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THE FRENCH AEROSPACE LAB

Département Aérodynamique Aéroélasticité Acoustique

Effet de la tension de surface sur les liquides

Application au ballottement en micro-gravité

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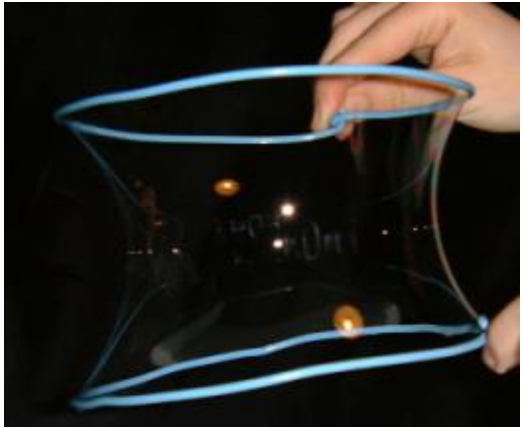
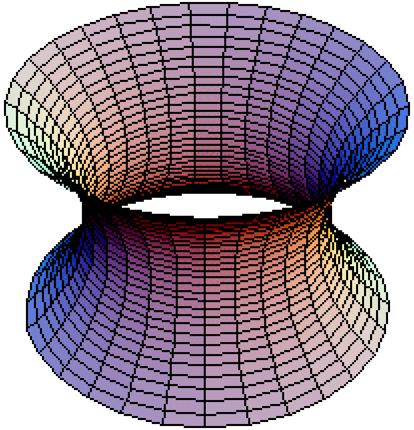
Plateau's problem: how to find a minimal surface shape?

find u such that $\mathcal{A}_\Gamma(u)$ minimal and $u = 0$ on $\partial\Gamma$ <=> $\delta\mathcal{A}_\Gamma(u, \delta u) = 0, \ \forall \ \delta u = 0$ on $\partial\Gamma$

$$\delta\mathcal{A}_\Gamma(u, \delta u) = \int_\Gamma \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \delta u . n \, \mathrm{dS} + \cancel{\oint_{\partial\Gamma} \delta u . \nu \, \mathrm{d}l}$$

$\Longrightarrow$ surface of mean curvature = 0

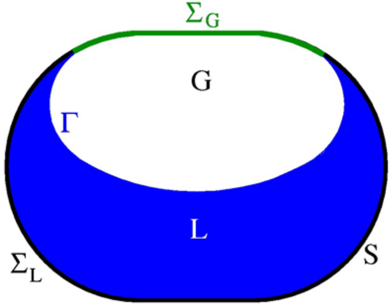




Liquids in micro-gravity: surface tension and meniscus

$$\mathbf{W} = \alpha\, \mathcal{A}_{\Gamma} + \alpha_L\, \mathcal{A}_{\Sigma L} + \alpha_G\, \mathcal{A}_{\Sigma G} + \boldsymbol{E}_{\boldsymbol{g}} + Cte$$

$$\Gamma^* \text{ such that: } \mathbf{W}(\Gamma^*) = \min_{\Gamma \in C} \mathbf{W} \text{ and } \mathbf{V}(\Gamma^*) = \mathbf{V}_0$$

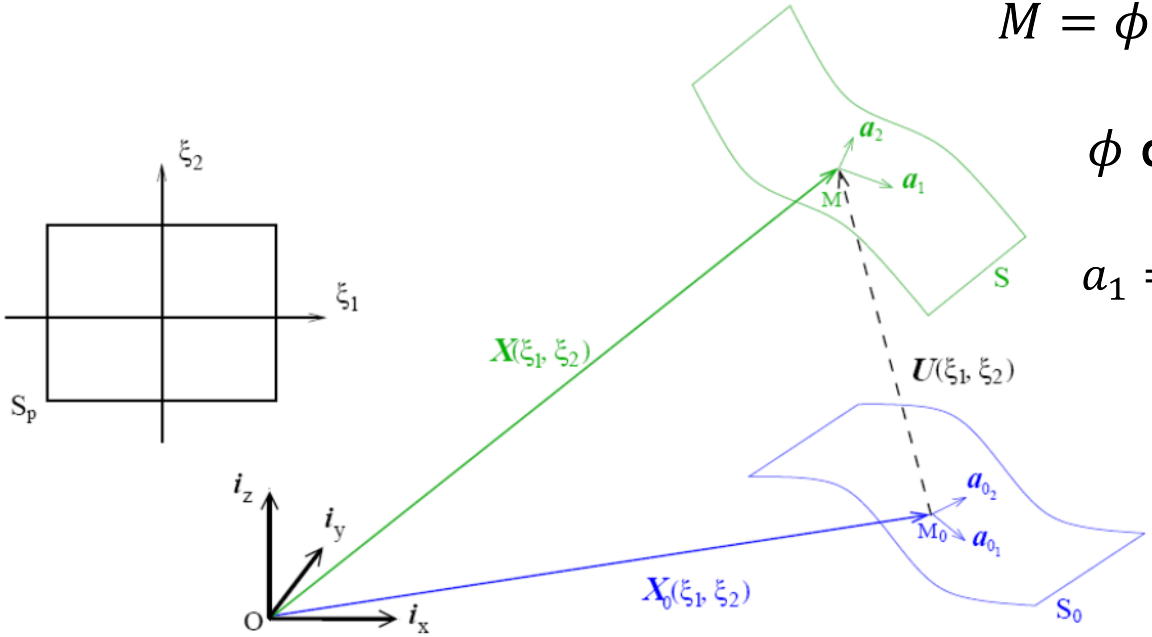


Lagrange multiplier Λ for incompressibility

Lagrangian stationarity:

$$\delta\mathcal{L}(U,\Lambda,\delta u) = \alpha\, \delta\mathcal{A}_{\Gamma} + \alpha_L\, \delta\mathcal{A}_{\Sigma L} + \alpha_G\, \delta\mathcal{A}_{\Sigma G} + \delta\, \boldsymbol{E}_{\boldsymbol{g}} + \Lambda\,\delta\mathbf{V} = 0, \quad \forall \boldsymbol{\delta u}$$

Un peu de géométrie différentielle …



$$M = \phi(\xi_1, \xi_2) = M_0(\xi_1, \xi_2) + U(\xi_1, \xi_2)$$

ϕ difféomorphisme

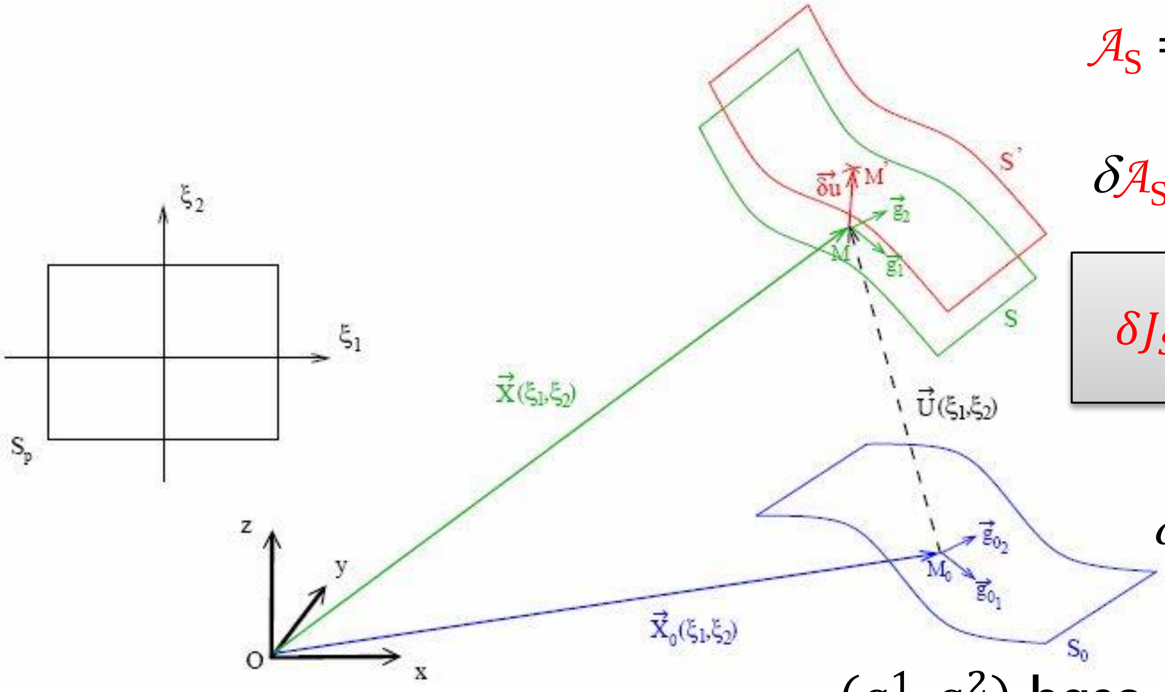
$$a_1 = \frac{\partial M}{\partial \xi_1}, a_2 = \frac{\partial M}{\partial \xi_2}$$ base covariante

$$n = \frac{a_1 \wedge a_2}{\|a_1 \wedge a_2\|}$$ normale

$$\mathrm{d}S = J_S \, d\xi_1 d\xi_2$$

avec $J_S = \|a_1 \wedge a_2\|$ jacobien

Surface area variation (1/2)



$$\mathcal{A}_S = \int_S \, \mathrm{d}S = \int_{\xi_1,\xi_2} J_S \, \mathrm{d}\xi_1 \mathrm{d}\xi_2$$

$$\delta\mathcal{A}_S(\delta u) = \int_{\xi_1,\xi_2} \delta J_S(\delta u) \, \mathrm{d}\xi_1 \mathrm{d}\xi_2$$

$$\boxed{\delta J_S(\delta u) = n.\left(\frac{\partial \delta u}{\partial \xi_1} \wedge a_2 + a_1 \wedge \frac{\partial \delta u}{\partial \xi_2}\right)}$$

$$\delta\mathcal{A}_S(\delta u) = \int_S \underbrace{\left(\frac{\partial \delta u}{\partial \xi_1} . a^1 + \frac{\partial \delta u}{\partial \xi_2} . a^2\right)}_{\mathrm{div}_S(\delta u)} \mathrm{d}S$$

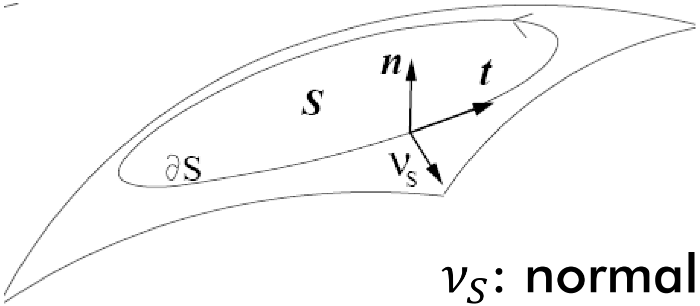
(a^1, a^2) base contravariante

Surface area variation (2/2)

normal and tangential components: $\delta u = (\delta u.n)n + \delta u_{\parallel}$

$$\Rightarrow \quad \mathrm{div}_S(\delta u) = (\delta u.n)\underbrace{\mathrm{div}_S(n)}_{2H} + \mathrm{div}_S(\delta u_{\parallel}) + \cancel{n.\nabla_S(\delta u.n)}$$

mean curvature $H = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$



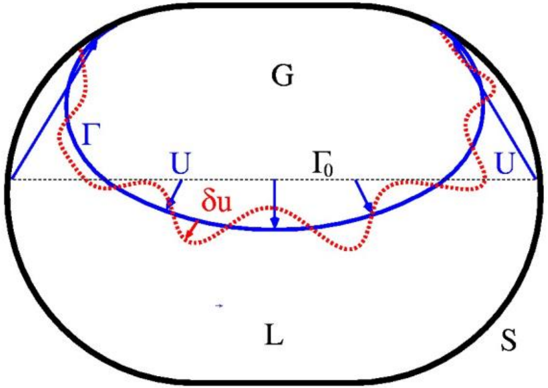
$$\delta\mathcal{A}_S(\delta u) = \int_S 2H\, n.\delta u\, \mathrm{d}S + \oint_{\partial S} \nu_S.\delta u\, \mathrm{d}\partial S$$

ν_S: normale géodésique extérieure au bord ∂S

Variation of the fluid gravity potential energy

$$E_g = \rho g \int_{\Omega_L} Z \,\mathrm{d}\Omega + Cte = \rho g \int_{\Omega_{L_0}} \overbrace{Z}^{OM.i_Z} \; J_V \,\mathrm{d}\Omega_0 + Cte$$

$$\Rightarrow \quad \delta E_g(\delta u) = \rho g \int_{\Omega_{L_0}} (\delta u.i_Z \; J_V + Z \;\mathrm{div}(\delta u)\, J_V)\,\mathrm{d}\Omega_0 = \rho g \int_{\Omega_L} \mathrm{div}(Z\,\delta u)\,\mathrm{d}\Omega$$



$$\delta E_g(\delta u) = \rho g \int_{\Gamma} Z\, n_\Gamma.\delta u \,\mathrm{d}\Gamma + \rho g \int_{\Sigma_L} \cancel{Z\, n_{\Sigma_L}.\delta u} \,\mathrm{d}\Sigma_L$$

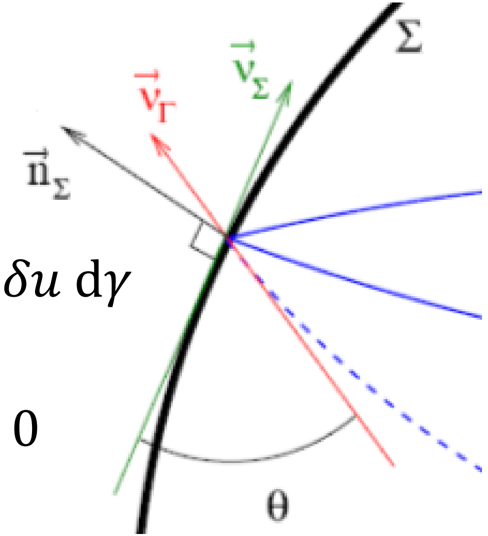
liquid slides on tank wall

idem for volume variation: $\delta V(\delta u) = \int_\Gamma n_\Gamma.\delta u \,\mathrm{d}\Gamma$

Variational equation of the static equilibrium

$$\delta\mathcal{L}(U,\Lambda,\delta u) = \alpha\ \delta\mathcal{A}_{\Gamma} + \alpha_{L}\ \delta\mathcal{A}_{\Sigma L} + \alpha_{G}\ \delta\mathcal{A}_{\Sigma G} + \delta E_{g} + \Lambda\delta V = 0, \quad \forall \boldsymbol{\delta u}$$

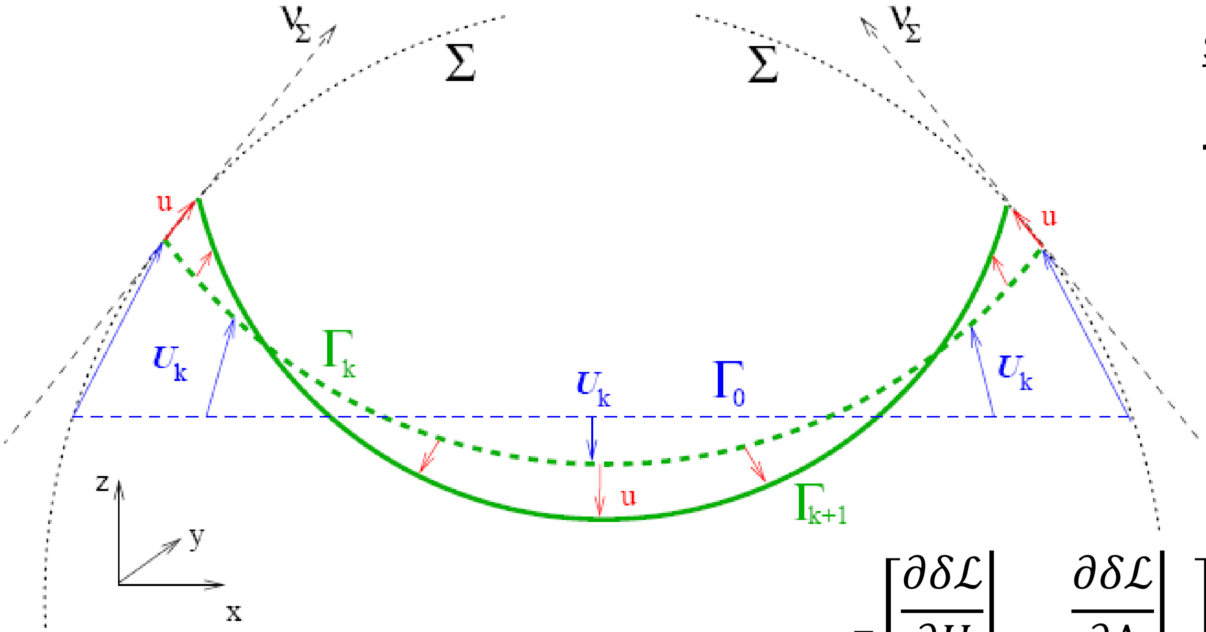
$$\forall\ \boldsymbol{\delta u}.\boldsymbol{n}_{\Sigma} = \mathbf{0}$$ with contact angle: $\cos\theta = \frac{\alpha_G - \alpha_L}{\alpha}$



$$\left\{\begin{array}{l} \delta\mathcal{L}(U,\Lambda,\delta u) = \alpha\displaystyle\int_{\Gamma} 2H\ \boldsymbol{n}_{\Gamma}.\delta u\ \mathrm{d}\Gamma + \alpha\oint_{\partial\Gamma}(\boldsymbol{\nu}_{\Gamma} - \cos\theta\ \boldsymbol{\nu}_{\Sigma}).\delta u\ \mathrm{d}\gamma \\ \qquad + \rho g\displaystyle\int_{\Gamma} Z\ \boldsymbol{n}_{\Gamma}.\delta u\ \mathrm{d}\Gamma + \Lambda\int_{\Gamma}\boldsymbol{n}_{\Gamma}.\delta u\ \mathrm{d}\Gamma = 0 \\ \forall\ \boldsymbol{\delta\lambda}, \quad \delta\lambda(V - V_0) = 0 \end{array}\right.$$

liquid position parameterized by $\boldsymbol{U}$ on Γ only

Newton-Raphson resolution



<u>step k</u>: U_k, Λ_k are known

find $u.n_\Sigma = 0, \lambda$ such that,

$\forall\ \delta u.n_\Sigma = 0$ and $\forall\ \delta\lambda$

$$\begin{cases} \delta\mathcal{L}(U_k+u, \Lambda_k+\lambda, \delta u) = 0 \\ \delta\lambda(V(U_k+u) - V_0) = 0 \end{cases}$$

linearization

$$\begin{pmatrix} \delta u \\ \delta\lambda \end{pmatrix}^T \begin{bmatrix} \left.\frac{\partial\delta\mathcal{L}}{\partial U}\right|_k & \left.\frac{\partial\delta\mathcal{L}}{\partial\Lambda}\right|_k \\ \left.\frac{\partial V}{\partial U}\right|_k & 0 \end{bmatrix} \begin{pmatrix} u \\ \lambda \end{pmatrix} = \begin{pmatrix} \delta u \\ \delta\lambda \end{pmatrix}^T \begin{bmatrix} -\mathcal{L}(U_k, \Lambda_k) \\ V_0 - V(U_k) \end{bmatrix}$$

+ fluid sliding condition !

Linearization of surface variation operator

$$\delta \mathcal{A}_S(\delta u) = \int_S 2H\, n.\delta u \,\mathrm{d}S + \oint_{\partial S} \nu_S.\delta u \,\mathrm{d}\partial S \quad \Longrightarrow \quad \delta H(u) = \ \ldots$$

$$\delta \mathcal{A}_S(\delta u) = \int_{\xi_1,\xi_2} \overbrace{J_S\, n}^{a_1 \wedge a_2}.\tau(\delta u)\,\mathrm{d}\xi_1 \mathrm{d}\xi_2 \qquad \text{with } \tau(\delta u) = J_S^{-1}\left(\frac{\partial \delta u}{\partial \xi_1} \wedge a_2 + a_1 \wedge \frac{\partial \delta u}{\partial \xi_2}\right)$$

$$\Longrightarrow \quad \delta[\delta \mathcal{A}_S(\delta u)](u) = \int_{\xi_1,\xi_2} \delta[a_1 \wedge a_2](u).\tau(\delta u) + (a_1 \wedge a_2).\delta\tau(u,\delta u)\,\mathrm{d}\xi_1 \mathrm{d}\xi_2$$

with $\delta\tau(u,\delta u) = -\dfrac{\delta J_S(u)}{J_S^2}\left(\dfrac{\partial \delta u}{\partial \xi_1} \wedge a_2 + a_1 \wedge \dfrac{\partial \delta u}{\partial \xi_2}\right) + J_S^{-1}\left(\dfrac{\partial \delta u}{\partial \xi_1} \wedge \dfrac{\partial u}{\partial \xi_2} + \dfrac{\partial u}{\partial \xi_1} \wedge \dfrac{\partial \delta u}{\partial \xi_2}\right)$

$\delta J_S(u) = J_S\, n.\tau(u)$

Linearization of other operators

$$\delta[\delta\mathcal{A}_S(\delta u)](u) = \dots$$

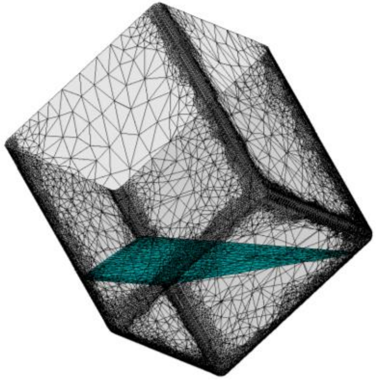
$$\int_{\xi_1,\xi_2} J_S \tau(u).\tau(\delta u) - J_S^{-1} n.\tau(u)\, n.\tau(\delta u) + n.\left(\frac{\partial \delta u}{\partial \xi_1} \wedge \frac{\partial u}{\partial \xi_2} + \frac{\partial u}{\partial \xi_1} \wedge \frac{\partial \delta u}{\partial \xi_2}\right) \mathrm{d}\xi_1 \mathrm{d}\xi_2$$

$\boldsymbol{\delta u}$ is assumed to be independent of u : not true since $\delta u.n_\Sigma(u) = 0$ on $\partial\Gamma$!

idem for edge term: $\delta\left[\oint_{\partial\Gamma} \nu_\Sigma.\delta u\, d\gamma\right](u) = \oint_\xi \delta u.\left(n_\Sigma \wedge \frac{\partial u}{\partial \xi}\right) \mathrm{d}\xi$

and gravity/volume terms: $\delta\left[\int_\Gamma Z\, n_\Gamma.\delta u\, \mathrm{d}\Gamma\right](u) = \int_\Gamma u.i_Z\, \delta u.n + Z\, \tau(u).\delta u\; \mathrm{d}\Gamma$

Singular problem and regularization



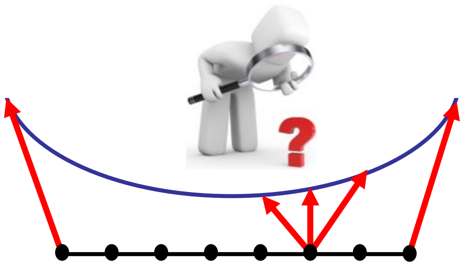
Finite Element discretization

Singular problem

Regularization

Bonet & Mahaney, 2001

ref. surface (ξ_1, ξ_2) = ref. FE

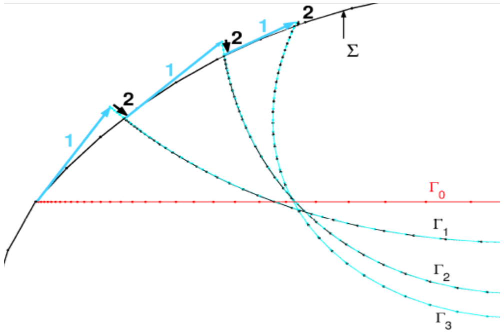


distortion energy with $\tilde{F}_S$ isochore part of surface transformation gradient

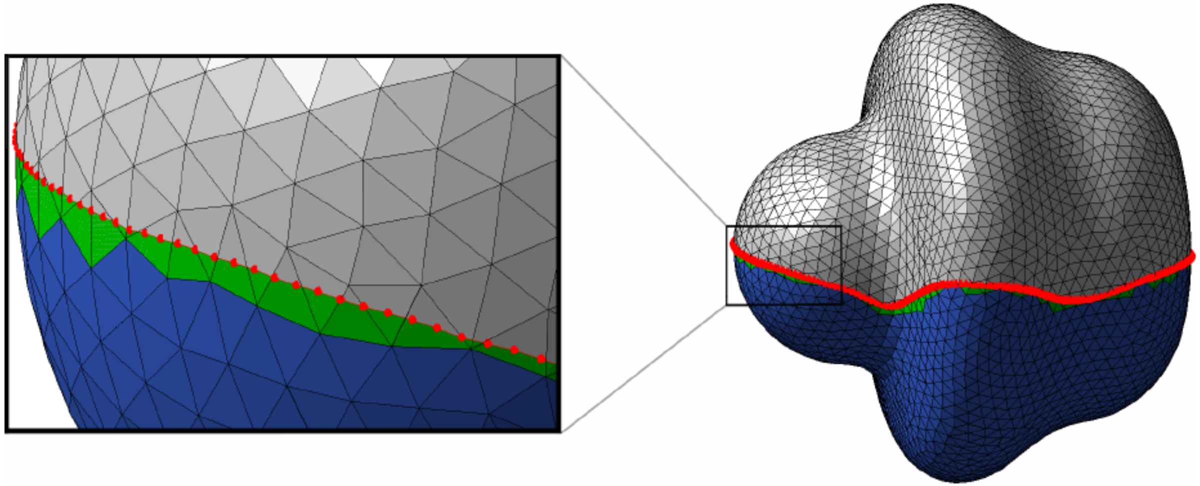
$$\mathbf{W} = \ldots + E_d = \frac{\mu}{2}\int_{\Gamma_0}\left(\tilde{F}_S : \tilde{F}_S - 3\right)\mathrm{d}\Gamma_0$$

⇒ raideur de membrane tangentielle

Implementation issues



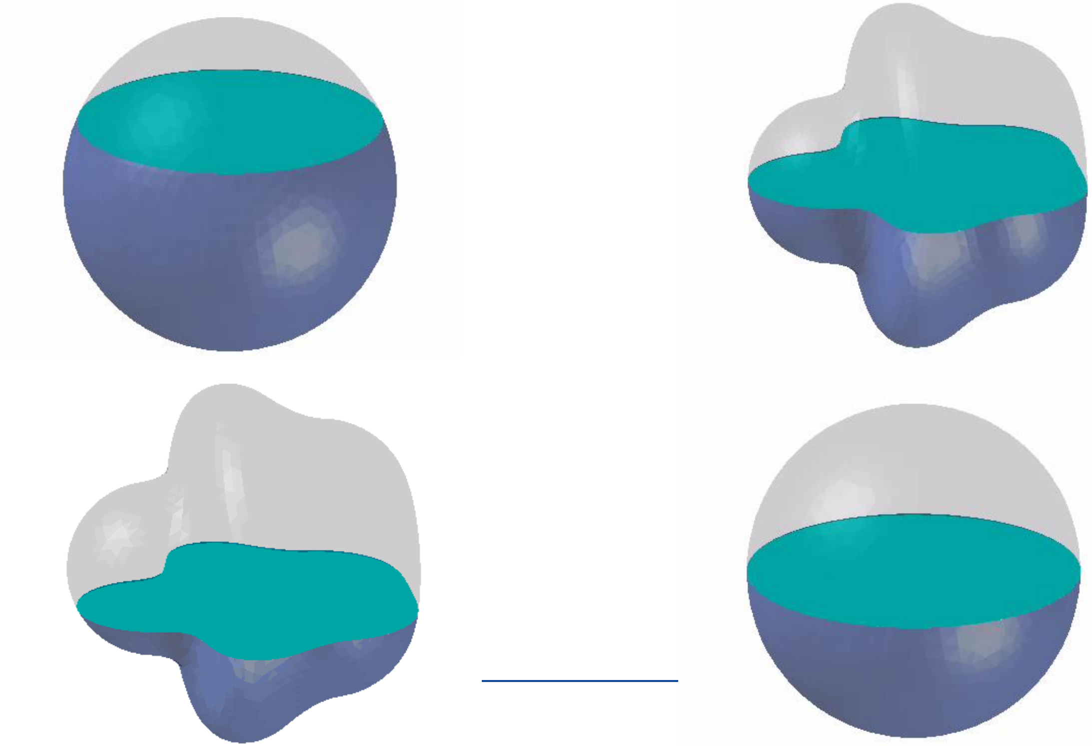
- How to correct the free surface in order to precisely follow the tank wall?



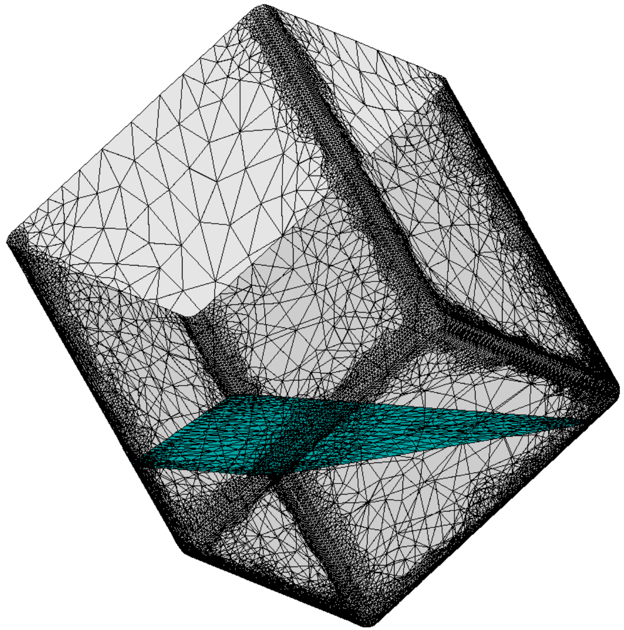
- How to precisely compute the liquid volume at each step?

Examples of wetting and non-wetting liquids

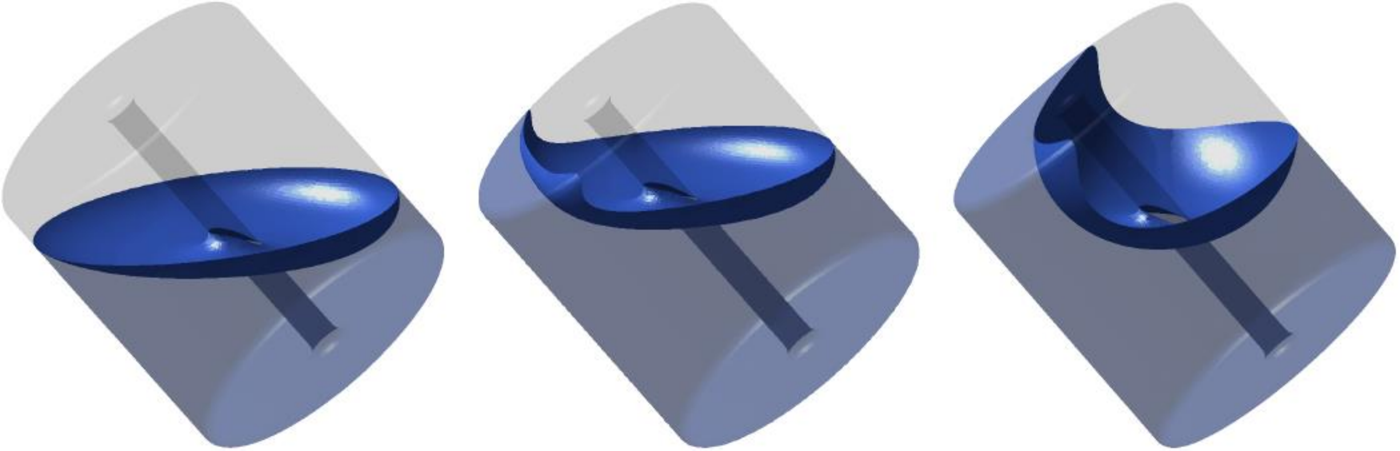
	wetting liquid (θ<90º)	non-wetting liquid (θ>90º)
low gravity		
microgravity		



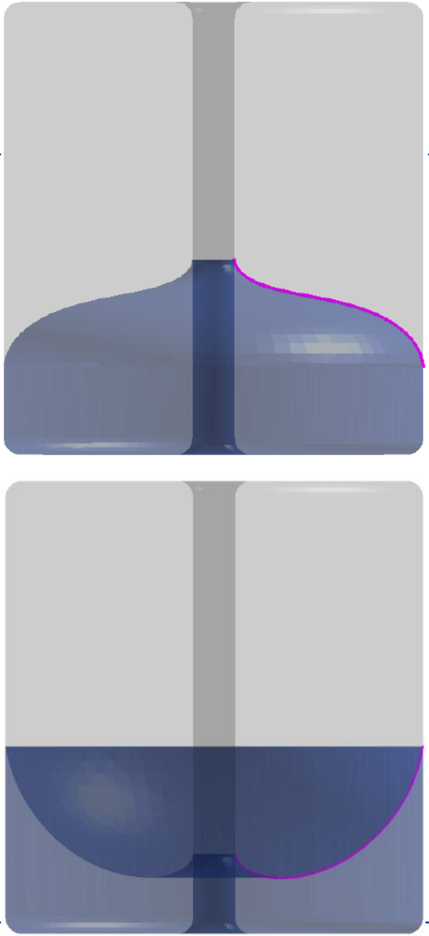
Example with corners



Free surface with multiple edges



validations on axisymetric cases



Extension to dynamic problem

φ : liquid displacement potential

wall curvature

dynamic pressure

$$\begin{cases} \left(\alpha \mathbf{K}_{\Gamma}^{\alpha} - \alpha\left(\cos(\theta)\mathbf{K}_{\partial\Gamma}^{\theta} + \sin(\theta)\,\mathbf{K}_{\partial\Gamma}^{\Sigma}\right) + \rho g \mathbf{K}_{\Gamma}^{g} + \Lambda\,\mathbf{K}_{\Gamma}^{\Lambda}\right) u - \rho\omega^2 \mathbf{C}\varphi - \lambda\,\mathbf{R} = 0 \\ \mathbf{M}\varphi - \mathbf{C}^T u = 0 \\ \mathbf{R}^T u = 0 \end{cases}$$

liquid mass

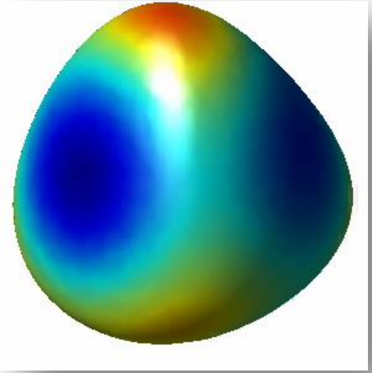
incompressibility

+ fluid sliding condition !

$$\Rightarrow \begin{bmatrix} \mathbf{K}^{\perp} & -\mathbf{R}_* \\ -\mathbf{R}_*^T & 0 \end{bmatrix} \begin{pmatrix} u^{\perp} \\ \lambda \end{pmatrix} = \omega^2 \begin{bmatrix} \rho \mathbf{C}_* \mathbf{M}_{**}^{-1} \mathbf{C}_*^T & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} \begin{pmatrix} u^{\perp} \\ \lambda \end{pmatrix}$$

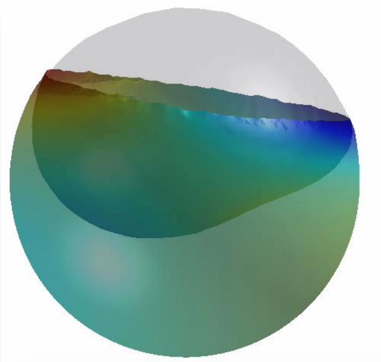
symmetric operators !

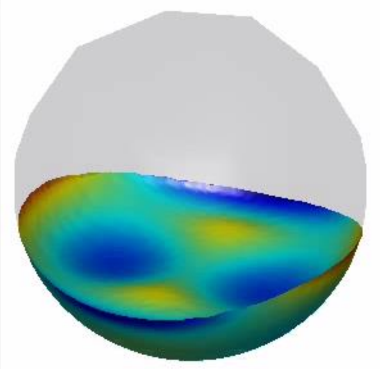
Examples of sloshing modes with capillarity

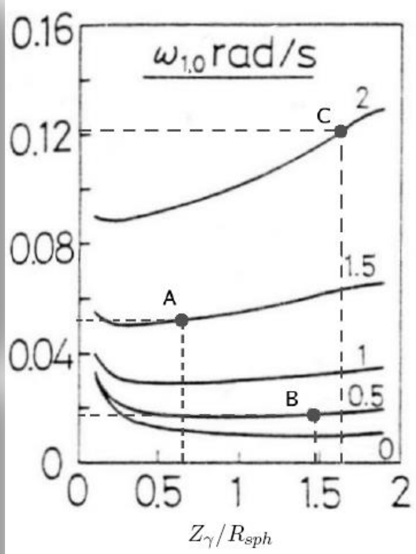


solution analytique (H. Lamb 1945) :

$$\omega_n^2 = \frac{\alpha}{\rho R^3} n(n-1)(n+2)$$







solution semi-analytique (M. Utsumi 1989)

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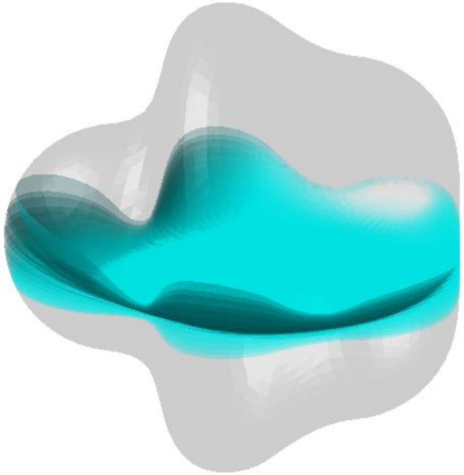
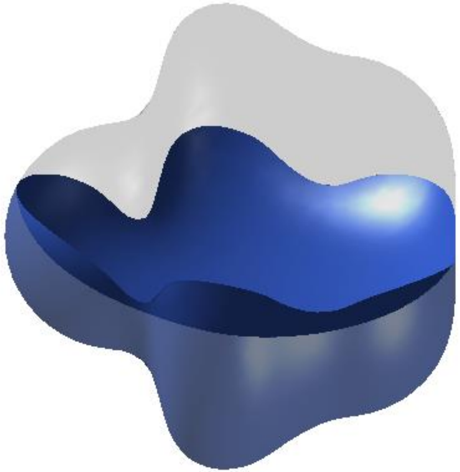
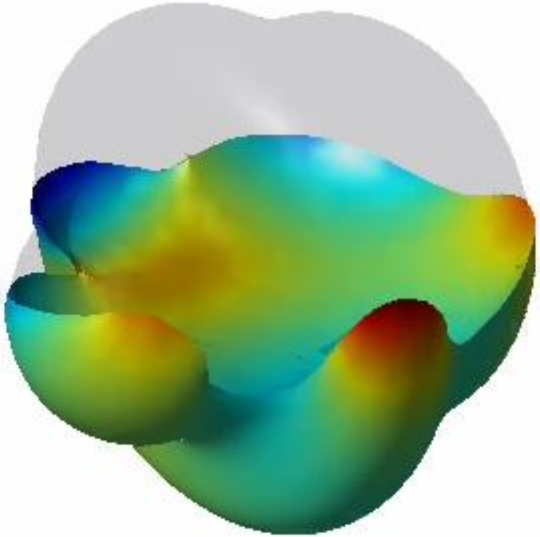
Merci pour votre attention !







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