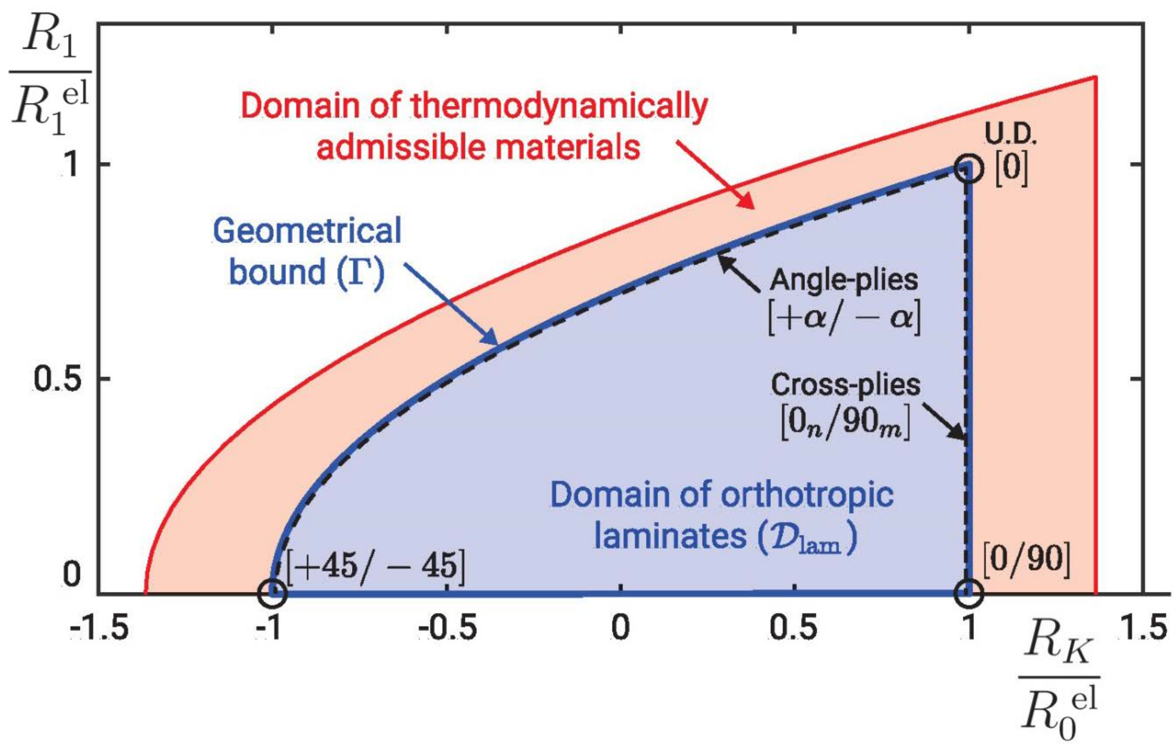

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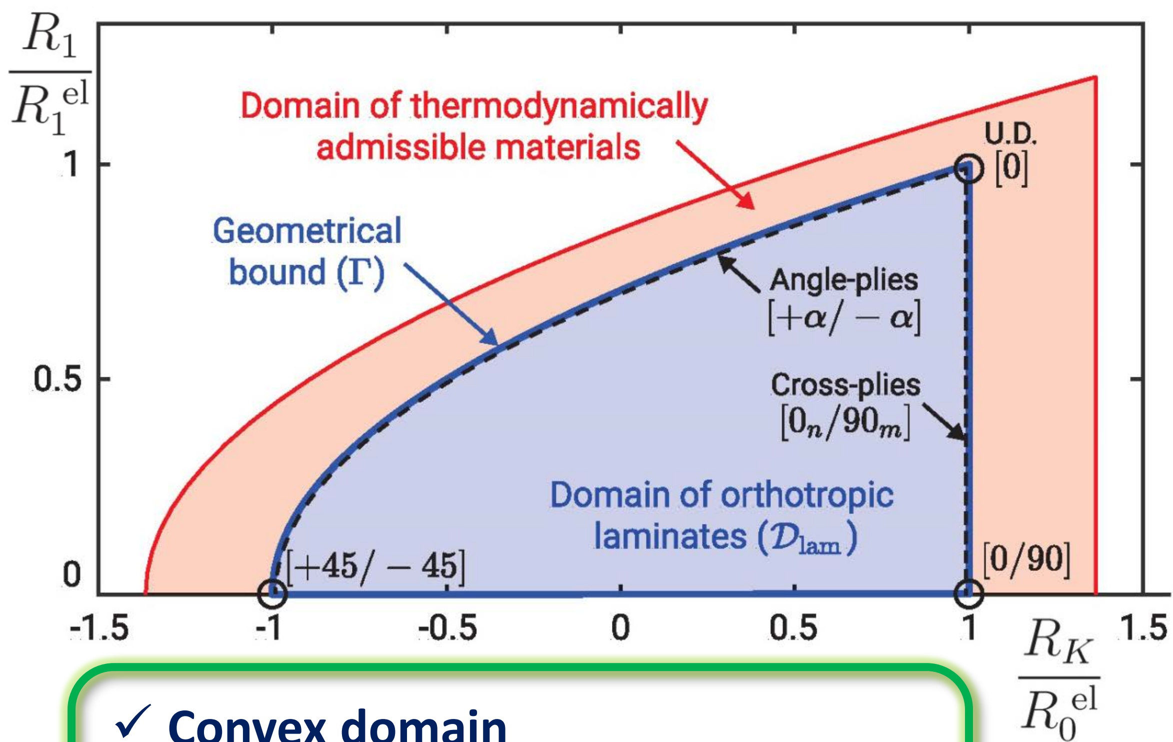

- ✓ **Convex domain**
- ✓ **All feasible orthotropic laminates**
- ✓ All elastic symmetries are included

Polar formulation of the two-level approach

Step 1 (macro-) : structural optimization

$$\min_{R_K, R_1, \Phi_1, \mathbf{x}} f\left(R_K, R_1, \Phi_1, \mathbf{x}\right)$$

subject to :

$$\begin{cases} g_i\left(R_K, R_1, \Phi_1, \mathbf{x}\right) \leq 0 \quad (i = 1, ..., r) \\ h_j\left(R_K, R_1, \Phi_1, \mathbf{x}\right) = 0 \quad (j = 1, ..., m \\ \mathbf{x}_L \leq \mathbf{x} \leq \mathbf{x}_U \\ R_K - R_0^{\mathrm{el}} \leq 0 \\ 2\left(\dfrac{R_1}{R_1^{\mathrm{el}}}\right)^2 - \dfrac{R_K}{R_0^{\mathrm{el}}} - 1 \leq 0 \end{cases}$$

(C. Julien, *PhD thesis* 2010 ; Jibawy et al, *IJSS* 2011 ; Montemurro et al, *JOTA* 2012 ; Montemurro, *PhD thesis* 2012)

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R_K^{opt} R_1^{opt} Φ_1^{opt}

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Solve the **identification problem :**

$$\min_{\boldsymbol{\delta}} F\left(\boldsymbol{\delta}\right)$$

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Optimization of composite structures via the polar formalism

2. Optimization of variable-stiffness composite structures using a two-level method based on the polar formalism

i. Compliance minimisation of tow-stereed composite structures

ii. Optimization of modular composite structures

Final remarks and open questions :

- **Formulation of optimization problems :** how to **take into account conditions depending on the stacking sequence**, such as strength, delamination, ...?
  How to express the correlation between **A** and **D** ?

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(Recent works : Montemurro et al, ENSAM Bordeaux ; Irisarri and Julien, ONERA)

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Influence of anisotropy on non-linearities and instabilities

Aeroelastic response of composite structures : optim & UQ

1. Optimization of composite laminated plates
2. Optimization of variable-stiffness composite structures
3. **Influence of anisotropy and uncertainties on non-linearities and instabilities**
   i. **Design of multistable composite shells**
      (Hamouche et al : *Meccanica* 2016, *PRSA* 2016 ; Brunetti et al : *IJSS* 2016, *Compos Struct* 2018)
      Collaboration with C. Maurini (d'Alembert) and S. Vidoli (Univ Roma La Sapienza)
      ANR project JC SLENDER 2014-18

   ii. **Aeroelastic optimization of composite plate wings**
   iii. **Influence of uncertainties on the aeroelastic response of composite plate wings**
      (Nitschke et al, *Compos Struct* 2019)
      PhD theses of C. Nitschke (2014-18) and M. Sharifi (2019-2023)
      Collaboration with J-C. Chassaing (d'Alembert)

Aeroelastic response of composite plate wings

- **Fluid-Structure Coupling : interaction** of **elastic**, **inertial** and **aerodynamic** forces
- **Coupled equations** of motion :

$$[\mathbf{M}]\ddot{w} + [\mathbf{C}]\dot{w} + [\mathbf{K}]w = F_{aero}(w, \dot{w}, \ddot{w})$$

- **Resolution by modal analysis** or time-integration

Crash of the Helios drone due to aeroelastic flutter in 2004

[NOLL et al : Investigation of the Helios prototype aircraft mishap. NASA Report, 9, 2004]

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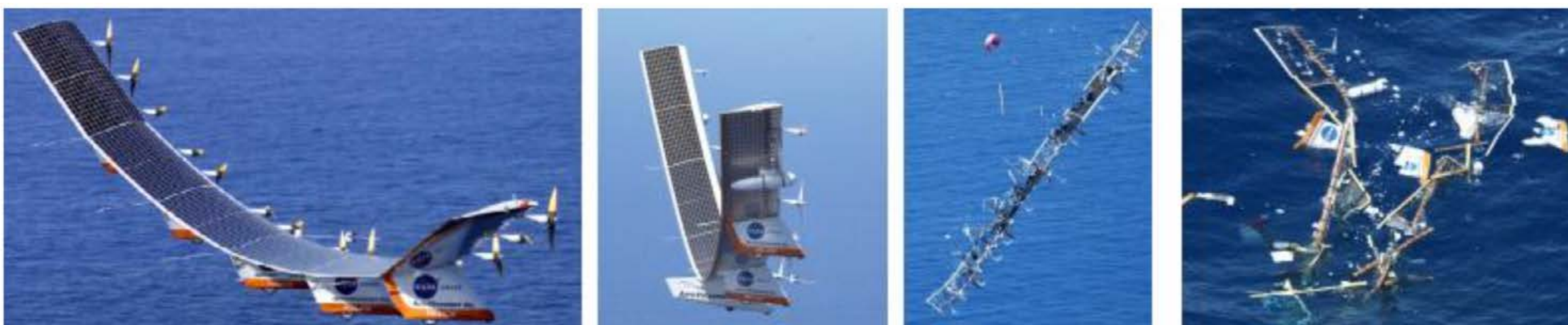

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- **Aeroelastic tailoring of composite wings : maximisation** of the **aeroelastic performances** (critical velocity, flutter margin) wrt **anisotropic elastic properties**
- **Limited to small number of plies** and/or **symmetric stacking sequences**

(Weisshar, *J Aircraft* 1981 ; Kameyama, *Comput & Struct* 2007 ; Thuwis et al, SMDO 2010 ; Stodieck et al, *AIAA J* 2015)

Grumman X-29

Aeroelastic optimization via the polar formalism

Two-level polar approach for aeroelastic analysis of composite plate wings

i. **Aeroelastic deterministic optimization**

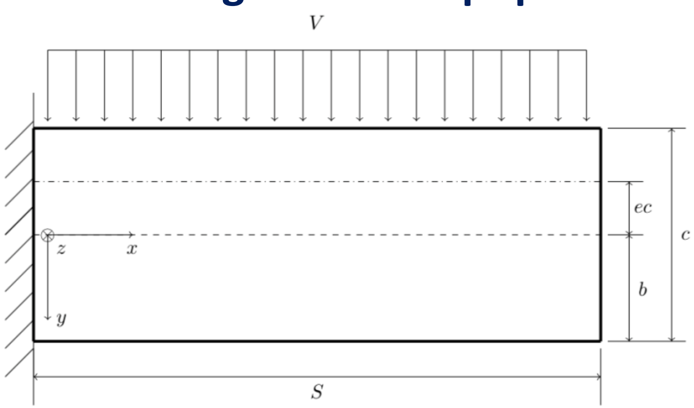
ii. **Influence of uncertainties** on the aeroelastic response

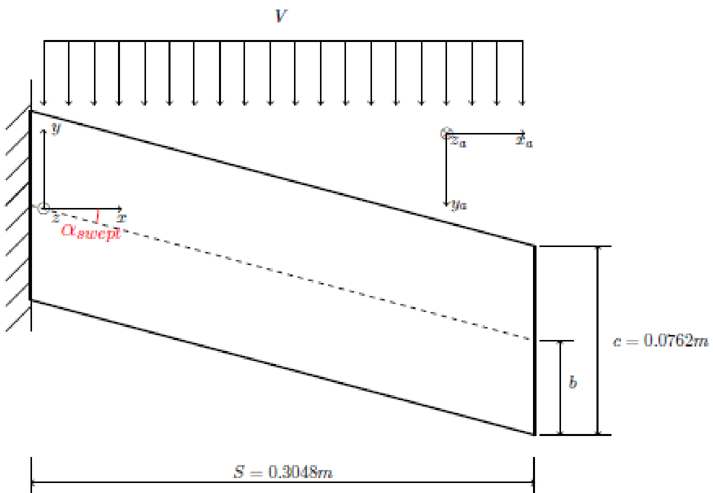
iii. **Aeroelastic optimization** in a **stochastic framework** (robust, RBDO)

Cases of study

- ✓ **Plate wing** made of a **16-ply orthotropic laminate : polar formalism** (R_K , R_1 and Φ_1)
- ✓ **Structural model :** FE orthotropic plate approximation (*Fenics*)
- ✓ **Aerodynamic model :** Doublet Lattice Method (Albano & Rodden, *AIAA J* 1969)
- ✓ **Aeroelastic solver : modal analysis** via the p-k method (Hassig, *J of Aircraft* 1971)

Models of straight and swept plate wings





Aeroelastic analysis of a plate composite wing

By coupling the FE model of the plate and the DLM approximation :

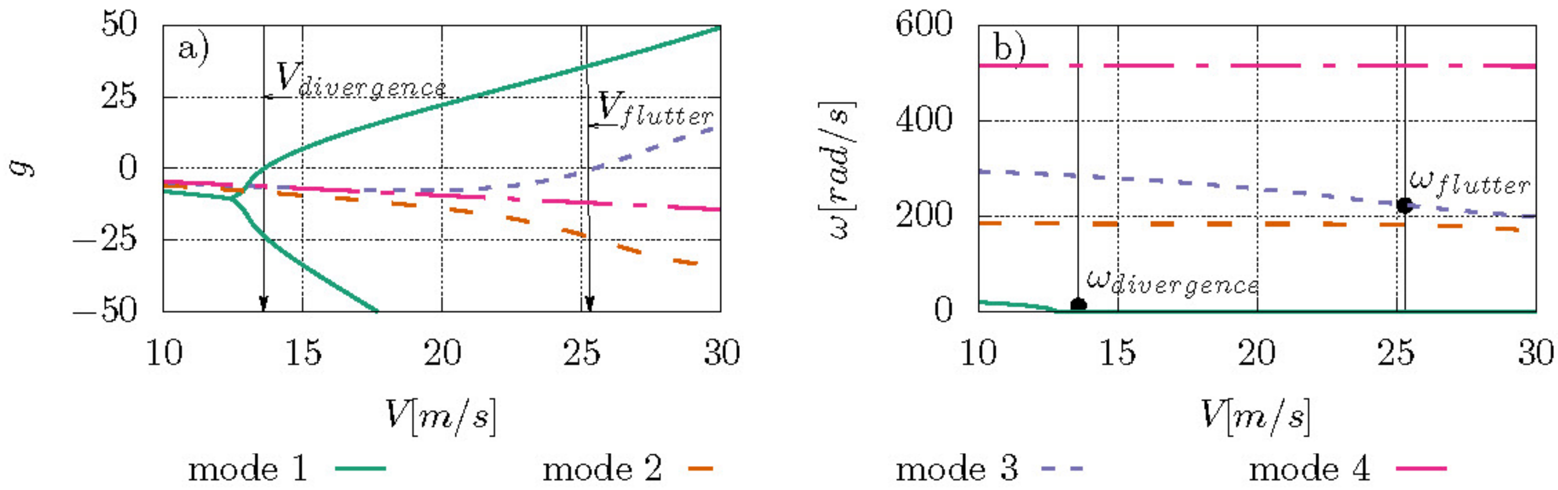
$$\hat{\mathbf{M}}\ddot{\mathbf{q}} - \omega \mathbf{A}^{I}_{aero}\dot{\mathbf{q}} + (\hat{\mathbf{K}} - \omega^2 \mathbf{A}^{R}_{aero})\mathbf{q} = 0$$

which can be transformed into a **1st-order eigenvalue problem :**

$$\begin{bmatrix} \mathbf{I} & 0 \\ 0 & \hat{\mathbf{M}} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}} \\ \ddot{\mathbf{q}} \end{Bmatrix} - \begin{bmatrix} 0 & \mathbf{I} \\ -(\hat{\mathbf{K}} - \omega^2 \mathbf{A}^{R}_{aero}) & \omega \mathbf{A}^{I}_{aero} \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

and the solved by the use of the **p-k method**.

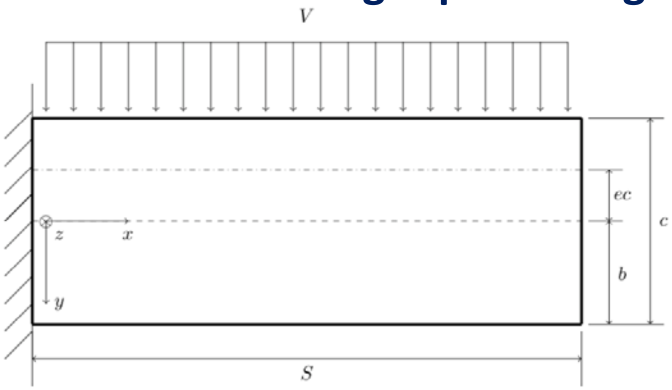
Example : stacking sequence $[-45_2/0]_{sym}$ (Hollowell et al., *J Aircraft*, 1984)



Aeroelastic optimization of a straight orthotropic wing

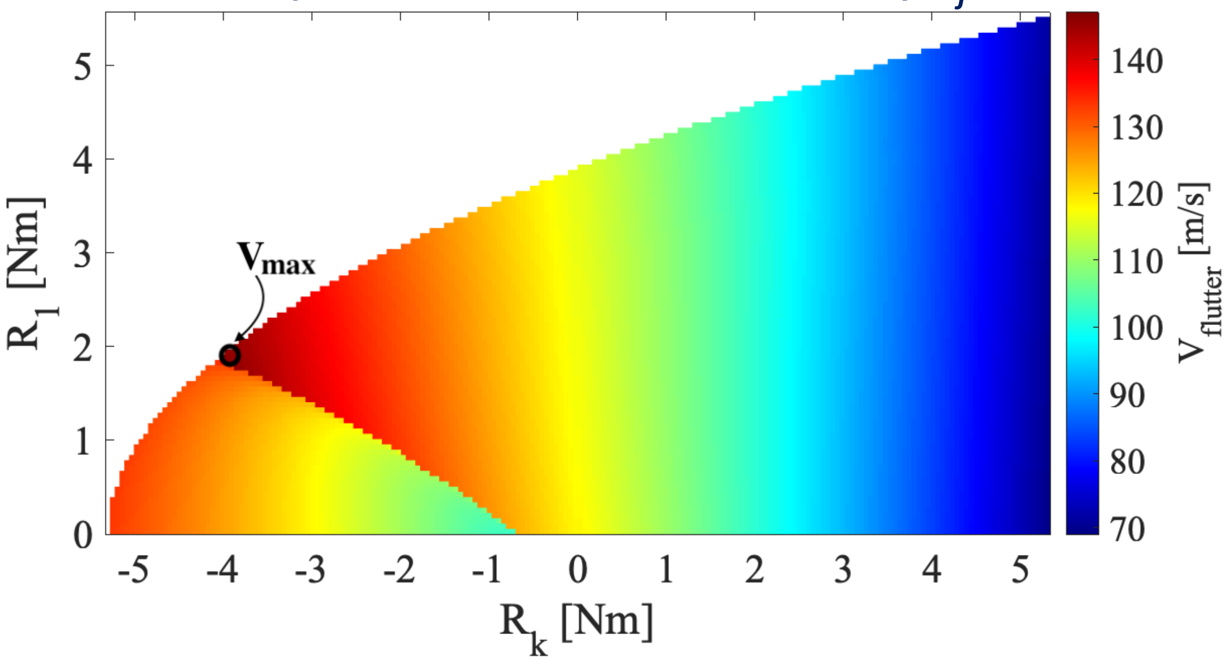
- ✓ **Case of a straight wing** made of an **uncoupled orthotropic 16-ply laminate : polar domain** (R_K - R_1) for the **bending tensor D**
- ✓ **Case Φ_1= 0°:** orthotropic axis aligned with the wing axis

Model of the straight plate wing



Maximum flutter velocity
V_{max} = 147 m/s

Response surface of flutter velocity V_f



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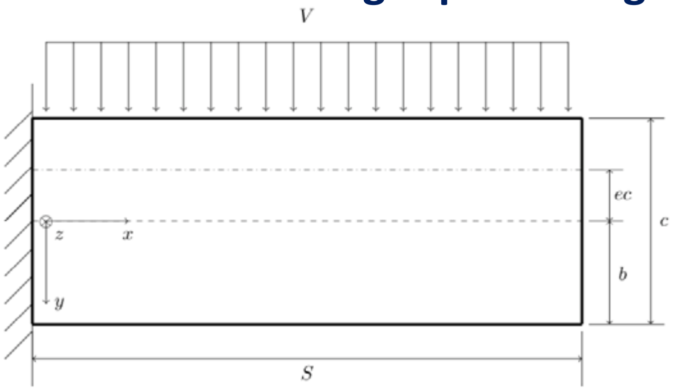

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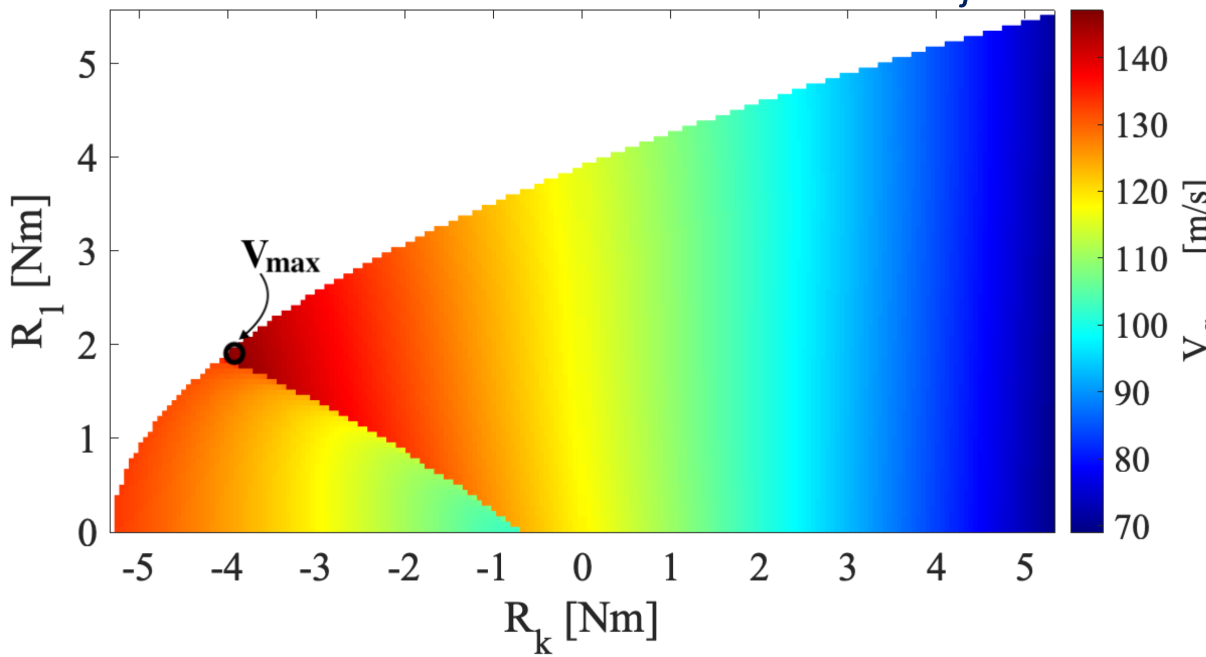

- ✓ **Maximum flutter velocity** for an **angle-ply orthotropic laminate** with $\alpha \approx 35°$
- ✓ **Improvement of V_f :** appr. 25% wrt isotropic material and 50% wrt cross-ply
- ✓ **Optimization wrt angle Φ_1,** cases of **swept and tapered plate wings** and **anisotropy**
- ✓ **Discontinuity** of critical flutter velocity due to **mode-switch**

Influence of uncertainties on the aeroelastic response

- **Composite laminated** wings : **multiple sources of errors** (angles, thicknesses)
- Optimal configurations lay close to **discontinuities** : strongly affected by uncertainties

Uncertainty quantification of flutter velocity wrt lamination errors

(Nitschke et al, *Compos Struct* 2019)

✓ Use of the **polar formalism** to **reduce the dimension of the uncertain problem**

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$$\begin{Bmatrix} \mathrm{N} \\ \mathrm{M} \end{Bmatrix} = \begin{bmatrix} \mathrm{A} & \mathrm{B} \\ \mathrm{B} & \mathrm{D} \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \chi \end{Bmatrix}$$

Modified bending tensor :

$$\tilde{\mathrm{D}} = \mathrm{D} - \mathrm{B}\mathrm{A}^{-1}\mathrm{B}$$

✓ Analysis of **general uncertain stacking sequences** (**unsymmetric** and **coupled**)

✓ Only **6 uncertain polar parameters** represent **errors on angles and thicknesses** (instead of 2x16 laminate's uncertain constitutive parameters)

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✓ Polar parameters are **correlated stochastic variables :** approximation of the uncertain function by a **polynomial chaos method** and **machine-learning techniques** for clustering configurations around the **discontinuity**

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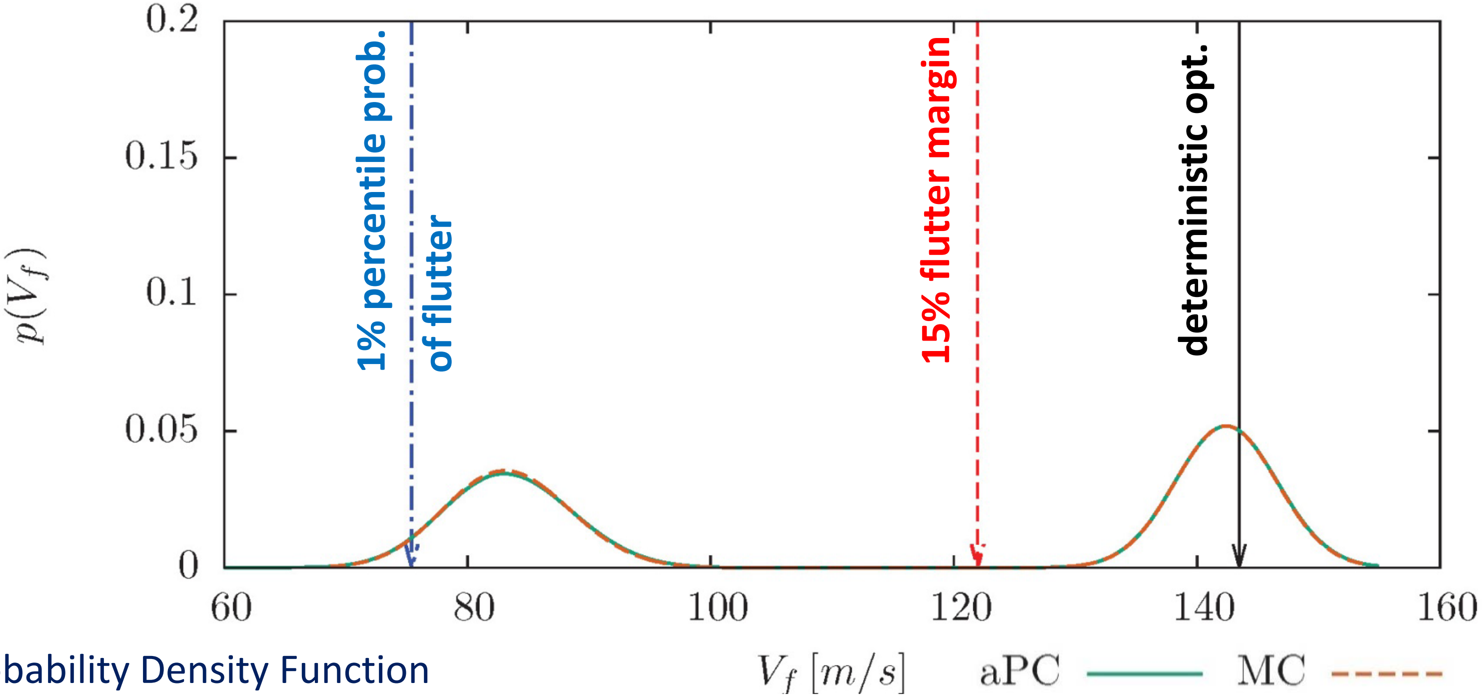
✓ **PDF(*) function of V_f for the optimal deterministic configuration**
Errors on angles (σ_α = 1°) **and thicknesses** (σ_t = 0.005 mm)



(*) PDF = Probability Density Function

Conclusions

Two-level polar approach to aeroelastic analysis of composite plate wings

- ✓ **Aeroelastic deterministic optimization :**
  - o Case of constant stiffness (straight, swept and tapered plate wings)
  - o Extension to the case of variable stiffness laminates (tow-steered)
- ✓ **Uncertainty quantification of the aeroelastic response :**
  - o **general** coupled and unsymmetric **configurations**
  - o influence of **errors in orientation angles and thicknesses** (similar approaches based on lamination parameters limited to symmetric distributions of errors on angles and do not consider errors on thicknesses)
- ✓ **Aeroelastic optimization** in a **stochastic framework** :
  - o **Formulation of the optimization problem :** choice of aeroelastic functions
  - o **Direct vs two-level approach :** how to take into account uncertainties at ply level ?
  - o **Effect of lamination errors on polar components :** how to describe the correlation ? How to take into account the non-univocal relation of angles and elastic moduli ?

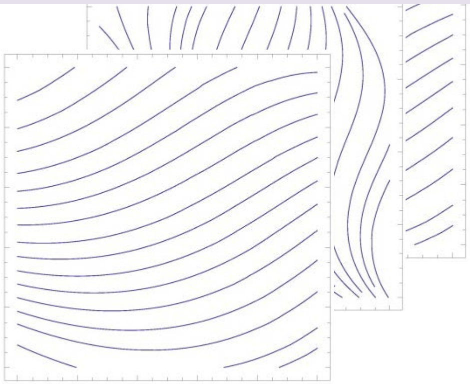
Thank you for your attention !

Application to the optimization of tow-steered composites

Tow-steered = reinforced with **curvilinear fibers**

Variable fields of anisotropic polar parameters

$R_K(x,y)$, $R_1(x,y)$ and $\Phi_1(x,y)$



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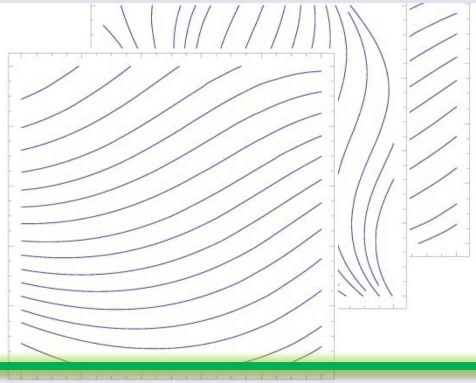
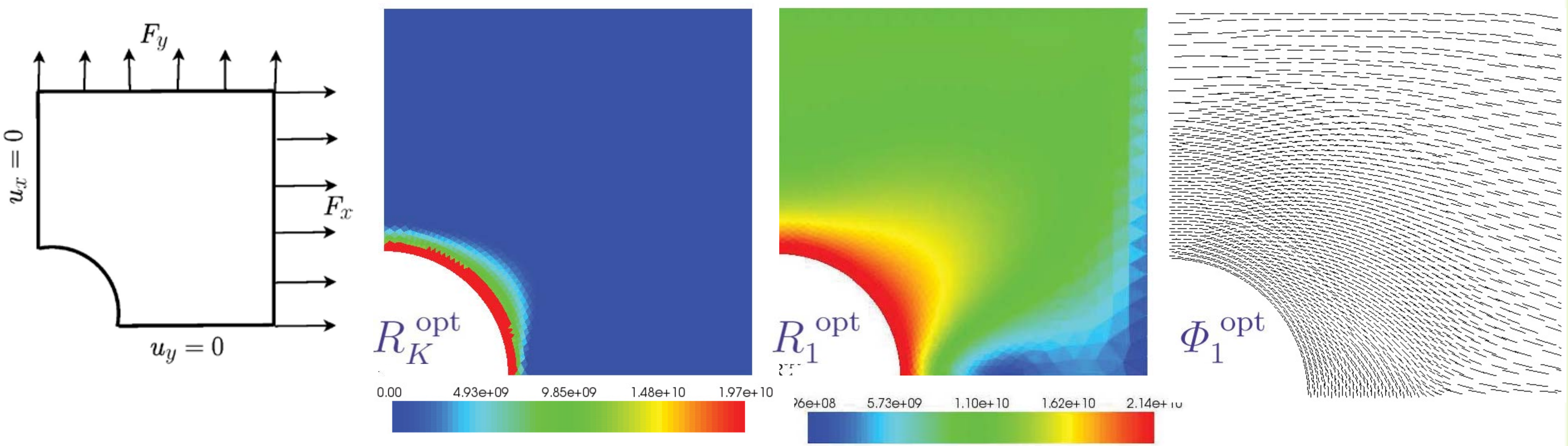


Compliance minimisation of tow-steered laminated structures



(C. Julien, *PhD thesis* 2010 ; Jibawy et al, *IJSS* 2011)

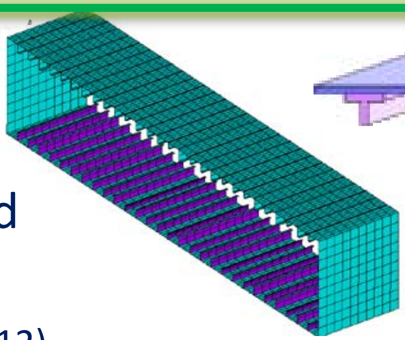
✓ Analytical minimisation of elastic energy for an orthotropic material [*JElas* 2010]

Application to the optimization of modular structures

Weight minimisation of composite stiffened panels under buckling constraint

Variables : <u>thickness</u>, <u>material properties</u> and **<u>number of constitutive modules</u>**

(Montemurro, *PhD Thesis 2012* ; Montemurro et al, *JOTA* 2012)



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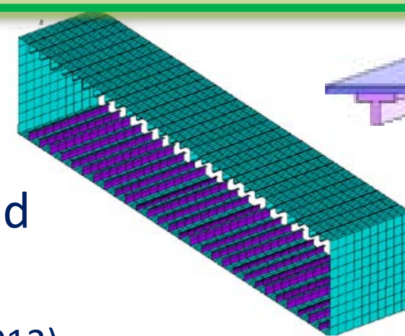
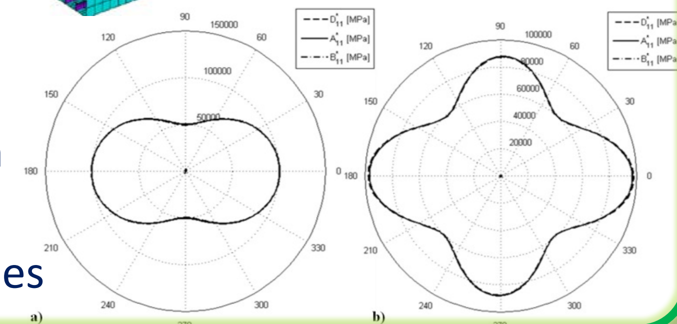


- ✓ Coupling FEM and **genetic algorithm with variable number of chromosomes**
- ✓ Optimal stacking sequence retrieval by polar-genetic approach

Stiffner and skin optimal elastic properties



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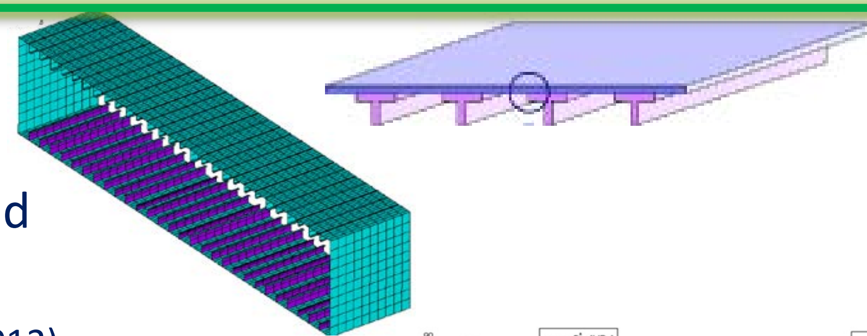

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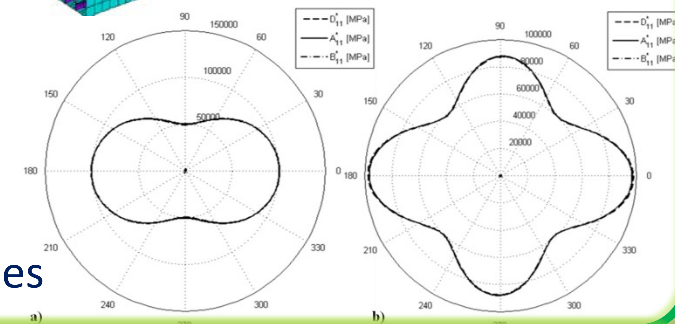
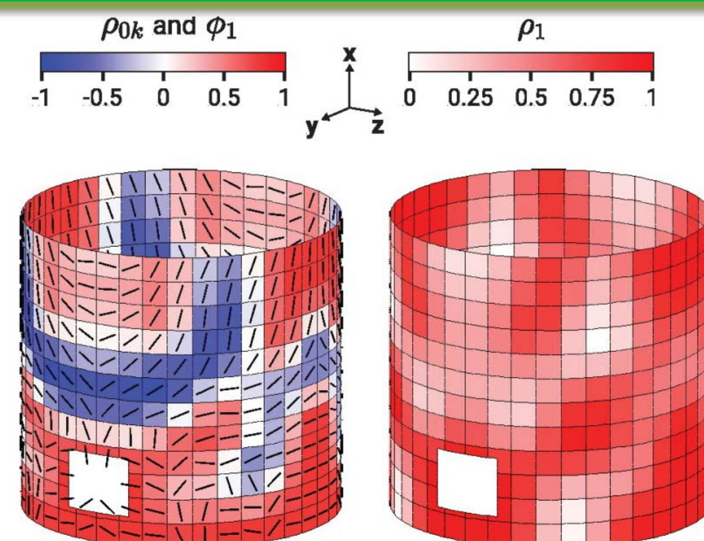


Compliance minimisation of large composite stiffened aerospace structures under buckling and mass constraints

Variables : <u>stiffening path</u>, **<u>laminate's thickness</u>** and **<u>material properties</u>**

(F. Savine, *PhD thesis* 2022, collaboration with ONERA and CNES)



My research contributions

1. Optimization of composite laminated plates
2. Optimization of variable-stiffness composite structures
3. **Influence of anisotropy and uncertainties on non-linearities and instabilities**

i. Design of multistable composite shells

(Hamouche et al : *Meccanica* 2016, *PRSA* 2016 ; Brunetti et al : *IJSS* 2016, *Compos Struct* 2018)

PhD thesis of W. Hamouche (2013-16), post-doc M. Brunetti (2015-16)

Collaboration with C. Maurini (d'Alembert) and S. Vidoli (Univ Roma La Sapienza)

ANR project JC SLENDER 2014-18

Design of multistable composite shells

- ✓ Reduced non-linear models of orthotropic shells (Vidoli and Maurini, PRSA 2008)
- ✓ Application of the two-level approach based on the polar formalism
- ✓ Design of prescribed prestress fields (e.g. thermo- or hygro-elastic couplings)

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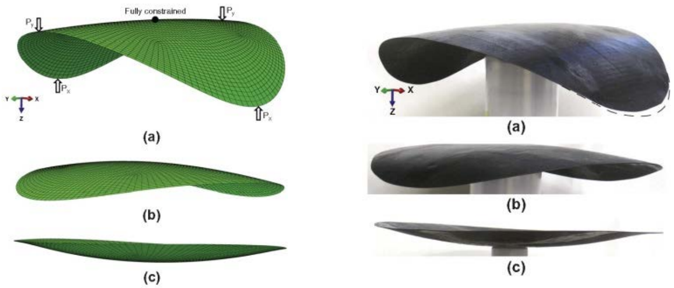
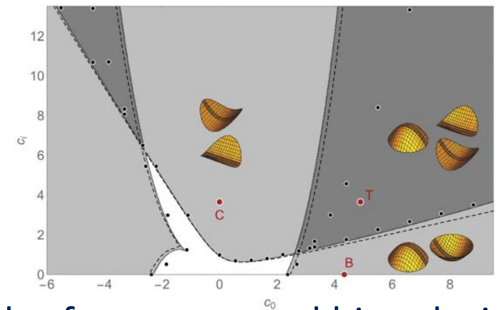


Design of a tristable composite shell at Bristol University (Coburn et al, *Compos Struct* 2013)

Effect of geometry and prestress



Design rules for prestressed bi-and tri-stable isotropic shells (Hamouche et al, *Meccanica* 2016)

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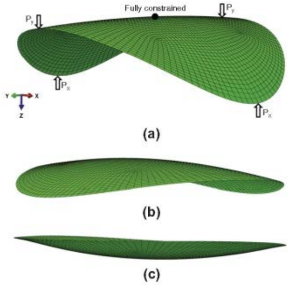

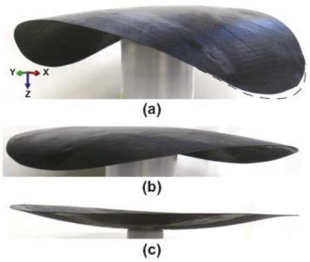

Design of a tristable composite shell
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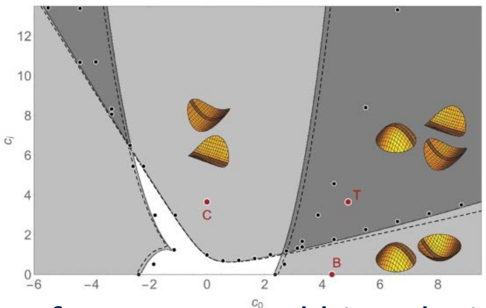
Effect of geometry and prestress



Design rules for prestressed bi-and tri-stable isotropic shells (Hamouche et al, *Meccanica* 2016)

Effect of orthotropy, geometry and prestress

Bistability of clamped orthotropic shells (Brunetti et al, *Compos Struct* 2018)