Ph.D CIFRE : Study of bifurcations in a two-fluid equation system of a thermohydraulic simulation code, and application to a natural circulation loop under two-phase flow conditions.

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PhD organisation

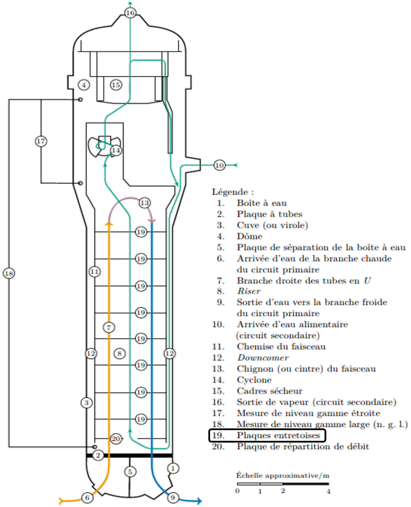
This PhD project is co-directed by Framatome and the LMFA laboratory (CNRS, École Centrale de Lyon, Univ. Lyon 1, INSA de Lyon), and conducted within the Doctoral School MEGA (ED 162). Supervised by

- Academic advisors
  - Benoît Pier : DR at LMFA
  - Frédéric Alizard : Maître de conférences at LMFA, université Lyon 1
- Advisors from industry
  - Cédric Pozzolini : Senior expert in numerical analysis of PDEs
  - Alexandre Muller : Specialist in thermohydraulics
  - Clément Grenat : Specialist in dynamics

Industrial context

Steam generator

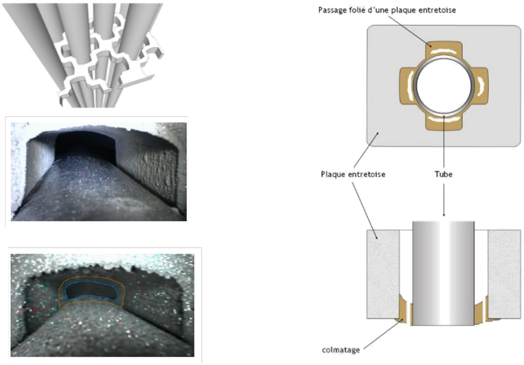
In a Pressurized Water Reactor (PWR), the steam generator transfers heat from the primary circuit to the secondary circuit to produce steam.
The primary coolant flows inside U-tubes, while the secondary water boils outside them.
Spacer grids maintain tube spacing and flow distribution.



Steam generator in a PWR

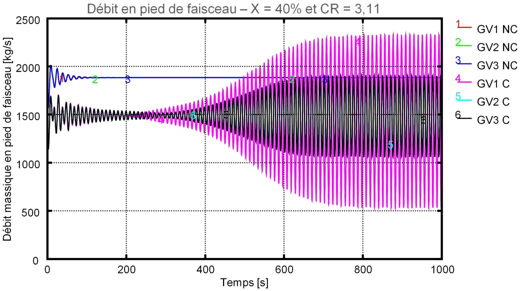
Clogging

The term *clogging* is used to designate the reduction of the flow passage cross-section between the tubes and the spacer grids, which is induced by deposits at the level of the drilled holes. To a lesser extent, tube fouling can also reduce the flow passage cross-section.



Steam generator in a PWR

Density wave oscillations (DWO)



Industrial study of DWO

Density wave oscillations (DWO) are self-sustained oscillations that arise from the dynamic coupling between variations in fluid density and the system's response, leading to periodic fluctuations in key flow variables such as mass flow rate and pressure.

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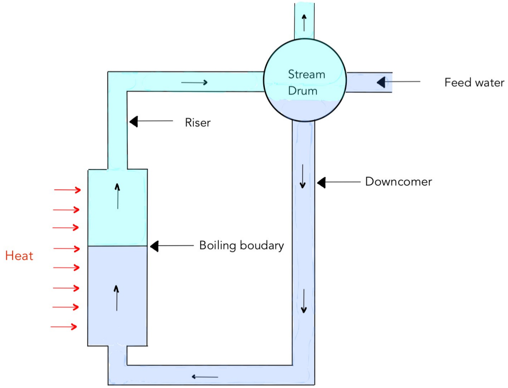
The Natural Circulation Loop

A natural circulation loop is a water–steam circuit where fluid motion is driven by density differences between the hot riser and the cold downcomer, and it forms the basic operating principle of steam generators in PWRs.



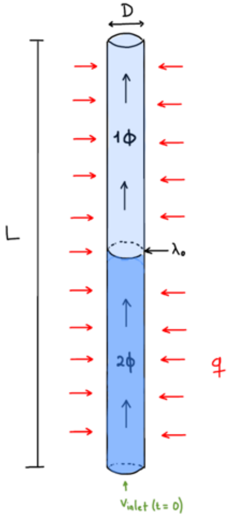
Natural Circulation Loop

Heated Channel Instability Mechanism

The idea is to perform a positive enthalpy perturbation δh_{inlet} at the entrance of the channel, and to observe how the propagation speed of the perturbation interacts with the global momentum balance, creating a closed feedback loop.

$$\Delta P = \Delta P_{1\phi}^{\text{grav}} + \Delta P_{2\phi}^{\text{grav}} + \Delta P_{1\phi}^{\text{drag}} + \Delta P_{2\phi}^{\text{drag}} + \Delta P^{\text{acc}}$$

$$\Delta P = P_{\text{in}} - P_{\text{out}}$$



Two-phase models for industrial codes

6-equation two-fluid model
two velocities, two energies, common pressure CATHARE

↓

5-equation drift-flux model
one mixture momentum + slip law
separate phasic mass and energy MANTA

↓

Homogeneous equilibrium model (HEM)
common velocity and thermodynamic equilibrium
(often shown as a 3-equation mixture model)

Simple heated channel model

We started with the study of the simplest model in which we think DWO will appear. The model corresponds to Euler gas dynamics with gravity and friction terms. This is a one-phase-flow model.

Conservation Laws

$$\begin{cases} \text{On } (t,x) \in \mathbb{R} \times (0,L), \\ \partial_t \rho + \partial_x(\rho v) = 0, \\ \partial_t(\rho v) + \partial_x\left(\rho v^2 + p\right) = -\dfrac{f_D}{2D} v|v| - \rho g, \\ \partial_t\left(\rho h - p\right) + \partial_x\left(\rho v h\right) - v\partial_x p = \dfrac{f_D}{2D}\rho v^2 |v| + q, \end{cases} \tag{1}$$

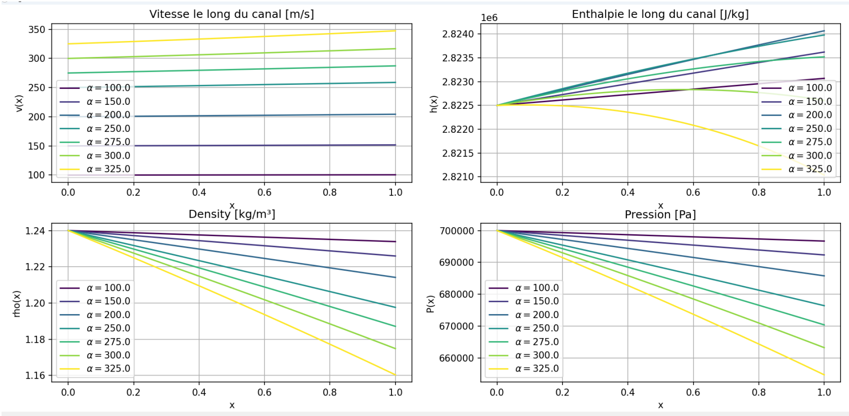
Initial Conditions

$$\begin{cases} \text{For } x \in (0, L), \\ \rho(0, x) = \rho_0(x), \\ \rho(0, x)v(0, x) = \rho_0(x)v_0(x), \\ h(0, x) = h_0(x), \end{cases} \tag{2}$$

Boundary Conditions

$$
\begin{cases}
\text{For } t \in \mathbb{R}, \\
p(t,0) = p_{\text{inlet}}, \\
h(t,0) = h_{\text{inlet}}, \\
p(t,L) = p_{\text{outlet}}.
\end{cases}
\tag{3}
$$

A shooting method was used, varying the inlet velocity α to match the boundary conditions and obtain the steady state.



Steady-state profiles

General mathematical statement of the problem

Heated Channel Stability Problem

A general formulation of the Heated Channel Stability Problem is given by the first-order initial boundary value problem (IBVP) :

$$\begin{cases} \partial_t U + \partial_x (F(U)) = S(U) & \text{in } (0,T) \times (0,L) \quad \text{(Blance Laws)} \\ U(0,x) = U_0(x), & x \in (0,L) \quad \text{(Initial conditions)} \\ \mathcal{B}(U(t,0), U(t,L)) = 0 & t \in \mathbb{R}^+ \quad \text{(Boundary conditions)} \end{cases} \tag{4}$$

The unknown function $U : \mathbb{R}^+ \to X$ takes values in a suitable functional space X, equipped with the norm $\|\cdot\|_X$.

Steady-state solution We are interested in cases where there exists a steady-state solution U^* of the balance laws and boundary conditions, independent of time. This means that we suppose the existence of a solution U^* to the problem

$$\begin{cases} \partial_x \left(F(U)\right) = S(U) & \text{in } (0, L) \quad \text{(Blance Laws)} \\ \mathcal{B}(U(0), U(L)) = 0 & \quad \text{(Boundary conditions)} \end{cases} \tag{5}$$

Mathematically, this allows us to remain under the assumption of small initial data when looking for solutions of the form

$$U(t, x) = U^*(x) + U_1(t, x). \tag{6}$$

From a physical point of view, this places us close to the true operating regimes.

Stability definition

We are interested in the stability of a given steady state U^* in the following sense. Suppose that the problem (4) defines a dynamical system $\mathcal{S} : \mathbb{R}^+ \times X \to X$ that takes any $U_0 \in X$ and gives $\mathcal{S}(t, U_0) = U(t, \cdot) \in X$.

Definition (Stability)

We say that U^* is stable if for any $\epsilon > 0$ ther is $\delta > 0$ such that

$$\|U_0 - U^*\|_X \leq \delta \Rightarrow \|U(t,\cdot) - U^*(\cdot)\|_X \leq \epsilon \quad \forall t \geq 0. \tag{7}$$

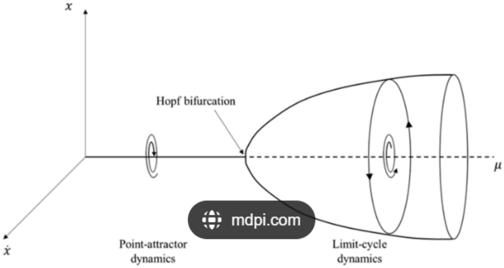
if in addition

$$\lim_{t\to\infty} \|U(t,\cdot) - U^*(\cdot)\|_X = 0, \tag{8}$$

we say that U^* is asymptotically stable. If the rate of convergence is exponential, we say that the state is exponentially stable.

Looking for a Hopf Bifurcation

Density wave oscillations have often been studied via reduced-order models (ROMs) and are commonly described as a Hopf bifurcation. A Hopf bifurcation appears to match the behavior we observe : a transition from a stable steady state to an unstable one, around which oscillations develop.



Our strategy for bifurcation identification.

- Identify the steady-state solutions.
- Establish the well-posedness of the evolution problem and identify the strongest available regularity framework.
- Apply the center manifold theorem to describe the local behavior near the steady state, (when possible).
- Analyze stability using alternative techniques when/if the center manifold theorem is out of reach.

Our main goal is to perform this identification process for the most complete model possible in order to compare it with the industrial code currently used to perform this analysis, CATHARE.

Well-posedness and stability results

The first mathematical framework we found corresponding exactly to the heated-channel stability problem was the one developed by Bastin and Coron in Bastin et Coron 2016. Stability and Boundary Stabilization of 1-D Hyperbolic Systems.

Theorem (Well-posedness of the IBVP)

There exists $\delta_0 > 0$ such that for any initial data $Z_0 \in H^2((0,L);\mathbb{R}^n)$ satisfying

$$\|Z_0\|_{H^2((0,L);\mathbb{R}^n)} \leq \delta_0 \tag{9}$$

and suitable compatibility conditions to satisfy the boundary conditions, there exists a unique maximal solution

$$Z \in C^0\left([0,T), H^2((0,L);\mathbb{R}^n)\right), \tag{10}$$

with $T \in (0,+\infty]$, of the following initial-boundary value problem

Theorem (... continuation)

$$\begin{cases} \partial_t Z + A(Z,x)\partial_x Z + B(Z,x) = 0 \quad t \in [0,+\infty)\,, \quad x \in [0,L]\,, & \\ Z(0,x) = Z_0(x) \quad x \in [0,L]\,, & \textit{(I.C)} \\ Z_{in}(t) = \mathcal{H}\left(Z_{out}(t)\right) & \textit{(B.C)}. \end{cases} \tag{11}$$

Moreover, if

$$\|Z(t,\cdot)\|_{H^2((0,L);\mathbb{R}^n)} \leq \delta_0, \quad \forall t \in [0,T) \tag{12}$$

then $T = +\infty$.

Démonstration.

The proof of this theorem is based on *Hille-Yosida's theorem* for the linearized system, combined with fixed-point arguments Bastin et Coron 2016. □

Assumptions for the validity of this theorem

- $A : \mathbb{R}^n \times [0, L] \to \mathcal{M}_{n,n}(\mathbb{R})$ is of class C^2 and such that

  $$A(0, x) = \mathrm{diag}\{\lambda_1(x), \dots, \lambda_m(x), -\lambda_{m+1}(x), \dots, -\lambda_n(x)\}, \quad \lambda_i(x) > 0,\ \forall x \in [0, L];$$

- $B : \mathbb{R}^n \times [0, L] \to \mathbb{R}^n$ is of class C^2 and such that $B(0, x) = 0$;
- $\mathcal{H} : \mathbb{R}^n \to \mathbb{R}^n$ is of class C^2 and such that $\mathcal{H}(0) = 0$.

These assumptions ensure that, for solutions close to a steady state, the system remains close to a diagonalizable one.

Theorem (Exponential stability)

The steady state $Z(t,x) \equiv 0$ of (11) is exponentially stable for the H^2-norm if there exists a map

$$Q \in C^1([0,L];\mathcal{D}_n^+)$$

such that the following matrix inequalities hold :

(i) *the matrix*

$$Q(L)\Lambda(L) - K^\top Q(0)\Lambda(0)K$$

is positive semi-definite ;

(ii) *the matrix*

$$-\big(Q(x)\Lambda(x)\big)_x + Q(x)M(0,x) + M^t(0,x)Q(x)$$

is positive definite for all $x \in [0,L]$.

Theorem (...)

Where

$$K = \mathcal{H}'(0)$$

and

$$M(0,x) = \frac{\partial B}{\partial Z}(0,x).$$

Démonstration.

It is based on the construction of a Lyapunov function. $\square$

Stability depends on the steady-state solution through the term $M(0,x)$ and depends on the boundary conditions through K.

Numerical approach

Riemann Problem

For a hyperbolic system of conservation laws

$$\partial_t U + \partial_x F(U) = 0,$$

the elementary building block of many numerical solvers is the Riemann problem. It consists in solving the problem with piecewise constant data

$$U(0,x) = \begin{cases} U_L, & x < 0, \\ U_R, & x > 0. \end{cases}$$

Finite volume methods

In a finite volume method, the solution is approximated by cell averages U_i^n. At each interface $x_{i+\frac{1}{2}}$, a local Riemann problem is solved between U_i^n and U_{i+1}^n. Integrating over the spatial cell and approximating the time derivative at first order gives the following discretization.

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\left(F_{i+\frac{1}{2}}^n - F_{i-\frac{1}{2}}^n\right).$$

Riemann solvers naturally capture wave propagation in hyperbolic systems.

Conclusion

Achivements

- To understand the fundamentals of two-fluid thermohydraulic models.
- To Formulate the problem within the IBVP framework for 1D conservation laws with source terms and started a stability analysis
- To Study and start developing finite volume methods based on Riemann solvers

Objectives :

- To prove that, in a one-dimensional system of balance laws modeling a monophasic flow with a perfect gas equation of state (EOS), a bifurcation can occur.
- To develop a numerical method to demonstrate this phenomenon in a real framework.
- To extend these results to more sophisticated models.

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