Weyl–Cosserat Elasticity and Gravitational Memory: An Effective Microstructured Model of Spacetime

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23 June 2026

GDR GDM INSA LYON

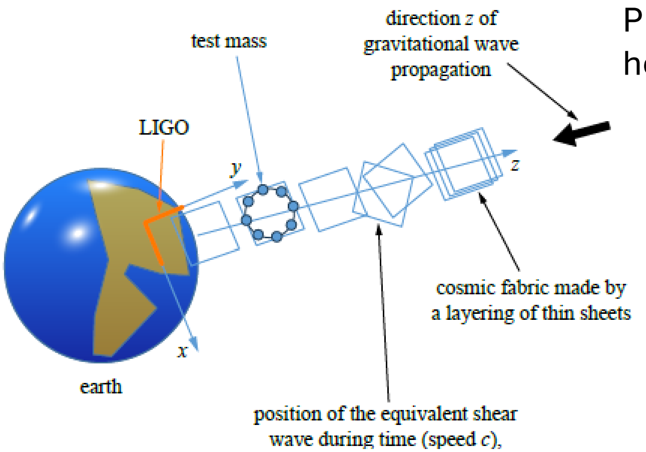
What are the physical implications of a permanent deformation of the geometry?

Agenda

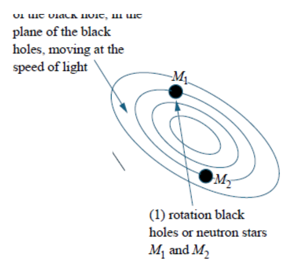
1) **Gravitational wave in elastic regime**
2) **Gravitational wave and memory effects in modern physics**
3) **Gravitational wave and memory effect=> possible connection with mechanics in plasticity**
4) **The problematic of the memory effect with the analogy of the elastic medium**
5) **Weyl–Cosserat Elasticity and Gravitational Memory: An Effective Microstructured Model of Spacetime**
6) **Difference between Torsion of Einstein Cartan and effective torsion in Weyl Cosserat**
7) **Weyl–Cosserat Elasticity and Gravitational Memory:Potential consequences on fundamental physics**
8) **Conclusion**

1) Gravitational wave in elastic regime

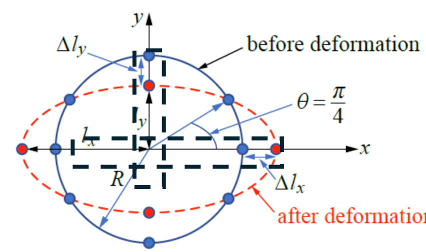
Gravitational Wave and its measured strains of space



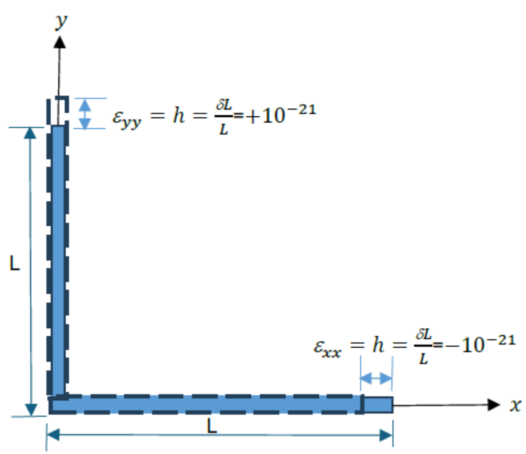
Physical phenomena: Black holes coalescence



Displacement of test masses following the GW theory



Light browse vacuum tube until to reach mirror and come back creating interferences on reception mirror => h

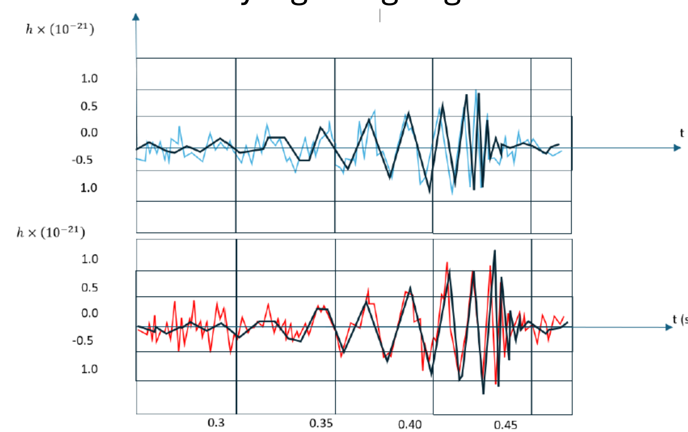


in gauge

$$\frac{\Delta L}{L}(t)$$

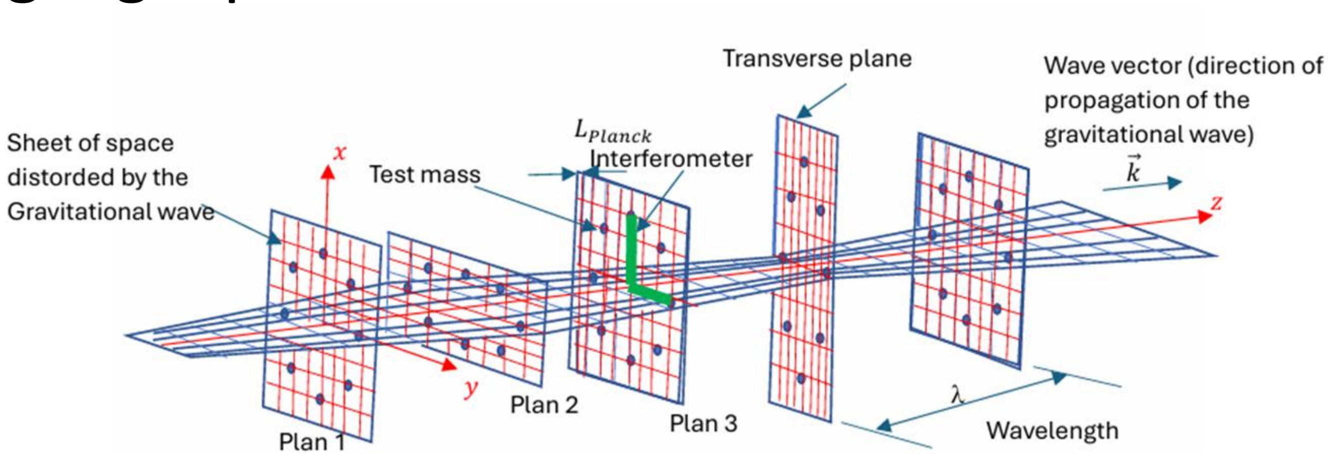
lengthening and shortening of space

Strains measured by Ligo/Virgo eg GW150914



Transverse Elastic Regime

- Propagation of the GW along z-direction
  - Independent transverse (x,y) planes
  - Strains concentrated in these planes xy
  - TT gauge: pure transverse shear



$$h_{\mu\nu} = A_+ \cos\Big(\frac{\omega}{c}(ct - z)\Big)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$\boxed{h_{ij}^{TT}(t - z)}$$

$$h_{\mu\nu} = A_\times \cos\Big(\frac{\omega}{c}(ct - z)\Big)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The essential displacement field

"...the essential achievement of general relativity, namely to overcome 'rigid' space (ie the inertial frame), is only indirectly *connected with the introduction of a Riemannian metric. The directly relevant conceptual element is the 'displacement field' (Γ^l_{ik}), which expresses the infinitesimal displacement of vectors. It is this which replaces the parallelism of spatially arbitrarily separated vectors fixed by the inertial frame (ie the equality of corresponding components) by an infinitesimal operation. This makes it possible to construct tensors by differentiation and hence to dispense with the introduction of 'rigid' space (the inertial frame). In the face of this, it seems to be of secondary importance in some sense that some particular Γ field can be deduced from a Riemannian metric..."*

A. Einstein (4 April 1955)[1]

If they are displacements, displacement of what?

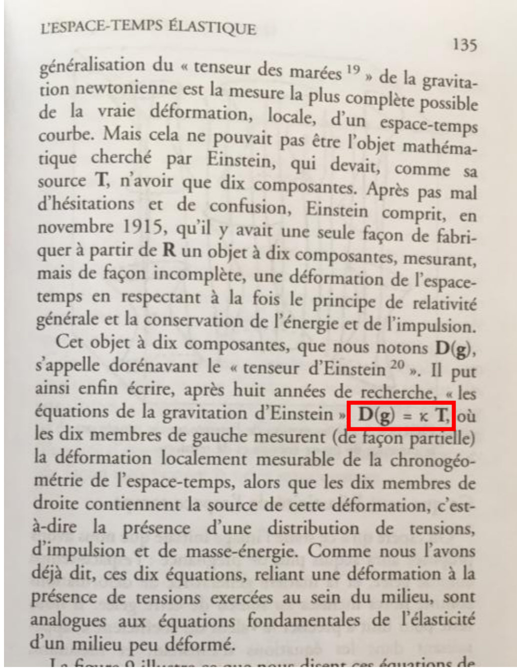
Continuum mechanics studies the deformation and motion of continuous media.

Effective Approach :Analogy of the elastic medium : State of the art :Einstein - Hooke’s law as a model of space-time behaviour -

A. Sakharov 1967

$$\boxed{S(R) = -\frac{1}{16\pi G}\int (dx)\,\sqrt{-gR}}$$

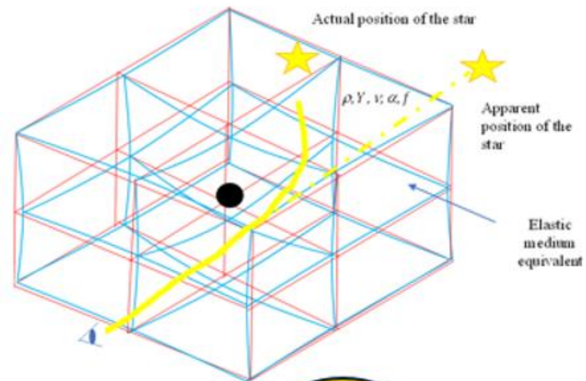
T. Damour book (If Einstein was told me)

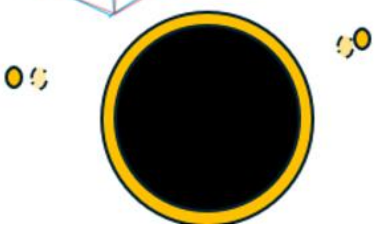


L’ESPACE-TEMPS ÉLASTIQUE 135

généralisation du « tenseur des marées [19] » de la gravitation newtonienne est la mesure la plus complète possible de la vraie déformation, locale, d’un espace-temps courbe. Mais cela ne pouvait pas être l’objet mathématique cherché par Einstein, qui devait, comme sa source **T**, n’avoir que dix composantes. Après pas mal d’hésitations et de confusion, Einstein comprit, en novembre 1915, qu’il y avait une seule façon de fabriquer à partir de **R** un objet à dix composantes, mesurant, mais de façon incomplète, une déformation de l’espace-temps en respectant à la fois le principe de relativité générale et la conservation de l’énergie et de l’impulsion.

Cet objet à dix composantes, que nous notons **D**(**g**), s’appelle dorénavant le « tenseur d’Einstein [20] ». Il put ainsi enfin écrire, après huit années de recherche, « les équations de la gravitation d’Einstein » $\boxed{\mathbf{D}(\mathbf{g}) = \kappa\,\mathbf{T},}$ où les dix membres de gauche mesurent (de façon partielle) la déformation localement mesurable de la chronogéométrie de l’espace-temps, alors que les dix membres de droite contiennent la source de cette déformation, c’est-à-dire la présence d’une distribution de tensions, d’impulsion et de masse-énergie. Comme nous l’avons déjà dit, ces dix équations, reliant une déformation à la présence de tensions exercées au sein du milieu, sont analogues aux équations fondamentales de l’élasticité d’un milieu peu déformé.



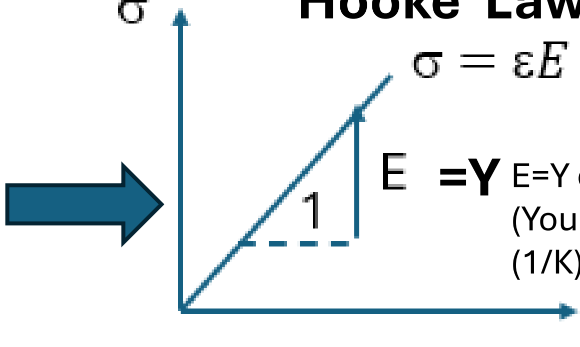


Elastic behaviour proven with A. Eddington 1919

Stress (T)

Hooke Law

$\sigma = \varepsilon E$



=Y E=Y of vacuum (Young’s modulus) (1/K)

Strains (D(g)

State of the art

$$\boxed{D(g) = K\,.T}$$

Analogy of the elastic medium in weak field: strains/stress of the vacuum= Σsheets

Metric of the space time $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

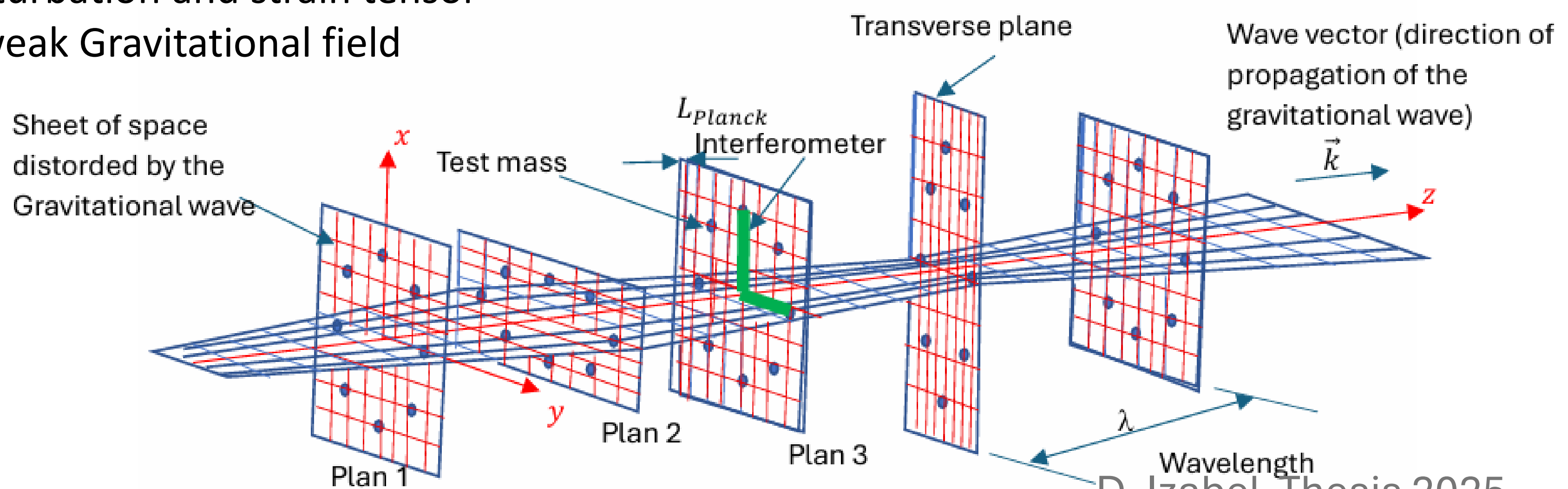
Independent successive plans $h_{ij}^{TT}(t-z)$

Analogy between metric $h_{\mu\nu} = 2\varepsilon_{\mu\nu}$ perturbation and strain tensor in weak Gravitational field

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4}(\cancel{T_{\mu\nu}} + \boxed{t_{\mu\nu,el}})$$

Curvature (1/m²) = Flexibility (1/N) x density of mass energy (N.m/m³)

New: necessary? Aim of the thesis => elastic analogy



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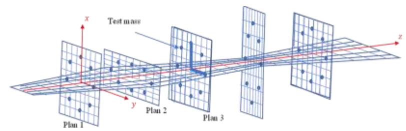
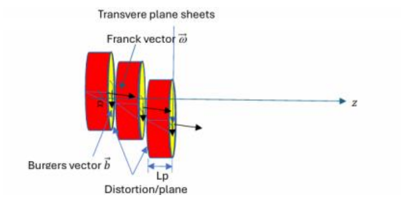
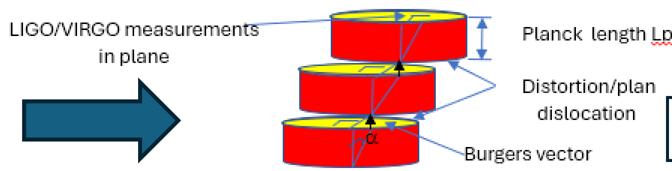
But to have an elastic medium need to reconnect the sheets together

Possible by going to plastic regime, defect theory , burger vectors,

Proposed solution: reconstitute a medium (gluing of the sheets) via addition of geometric torsion in GR => image crystallography defect theory







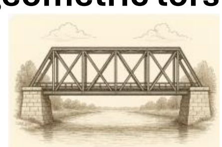
Space time

Riemann curvature corrected with **geometric torsion**

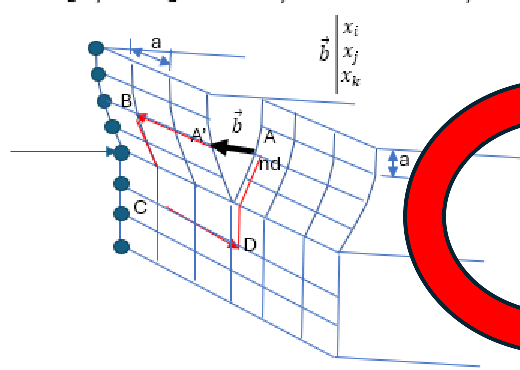
$$[D_\mu , D_\nu]V^\rho = D_\mu D_\nu V^\rho - D_\nu D_\mu V^\rho = R^\rho_{\sigma\mu\nu}V^\sigma \boxed{-2\Gamma^\lambda_{[\mu\nu]}} D_\lambda V^\rho$$

Burger's vector

$$\boxed{db^\mu = -\Gamma_{\nu\lambda}^{\ \ \mu} dA^{\nu\lambda}}$$



Atom network



Geometric torsion

Cosmological crystal

$$\boxed{-T^\lambda_{\mu\nu} = -2\Gamma^\lambda_{[\mu\nu]}}$$

With defects

Geometric torsion to connect the sheets (image of the **defect theory in crystallography**

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GR modification : Einstein-Cartan approach with Torsion Field added

2) Gravitational wave and memory effect in modern physics

It's remain residual strains in the Riemann Geometry **(in classical GR)** after the passage of a Gravitational wave!=> Effective approach, Plasticity of the space time

2.1) The measurements and associated physical facts

A memory effect is effectively measured

EDITORS' SUGGESTION

Detecting Gravitational-Wave Memory with LIGO: Implications of GW150914

Paul D. Lasky[1,*], Eric Thrane[1], Yuri Levin[1], Jonathan Blackman[2], and Yanbei Chen[3]

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Phys. Rev. Lett. **117**, 061102 – **Published 5 August, 2016**

DOI: https://doi.org/10.1103/PhysRevLett.117.061102

Detecting gravitational-wave memory with LIGO: implications of GW150914

Paul D. Lasky,[1,*] Eric Thrane,[1] Yuri Levin,[1] Jonathan Blackman,[2] and Yanbei Chen[3]

[1] *Monash Centre for Astrophysics, School of Physics and Astronomy, Monash University, VIC 3800, Australia*
[2] *TAPIR, Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California 91125, USA*
[3] *Theoretical Astrophysics, California Institute of Technology, Pasadena, CA 91125, USA*

It may soon be possible for Advanced LIGO to detect hundreds of binary black hole mergers per year. We show how the accumulation of many such measurements will allow for the detection of gravitational-wave memory: a permanent displacement of spacetime that comes from strong-field, general relativistic effects. We estimate that Advanced LIGO operating at design sensitivity may be able to make a signal-to-noise ratio 3(5) detection of memory with $\sim 35\,(90)$ events with masses and distance similar to GW150914. We highlight the importance of incorporating higher-order gravitational-wave modes for parameter estimation of binary black hole mergers, and describe how our methods can also be used to detect higher-order modes themselves before Advanced LIGO reaches design sensitivity.

The memory effect is effectively measured

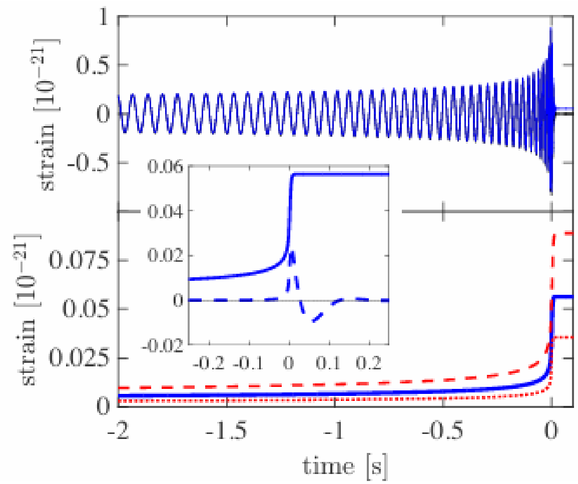

FIG. 2: Gravitational-wave strain time series using parameters consistent with GW150914 [1, 17]. The top panel shows the strain time series with GW memory (blue curve) and without (black). The bottom panel shows only the memory-induced strain series, where the blue curve uses the maximum likelihood parameters for GW150914 [1, 17]. The red dotted and dashed curves are binaries at the same distance (410 Mpc) and with the same orientation ($\theta = 140°$), but equal mass binaries with $m_{1,2} = 20\,M_{\odot}$ and $50\,M_{\odot}$ respectively (cf. $65\,M_{\odot}$ for the blue curve). Inset: the solid blue curve shows a zoomed-in version of the blue curve from the bottom panel, while the dashed curve is after a high-pass filter to show the signal visible in aLIGO.

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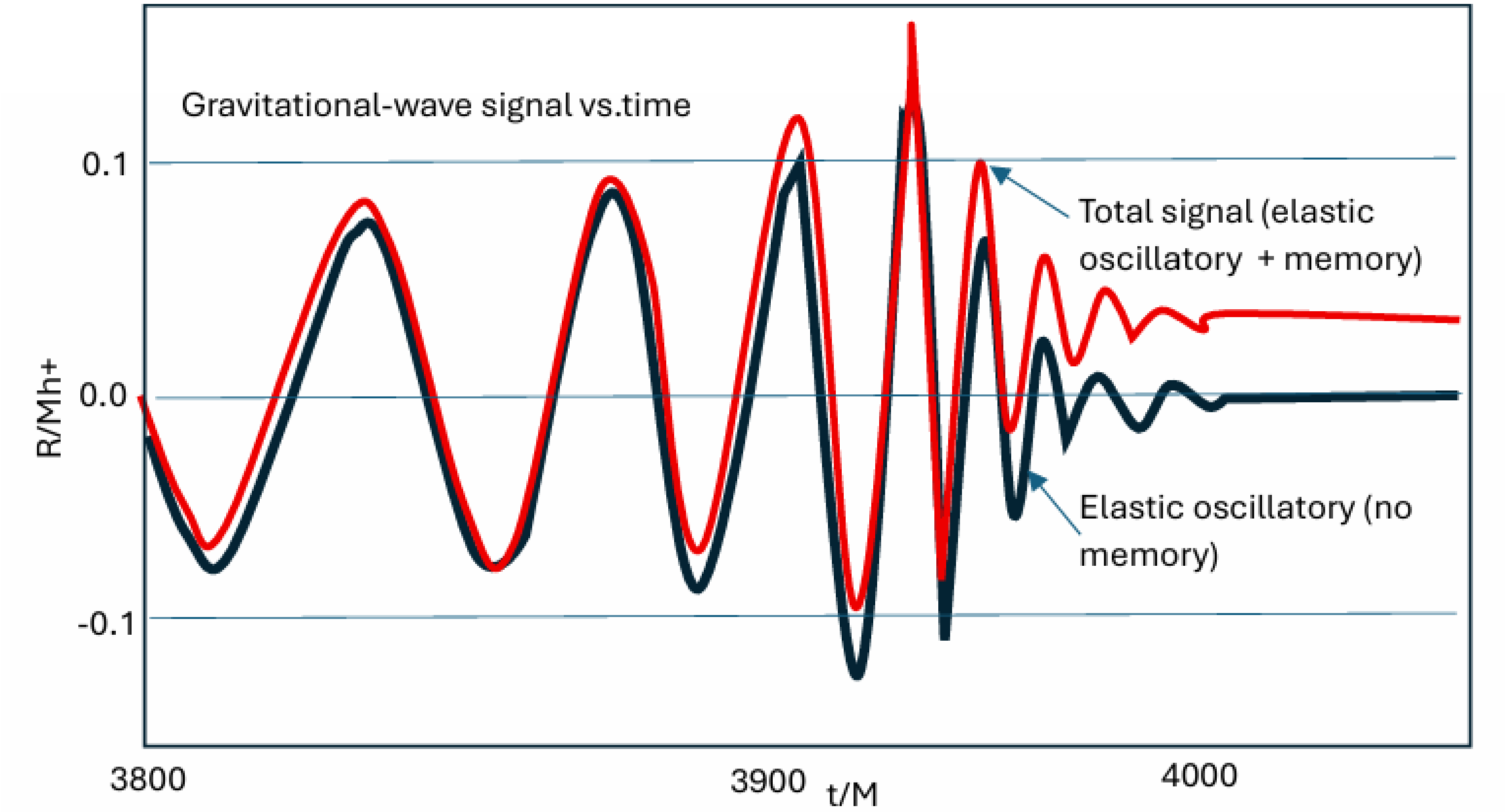
It's verry small but not 0 this memory

A memory effect is predicted by physicists



Now there is a storage of permanent strains …in geometry or in Elasther???

2.2 The physical theory of the memory effect: Bondy Sachs metric

Bondi-Sachs coordinates used for GW +memory effect by physicists

The Bondi-Sachs metric

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} + \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + D^2C_{zz}dudz + \frac{1}{r}\left(\frac{4}{3}N_z - \frac{1}{4}\partial_z(C_{zz}C^{z\bar{z}})\right)dudz + c.c. + \cdots$$

Different terms of the metric

- $u = t - r$ retarded time
- $\gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$ unit metric of S^2
- r radial coordinate frome the source

Classical metric → 📷 snapshot
Bondi–Sachs → 🎥 movie with a beginning and an end

$z = e^{i\phi}\tan(\theta/2)$ complex stereographic coordinate and $\bar{z}$ its conjugate $z = e^{-i\phi}\tan(\theta/2)$

In analogy with the transverse-traceless (TT) gauge, the Bondi-Sachs metric near future null infinity $\mathcal{I}^+$ can be written as:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Where :

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & r^2\gamma_{z\bar{z}} \\ 0 & 0 & r^2\gamma_{z\bar{z}} & 0 \end{pmatrix}$$

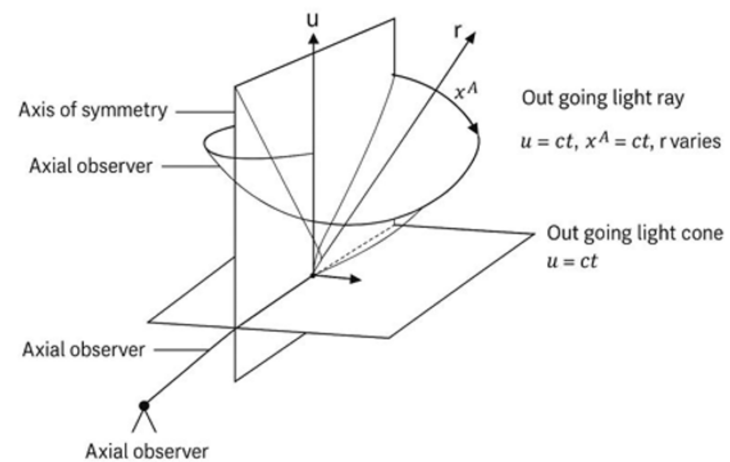
The perturbation $h_{\mu\nu}$ contains:

$$h_{zz} = rC_{zz}, h_{\bar{z}\bar{z}} = rC_{\bar{z}\bar{z}}$$

which correspond to spin +2 and -2 modes respectively.

The news tensor $N_{zz} = \partial_u C_{zz}$ acts as a strain-rate (u associates at time), and the memory is:

$$\Delta C_{zz} = \int_{-\infty}^{+\infty} N_{zz}\, du$$



Physics – metric – geometric view

- $u = t - r$ retarded time
- $\gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$ unit metric of S^2
- r radial coordinate frome the source

- the transverse plane x, y = plane tangent to the sphere at a point

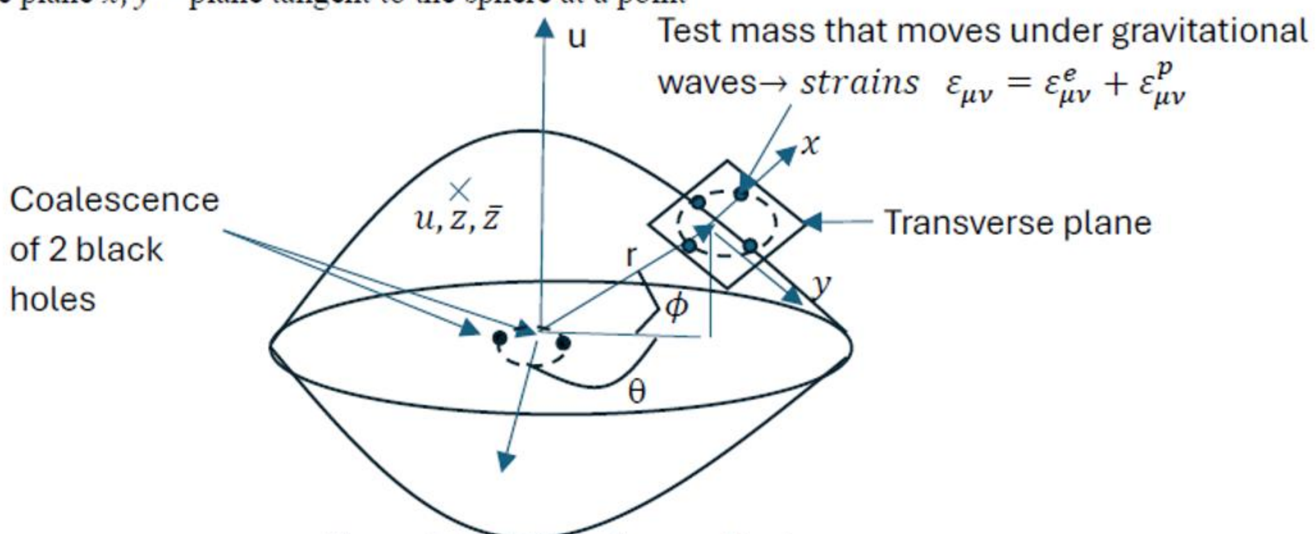


Figure 8: Bondi-Sachs coordinates

Tables of corespondances

Bondy Sachs metric / perturbation metrics

TT gauge	Bondi–Sachs
$h_{11} = h_+$	$C_{\theta\theta}$
$h_{22} = -h_+$	$-C_{\theta\theta}$
$h_{12} = h_\times$	$C_{\theta\phi}$

$$\Delta C_{AB} = C_{AB}(+\infty) - C_{AB}(-\infty)$$

$$\Delta h_+, \Delta h_\times$$

TT gauge	Bondi–Sachs
h_+	partie réelle de C_{zz}
$h_\times$	partie imaginaire de C_{zz}
h_{11}, h_{22}, h_{12}	codés dans C_{AB}

$$C_{zz} = h_+ - ih_\times$$

$$C_{\bar{z}\bar{z}} = h_+ + ih_\times$$

2 memory effects predicted in classical GR:

In the transverse planes

Displacements

$$\Delta^+ s^{\bar z} = \frac{\gamma^{z\bar z}}{2r}\Delta^+ C_{zz} s^{\bar z},$$

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Phys. Rev. Lett. **67**, 1486 – **Published 16 September, 1991**

DOI: https://doi.org/10.1103/PhysRevLett.67.1486

Spin memory and angular momentum flux

$$\Delta^+ u = \frac{1}{2\pi L}\int du \oint_C \left(D^z C_{zz} dz + D^{\bar z} C_{\bar z\bar z} d\bar z\right).$$

New Gravitational Memories

Sabrina Pasterski, Andrew Strominger and Alexander Zhiboedov

Center for the Fundamental Laws of Nature
Harvard University, Cambridge, MA 02138 USA

3) The problematic of the memory effect with the analogy of the elastic medium

Why Plasticity Is Required

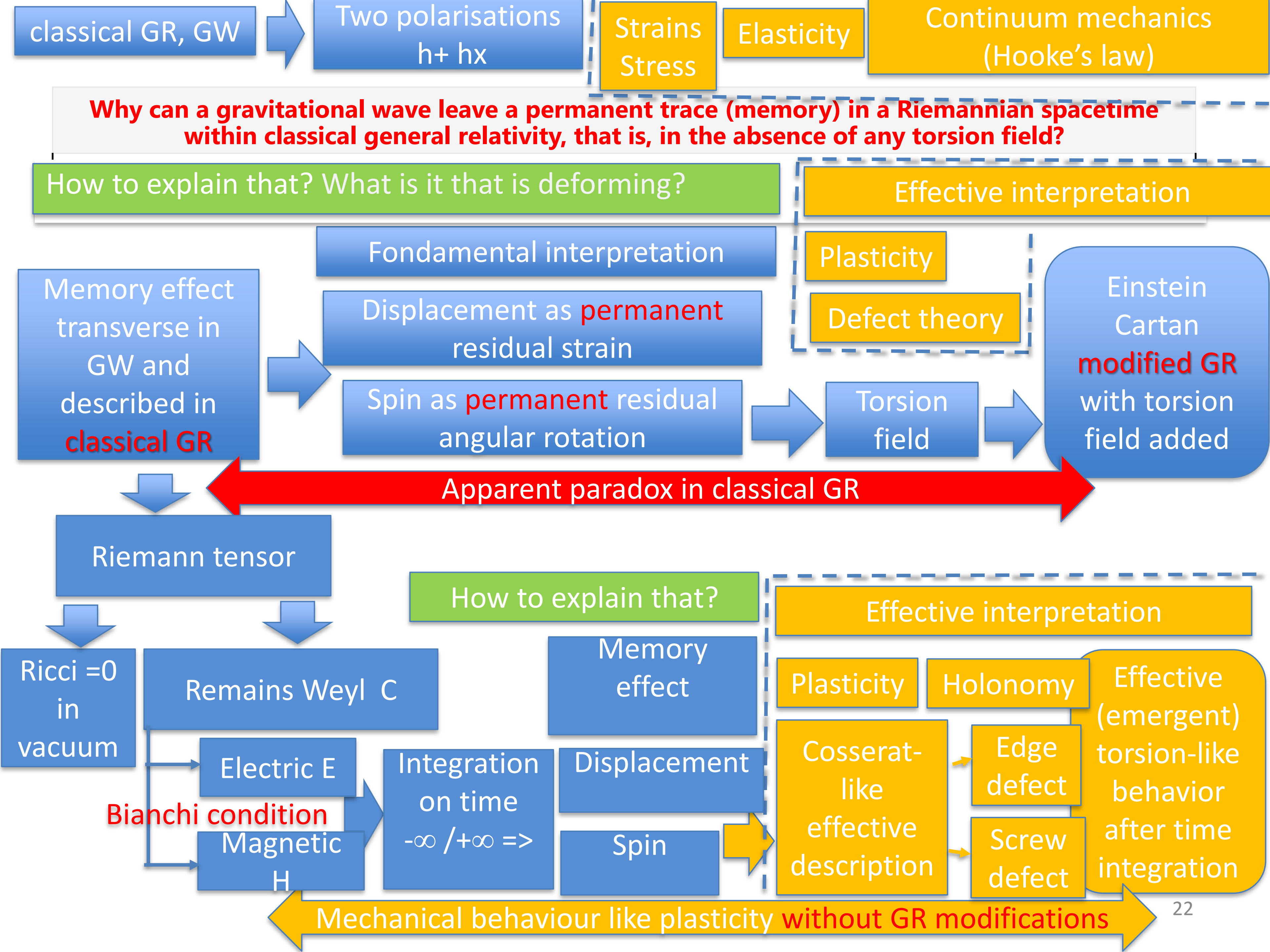
- Elastic response would relax to zero the strains

Fondamental

  – Memory effects are irreversible

Effective

  – Plastic spacetime behavior


classical GR, GW
Two polarisations h+ hx
Strains Stress
Elasticity
Continuum mechanics (Hooke's law)
Why can a gravitational wave leave a permanent trace (memory) in a Riemannian spacetime within classical general relativity, that is, in the absence of any torsion field?
How to explain that? What is it that is deforming?
Effective interpretation
Fondamental interpretation
Plasticity
Defect theory
Memory effect transverse in GW and described in classical GR
Displacement as permanent residual strain
Spin as permanent residual angular rotation
Torsion field
Einstein Cartan modified GR with torsion field added
Apparent paradox in classical GR
Riemann tensor
How to explain that?
Effective interpretation
Ricci =0 in vacuum
Remains Weyl C
Memory effect
Plasticity
Holonomy
Effective (emergent) torsion-like behavior after time integration
Electric E
Bianchi condition
Magnetic H
Integration on time -∞ /+∞ =>
Displacement
Spin
Cosserat-like effective description
Edge defect
Screw defect
Mechanical behaviour like plasticity without GR modifications

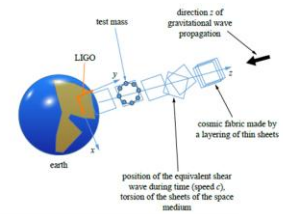
4) Gravitational wave and memory effect=> possible connection with mechanics in plasticity

Gravitational Memory as Elastic–Plastic Spacetime

- Gravitational waves: transverse elastic shear
  - Gravitational memory:

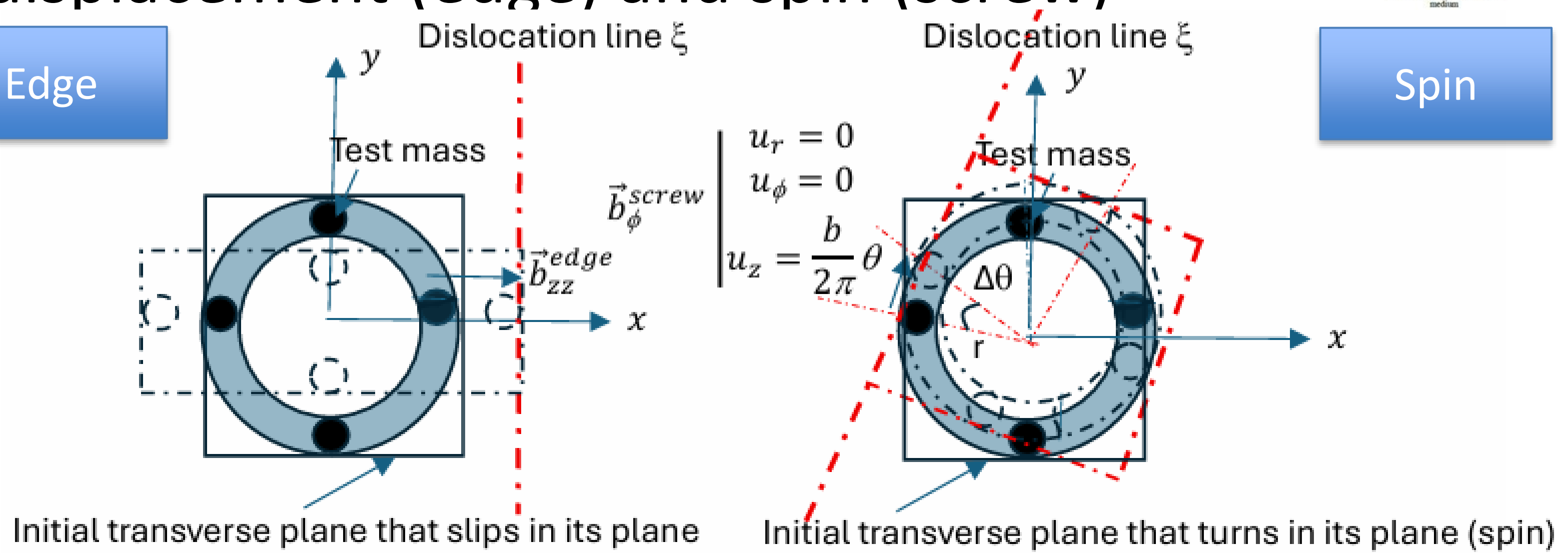
    Analogy with permanent (plastic) deformation
  - Two distinct memory effects:

    displacement (edge) and spin (screw)

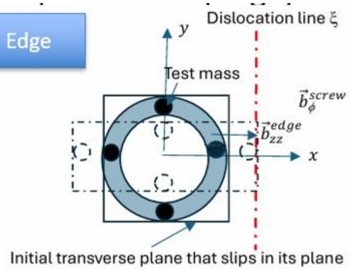


Edge

Spin



Edge



Displacement memory and effective edge dislocation

We postulate the following Key correspondence: The permanent displacement $\Delta^+ s^{\bar{z}}$ corresponds to an edge-type Burgers vector in the transverse plane (Figure 4a). From the strain analogy $\epsilon_{zz} = \frac{1}{2} h_{zz} = \frac{r}{2} C_{zz}$, we define an effective in-plane displacement $u_{zz} \equiv \epsilon_{zz}$. Applying the Burgers definition:

We define burger vector and edge dislocation in Bondi/Sachs metric

$$\boxed{B_{zz} = \oint_C \partial\, u_{zz} = \frac{r}{2} \oint_C \partial\, C_{zz} = \frac{r}{2} \Delta C_{zz}}$$

The normalized in-plane Burgers vector is :

$$b_{zz}^{\text{edge}} \equiv \frac{2}{r} B_{zz} = \Delta C_{zz}$$

Hypothesis in an effective medium in plasticity state

The derivation of this postulate is given in appendix A.

In the Bondi framework, displacement memory can be written as:

$$\Delta x^A = \frac{1}{2} \int_{-\infty}^{+\infty} \partial_u h_{AB}\, x^B\, du$$

In the TT gauge, $h_{AB} \sim C_{AB}/r$, so:

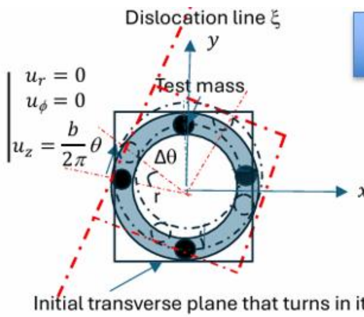
$$\boxed{\Delta^+ s^{\bar{z}} = \frac{\gamma_{z\bar{z}}}{2r} \Delta^+ C_{zz} s^{\bar{z}}}$$

The physical interpretation is so the following. A translation perpendicular to the propagation direction within the transverse plane (Figure 4a), exactly analogous to an edge dislocation's Burgers vector perpendicular to its line.

Fact: mathematical analogy between the edge Burger vector and memory displacement

	Field	Formula	Interpretation	Nature of Holonomy
Fondamental	Relativity – displacement memory	$\Delta x^A = \frac{1}{2}\int \partial_u h_{AB} x^B du$	Permanent displacement after gravitational wave	Time holonomy of metric
Effective	Defectology – edge dislocation	$b^i = \int \epsilon^{\mu\nu}\, \partial_\mu\, \partial_\nu u^i dA$	Failure to close contour in crystal	Spatial holonomy of displacement field

Spin memory and the screw dislocation



Spin

ılate the following key correspondence: Spin memory $\Delta^{+}u$ corresponds to a screw-type Burgers vector
l in the transverse plane (Figure 4b).
a circular contour of radius r in the transverse plane. The spin memory produces a tangential
nent:

$$s_\phi = r\Delta\theta$$

where $\Delta\theta$ is the effective rotation angle encoded in the spin memory. This defines an effective tangential Burgers vector (see Appendix B for derivation):

$$b_\phi^{\mathrm{screw}} = r\Delta\theta$$

For a crystal screw dislocation in the transverse plane:

We define burger vector and screw dislocation in Bondi/Sachs metric

$$\boxed{b_\phi = \oint_C \partial_\phi u_z \, d\phi = \int_0^{2\pi} \frac{b}{2\pi} \, d\phi = b}$$

where $u_z = \frac{b}{2\pi}\phi$ is the displacement along z.
For gravitational spin memory:

$$b_\phi^{\mathrm{eff}} = r\Delta\theta = \frac{r}{2\pi}\int du \oint_C D^z \, C_{zz} dz$$

The derivation of this postulate is given in appendix B.

Both are tangential Burgers vectors, measuring an accumulated translation around a closed contour, not a rotation (which would correspond to a Frank vector).
From equation (2), we have:

$$\Delta\theta \sim \frac{1}{2\pi r}\int du \oint_C D^z \, C_{zz} dz$$

Thus:

$$b_\phi^{\mathrm{screw}} \sim \frac{1}{2\pi}\int du \oint_C D^z \, C_{zz} dz$$

The spin memory operational formula is:

$$\Delta^{+}u = \frac{1}{2\pi L}\int du \oint_C (D^z C_{zz} dz + D^{\bar{z}} C_{\bar{z}\bar{z}} d\bar{z})$$

This can be rewritten as a tangential holonomy:

$$\boxed{\Delta\tau = \oint_C \Delta\,\Gamma_A dx^A \sim \int du \, \epsilon^{AB} \, \partial_A N_B}$$

Hypothesis in an effective medium in plasticity state

Fact: mathematical analogy between screw dislocation and spin memory

	Field	Formula	Interpretation	Nature of Holonomy
Fondamental	Relativity – Spin Memory	$\Delta\tau = \frac{1}{2\pi L}\int du \oint_{C} D^{z}\, C_{zz}\, dz$	Permanent time delay between counter-circling beams after a gravitational wave	Spacetime tangential holonomy
Effective	Defectology – Screw Dislocation	$b_{\phi} = \oint_{C} \partial_{\phi}\, u_z\, r d\phi = \int_{\Sigma} \epsilon^{\mu\nu}\, \partial_{\mu}\, \partial_{\nu} u_z\, dA$	Tangential closure failure around a dislocation line	Spatial tangential holonomy

Table 2: Analogy between Screw Burgers Vector and Spin Memory

Proposed solution

5) Weyl–Cosserat Elasticity and Gravitational Memory: An Effective Microstructured Model of Spacetime

Weyl Tensor in Vacuum

On any $n \geq 4$pseudo-Riemannian manifold (M, g),

Riemann=Weyl (vacuum strain)+Ricci ((loaded strains)

$$R_{\mu\nu\alpha\beta} = C_{\mu\nu\alpha\beta} + \frac{1}{n-2}\left(g_{\mu[\alpha}R_{\beta]\nu} - g_{\nu[\alpha}R_{\beta]\mu}\right) - \frac{R}{(n-1)(n-2)}\, g_{\mu[\alpha}g_{\beta]\nu}$$

Here, $C_{\mu\nu\alpha\beta}$is the traceless, conformally invariant part of curvature. In vacuum in General Relativity the Ricci tensor is null ($R_{\mu\nu} = 0$), the full curvature reduces to Weyl, and gravitational shear waves are pure Weyl.

In vacuum , Ricci tensor =0,
remain Weyl tensor

$$R_{abcd} = C_{abcd} \;+\; \left(g_{a[c}R_{d]b} - g_{b[c}R_{d]a}\right) \;-\; \frac{1}{3}R\, g_{a[c}g_{d]b}$$

où :

- C_{abcd} = Weyl tensor,
- R_{ab} = Ricci tensor,
- R = Ricci scalar.

- Ricci-flat spacetime: $R_{abcd} = C_{abcd}$

Gravitational waves entirely encoded in Weyl tensor

The Weyl tensor

La courbure n'agit pas sur des vecteurs, mais sur des plans.

In 4 spacetime dimensions, the space of 2-forms has dimension **6**, therefore:

$$\boxed{\text{Weyl tensor} \;\equiv\; 6 \times 6 \text{ matrix}}$$

We introduce a canonical basis of bivectors:

$$(01,\ 02,\ 03 \mid 23,\ 31,\ 12)$$

- The **first three** components are time–space bivectors
- The **last three** are purely spatial bivectors

By definition:

Equivalent electric field

$$\boxed{E_{ij} = C_{0i0j}}$$

Equivalent magnetic field

$$\boxed{H_{ij} = \tfrac{1}{2}\,\epsilon_{ikl}\,{C^{kl}}_{0j}}$$

- E_{ij}: **tidal / shear deformations**
- H_{ij}: **rotation / frame dragging**

Both E and H are:

- symmetric
- traceless
- 3×3 matrices

Type de bivecteur	Sens physique
$dt \wedge dx^i$	Déformation (partie électrique E)
$dx^i \wedge dx^j$	Rotation (partie magnétique H)

Similar structure between The Weyl tensor/stress tensor

Tensor indices	Voigt index
11	1
22	2
33	3
$23, 32$	4
$31, 13$	5
$12, 21$	6

$$\varepsilon = \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{pmatrix}, \qquad \sigma = \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix}$$

Weyl tensor

stress tensor

$$\boxed{\sigma_I = C_{IJ}\,\varepsilon_J}$$

$$\boxed{\mathcal{C} = \begin{pmatrix} E & H \\ H & -E \end{pmatrix}}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \qquad \mu = \frac{E}{2(1+\nu)}$$

$$\boxed{C_{IJ} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{pmatrix}}$$

$$\mathcal{C} = \begin{pmatrix} E_{11} & E_{12} & E_{13} & H_{11} & H_{12} & H_{13} \\ E_{21} & E_{22} & E_{23} & H_{21} & H_{22} & H_{23} \\ E_{31} & E_{32} & E_{33} & H_{31} & H_{32} & H_{33} \\ H_{11} & H_{12} & H_{13} & -E_{11} & -E_{12} & -E_{13} \\ H_{21} & H_{22} & H_{23} & -E_{21} & -E_{22} & -E_{23} \\ H_{31} & H_{32} & H_{33} & -E_{31} & -E_{32} & -E_{33} \end{pmatrix}$$

Same structure

$$\boxed{C = \begin{pmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{pmatrix}}$$

"The Weyl tensor has the same mathematical structure as the stiffness tensor C_{ijkl} in continuum mechanics."

Practically the components of the Weyl tensor

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$C_{abcd} \sim \partial_a \partial_c h_{bd} + \partial_b \partial_d h_{ac} - \partial_a \partial_d h_{bc} - \partial_b \partial_c h_{ad}$$

$$E_{ab} = C_{acbd} u^c u^d \sim \partial_a \partial_c h_{bd}\, u^c u^d + \ldots$$

Electric and Magnetic Weyl Parts

Weyl tensor can be split in equivalent electric and magnetic tensor

- Electric part (tidal shear):

$E_{ij} = C_{i0j0}$

$$E_{ab} = C_{acbd}u^c u^d.$$

These are the components of the 4-velocity vector of an observer:

$$u^a = \frac{dx^a}{d\tau}$$

Physically:

- u^a is the tangent vector to the worldline
- It is normalized: $u^a u_a = -1$
- It represents the observer who "measures" the field

- Magnetic part (frame dragging):

$H_{ij} = (1/2)\, \varepsilon_{ikl}\, C_{klj0}$

$$H_{ab} = \frac{1}{2}\epsilon^{mn}_{ac} C_{mnbd}u^c u^d$$

u^a is a 4-velocity field, that is a field of vectors tangent to the worldlines of observers

The structure of electric and magnetic field are similar to C_{ijkl} used in mechanics

In 3D elasticity, minor symmetries $C_{ijkl} = C_{jikl} = C_{ijlk}$ and major symmetry $C_{ijkl} = C_{klij}$ allow one to represent C_{ijkl} as a symmetric 6×6 matrix in Voigt notation, acting on the 6D space of symmetric strains. The energy density $E = \frac{1}{2} C_{ijkl}\varepsilon_{ij}\varepsilon_{kl}$ ensures symmetry and positive-definiteness for stable materials; only 21 independent components remain in the general anisotropic case.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Effective

Continuum mechanics

Space strain from displacement field

$$\varepsilon_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$$

$$\sigma_{ij} = \boxed{C_{ijkl}}\,\varepsilon_{kl}$$

$$C_{ijkl} = C_{jikl} = C_{ijlk} = C_{klij}$$

Similar tensorial structure

Fondamental

$$C_{abcd} \sim \partial_a\partial_c h_{bd} + \partial_b\partial_d h_{ac} - \partial_a\partial_d h_{bc} - \partial_b\partial_c h_{ad}$$

Weyl electric and magnetic parts

$$E_{ab} = C_{acbd}u^c u^d \sim \partial_a\partial_c h_{bd}\, u^c u^d + \dots$$

Electric part (tidal deformation)

$$E_{ab} = \boxed{C_{acbd}}\,u^c u^d$$

Magnetic part (frame-dragging / rotational effects)

$$H_{ab} = \frac{1}{2}\epsilon_{ac}^{mn}\boxed{C_{mnbd}}\,u^c u^d \qquad H_{ab} = \frac{1}{2}\epsilon_{acmn}C^{mn}{}_{bd}u^c u^d$$

The Weyl tensor in general relativity (in a vacuum) and the C_{ijkl} stiffness tensor in continuum mechanics have exactly the same mathematical structure: they are self-adjoint operators acting on a 6-dimensional space.

$$H_{ab} \sim \epsilon\,\partial^2 h$$

While strain in elasticity arises from first derivatives of the displacement field, the Weyl tensor encodes second-order geometric deformations of spacetime.

$$H_{ij} \sim \epsilon_{ikl}\,\partial_k \dot{h}_{lj}$$

Maxwell-like Weyl Equations

- In vacuum Time evolution from Bianchi identities in vacuum $\nabla^d C_{abcd} = 0$ curvature conservation applied at the Weyl tensor:we calculate

$$\nabla^d C_{abcd} = \nabla_{[b} R_{a]c} + \frac{1}{6} g_{c[b} \nabla_{a]} R$$

  – $\dot{E}_{ij}$ = curl(H_{ij})
  – $\dot{H}_{ij}$ = – curl(E_{ij})

$$(\mathrm{curl}\, S)_{ij} = \varepsilon_i{}^{kl} \nabla_k S_{lj}$$

$$(\mathrm{curl}\, S)_{\langle ij \rangle} = \frac{1}{2} \left(\varepsilon_i{}^{kl} \nabla_k S_{lj} + \varepsilon_j{}^{kl} \nabla_k S_{li} \right)$$

(.) = Projected covariant derivative along the 4-velocity u^a

Projected covariant derivative along the 4-velocity u^a

In vacuum, the Bianchi identities $\nabla^d C_{abcd} = 0$ yield a set of Maxwell-like propagation equations [8]:

$$\dot{E}_{\langle ab \rangle} = \mathrm{curl}\, H_{ab}, \dot{H}_{\langle ab \rangle} = -\mathrm{curl}\, E_{ab}$$

In vacuum, the Bianchi identities ensure that spacetime curvature cannot be locally created or destroyed. They force the electric and magnetic parts of the Weyl tensor to be dynamically coupled: variations in tidal shear necessarily induce frame-dragging, and vice versa. This conservation-type structure is the gravitational analogue of compatibility conditions in continuum mechanics.

Equivalent Maxwell equation

$$\boxed{\dot{E}_{ij} = \frac{1}{2} \left(\varepsilon_i{}^{kl}\, \partial_k H_{lj} + \varepsilon_j{}^{kl}\, \partial_k H_{li} \right)}$$

$$\boxed{\dot{E}_{ij} = \mathrm{curl}\, H_{ij}, \qquad \dot{H}_{ij} = -\mathrm{curl}\, E_{ij}}$$

$$\boxed{\dot{H}_{ij} = -\frac{1}{2} \left(\varepsilon_i{}^{kl}\, \partial_k E_{lj} + \varepsilon_j{}^{kl}\, \partial_k E_{li} \right)}$$

Gravitational Memory Effects

- Memory = permanent metric change after wave passage
  - Two independent but related effects
  - Both arise from time-integrated Weyl curvature

Fondamental

Mémoire	Observable
Displacement (edge)	$\Delta h_{ij}^{TT} = 2\int E_{ij}\, dt$
Spin (screw)	$\int H_{ij}\, dt \neq 0$

Electric part of Weyl

Magnetic part of Weyl

Displacement Memory

Connection between Electric Weyl equivalent tensor and Displacement Memory (Edge-Type)

- Permanent elongation / shortening of transverse distances

Electric part of Weyl tensor

↓

Connected with a permanent displacement

Fondamental

$$\ddot{h}_{ij}^{TT} = -2E_{ij}$$

$$\Delta h_{ij}^{TT} = 2\int E_{ij}\,dt$$

↓

$$\Delta x_i = \frac{1}{2}\Delta h_{ij}x^j$$

Derivation of $\boxed{\ddot{h}_{ij}^{TT} = -2E_{ij}}$

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\partial_\rho\partial_\nu h_{\mu\sigma} + \partial_\sigma\partial_\mu h_{\nu\rho} - \partial_\sigma\partial_\nu h_{\mu\rho} - \partial_\rho\partial_\mu h_{\nu\sigma}\right)$$

$$R_{i0j0} = \frac{1}{2}\left(\partial_j\partial_0 h_{i0} + \partial_0\partial_i h_{0j} - \partial_0\partial_0 h_{ij} - \partial_j\partial_i h_{00}\right)$$

Gauge condition

$$h_{0\mu} = 0 \quad \text{et} \quad h_{ij} = h_{ij}^{TT}$$

- $h_{00} = 0$
- $h_{0i} = 0$

$$R_{i0j0} = \frac{1}{2}\left(\partial_j\partial_0 h_{i0} + \partial_0\partial_i h_{0j} - \partial_0^2 h_{ij} - \partial_j\partial_i h_{00}\right)$$

- $h_{i0} = 0 \Rightarrow$ premier terme = 0
- $h_{0j} = 0 \Rightarrow$ deuxième terme = 0
- $h_{00} = 0 \Rightarrow$ dernier terme = 0

$$\partial_0 = \frac{\partial}{\partial t} \Rightarrow \partial_0^2 = \frac{\partial^2}{\partial t^2}$$

$$R_{i0j0} = -\frac{1}{2}\partial_0^2 h_{ij}$$

Classical GR

$$\boxed{R_{i0j0} = -\frac{1}{2}\ddot{h}_{ij}^{TT}}$$

How to connect Weyl with burger vector ?

1. Starting point: Weyl tensor (vacuum)

$$\boxed{E_{ij} = R_{i0j0}}$$

- The electric part of the Weyl tensor equals the tidal Riemann component.

2. Relation with the metric perturbation (TT gauge)

$$\boxed{R_{i0j0} = -\frac{1}{2}\,\ddot{h}_{ij}^{TT}}$$

Therefore:

$$\boxed{H_{ij}^{TT} \equiv H_{ij}}$$

$$\boxed{E_{ij} = -\frac{1}{2}\,\ddot{h}_{ij}^{TT}}$$

3. Physical meaning

$$E_{ij} \sim \partial_t^2 h_{ij}^{TT}$$

The Weyl tensor represents the **acceleration of spacetime deformation**.

4. First time integration (strain rate)

$$\int_{-\infty}^{+\infty} E_{ij}(t)\,dt = -\frac{1}{2}\int_{-\infty}^{+\infty} \ddot{h}_{ij}^{TT}(t)\,dt$$

$$\int \ddot{h}(t)\,dt = \dot{h}(t) \qquad \int_{-\infty}^{+\infty} \ddot{h}(t)\,dt = \dot{h}(+\infty) - \dot{h}(-\infty)$$

$$\boxed{\int E_{ij}dt = -\frac{1}{2}\left[\dot{h}_{ij}^{TT}(+\infty) - \dot{h}_{ij}^{TT}(-\infty)\right]}$$

Usually (physical GW case)

$$\dot{h}_{ij}^{TT}(\pm\infty) = 0 \Rightarrow \int E_{ij}dt = 0$$

So the first integral does **not capture memory**

5. Gravitational memory (key step)

$$\boxed{\Delta h_{ij}^{TT} = h_{ij}^{TT}(+\infty) - h_{ij}^{TT}(-\infty)}$$

Equivalent form:

$$\boxed{\Delta h_{ij}^{TT} = \int_{-\infty}^{+\infty} \partial_u h_{ij}^{TT}\,du}$$

6. Link with Weyl (double integration)

Using:

$$E_{ij} = -\frac{1}{2}\ddot{h}_{ij}^{TT}$$

7. Physical displacement of test masses

$$\boxed{\Delta x_i = \frac{1}{2}\Delta h_{ij}^{TT} x^j}$$

8. Cosserat identification (key bridge)

$$\boxed{\beta_{ij} = \Delta h_{ij}^{TT}}$$

- Distortion tensor = gravitational memory

9. Burgers vector (defect closure)

$$\boxed{b_i = \oint \beta_{ij}\,dx^j}$$

Therefore:

$$\boxed{b_i = \oint \Delta h_{ij}^{TT}\,dx^j}$$

10. Full expression in terms of Weyl

$$\boxed{b_i = \oint \left(\int_{-\infty}^{+\infty}\int_{-\infty}^{t} E_{ij}(t')\,dt'\,dt\right) dx^j}$$

Edge Dislocation Interpretation

- Burgers vector lies in the transverse plane
  - Exact analogue of an edge dislocation
  - Generated by « **electric part » of Weyl Tensor**

Displacement memory = edge

Displacement memory = EDGE (Burgers)

Physical effect

- Permanent **elongation / shortening**
- Relative displacement of test masses

Fondamental

Equations

$$\varepsilon_{ikl} = \begin{cases} +1 & \text{permutation paire} \\ -1 & \text{permutation impaire} \\ 0 & \text{sinon} \end{cases}$$

$$\ddot{h}^{TT}_{ij} = -2E_{ij}$$

$$\Delta h^{TT}_{ij} = 2\int E_{ij}\,dt$$

$$\Delta x_i = \tfrac{1}{2}\Delta h_{ij} x^j$$

Defect interpretation

$$b_i = \oint \Delta h^{TT}_{ij}\, dx^j$$

Edge dislocation

Burgers vector **lies in the transverse plane**

$$H^{TT}_{ij} \equiv H_{ij}$$

$$\Delta H^{TT}_{ij} = H^{TT}_{ij}(+\infty) - H^{TT}_{ij}(-\infty)$$

$$b_i = \oint \Delta h^{TT}_{ij}\, dx^j$$

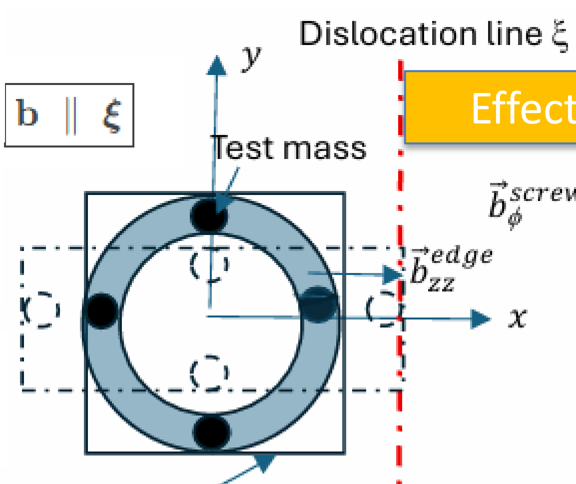
$$H_{ij} = \frac{1}{2}\,\varepsilon_{ikl}\, C^{kl}{}_{j0}$$

$$H_{ij} = \frac{1}{2}\,\varepsilon_{ikl}\,\partial_k \dot{h}^{TT}_{lj}$$

$$b_i = \oint \Delta h^{TT}_{ij}\, dx^j$$

$\mathbf{b} \parallel \xi$

Effective



Where is the correspondance with displacement memory?

(Weyl)

$E_{ij}(t)$

$$\int_{-\infty}^{+\infty} E_{ij}(t)\,dt$$

Bondi–Sachs

$$\boxed{E_{ij} \longleftrightarrow \partial_u N_{AB}}$$

$$N_{AB}(u) = \partial_u C_{AB}$$

$$\boxed{\int E_{ij}dt \longleftrightarrow N_{AB}}$$

$$\int_{-\infty}^{+\infty} \partial_u N_{AB}(u)\,du = N_{AB}(+\infty) - N_{AB}(-\infty)$$

$$\int \left(\int E_{ij}dt \right) dt \sim h_{ij}$$

$$\boxed{\Delta h_{ij}^{TT} \longleftrightarrow \Delta C_{AB}}$$

$$\Delta C_{AB} = \int_{-\infty}^{+\infty} N_{AB}(u)\,du$$

$$b_i = \oint \left(\int E_{ij}dt \right) dx^j$$

$$\int E_{ij}dt \longrightarrow N_{AB}$$

$$\boxed{b_i = \oint N_{AB}\,dx^B} \qquad \boxed{\Delta C_{AB} = \int N_{AB}du}$$

Étape	Ton modèle	Bondi
1	E_{ij}	$\partial_u N_{AB}$
2	$\int E_{ij}dt$	N_{AB}
3	$\int\int Edt$	C_{AB}
4	$b_i = \oint(\cdot)dx$	(pas présent)

Spin Memory

Magnetic part (completeness)

$$\boxed{H_{ij} = \frac{1}{2}\varepsilon_{ikl}C_{klj0}}$$

Maxwell-like equations:

$$\boxed{\dot{E}_{ij} = \mathrm{curl}(H) \quad , \quad \dot{H}_{ij} = -\mathrm{curl}(E)}$$

Spin memory:

$$\boxed{\omega_{ij} \sim \int H_{ij} dt}$$

Spin Memory (Screw-Type)

- Permanent relative rotation between worldlines
  - Generated by **« magnetic part » of the Weyl tensor**

Spin memory = screw

Fondamental

$$\boxed{\Delta\phi_i \propto \int H_{ij} x^j \, dt}$$

$$\boxed{H_{ij} = \frac{1}{2}\,\varepsilon_{ikl}\,C^{kl}{}_{j0}}$$

Screw Dislocation Interpretation

- Tangential Burgers vector around propagation axis

7 Spin memory = SCREW

Physical effect

- Permanent **relative rotation**
- Time delay between counter-orbiting worldline

Equation Fondamental

$$H_{ij} = \frac{1}{2}\,\varepsilon_{ikl}\,C^{kl}{}_{j0}$$

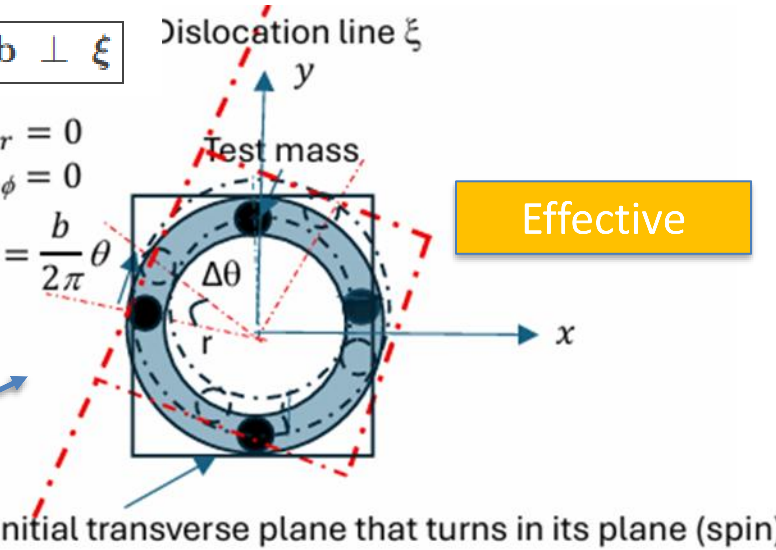
$$H_{ij} = \frac{1}{2}\,\varepsilon_{ikl}\,\partial_k \dot{h}^{TT}_{lj}$$

$$\Delta\phi_i \propto \int H_{ij} x^j \, dt$$

$$\mathbf{b} \perp \xi$$

$$\vec{b}^{\,screw}_{\phi} \;\Bigg|\; \begin{aligned} u_r &= 0 \\ u_\phi &= 0 \\ u_z &= \frac{b}{2\pi}\theta \end{aligned}$$

Effective



Defect interpretation

- Tangential Burgers vector
- Helical torsional structure

➡ **Screw dislocation**

➡ Rotation around the propagation axis

$$\Delta\phi_i \sim \int H_{ij}(t)\, x^j \, dt$$

$$b = \int r \, d\phi$$

Both displacement and spin memory connected in effective Cosserat medium: birth of effective torsion in classical GR

Link with Cosserat (Micropolar) Medium

Degrees of freedom:

Effective

$$u_i \quad \text{(displacement)}, \qquad \phi_i \quad \text{(micro-rotation)}$$

Distortion:

$$\boxed{\beta_{ij} = \partial_j u_i + \varepsilon_{ijk}\phi_k}$$

$$\boxed{\beta_{ij} = \varepsilon_{ij} + \omega_{ij}}$$

$$\boxed{\varepsilon_{ij} = \frac{1}{2}(\partial_j u_i + \partial_i u_j)} \qquad \boxed{\omega_{ij} = \frac{1}{2}(\partial_j u_i - \partial_i u_j)}$$

Connexion between β_{ij} and burger vector and Weyl

Cosserat medium

Defects & Burgers vector

Dislocation density:

$$\beta_{ij} = \partial_j u_i + \varepsilon_{ijk}\phi_k$$

Burgers vector:

$$\beta_{ij} = \Delta h_{ij}^{TT}$$

Effective

$$\alpha_{ij} = \varepsilon_{jkl}\partial_k\beta_{il}$$

$$b_i = \oint \beta_{ij}\, dx^j$$

GW

$$b_i = \int_S \left(\int du\, \dot{H}_{ij}\right) dS^j$$

$$b_i = \int_S \Delta H_{ij}\, dS^j$$

$$b_i = \oint \Delta h_{ij}^{TT}\, dx^j$$

Memory effect in displacement

$$\Delta h_{ij}^{TT} = \int_{-\infty}^{+\infty} \partial_u h_{ij}^{TT}\, du$$

Connection with burger vector version memory effect

$$b_i = \oint \Delta h_{ij}^{TT} dx^j = \int du \oint \partial_u h_{ij}^{TT} dx^j = \int_S \Delta H_{ij} dS^j$$

$$\Delta^+ s^{\bar{z}} = \frac{\gamma^{z\bar{z}}}{2r}\Delta^+ C_{zz} s^{\bar{z}},$$

$$\Delta x^A = \frac{1}{2}\int \partial_u h_{AB} x^B du$$

$$\beta_{ij} = \varepsilon_{ij} + \omega_{ij}$$

Cosserat / coframe

$$e^a = e^a_{\ \mu}(x)\, dx^\mu$$

a = indice interne (repère local)

μ = coordonnée espace-temps

$e^a_{\ \mu}$ = champ dynamique

Relation with Weyl tensor and memory

In vacuum $R_{\mu\nu} = 0$, the dynamics is entirely encoded in the Weyl tensor.

Time-integrated Weyl gives memory:

$$g_{\mu\nu} = \eta_{ab}\, e^a_\mu\, e^b_\nu$$

$$\varepsilon_{ij} \sim \int_{-\infty}^{+\infty} E_{ij}\, dt \quad \text{(displacement memory)}$$

$$\omega_{ij} \sim \int_{-\infty}^{+\infty} H_{ij}\, dt \quad \text{(spin memory)}$$

Therefore, the permanent coframe change is:

$$\Delta e^a \sim \int \delta e^a\, dt \Rightarrow \boxed{\Delta e^a = \int_{-\infty}^{+\infty} (\varepsilon^a{}_b + \omega^a{}_b)\, \bar{e}^b\, dt}$$

Coframe = time-integrated deformation.

Explanation effective torsion from coframe

Élie Cartan's torsion in geometry and in field theory, an essay

Friedrich W. Hehl[1] and Yuri N. Obukhov[2]
email: hehl@thp.uni-koeln.de, yo@thp.uni-koeln.de

$$g_{\mu\nu} = \eta_{ab}\, e^a_\mu\, e^b_\nu$$

$$\boxed{e^a = \text{base locale de 1-formes qui décrit la géométrie de l'espace-temps}}$$

$$\Delta g_{\mu\nu} \neq 0 \quad \Longrightarrow \quad \Delta e^a \neq 0$$

$$\boxed{\text{Memory} \;\Rightarrow\; \Delta e^a \;\Rightarrow\; T^a_{\text{eff}} = d(\Delta e^a)}$$

$$\varepsilon_{ij} \sim \int E_{ij} dt \quad , \quad \omega_{ij} \sim \int H_{ij} dt$$

5 Emergent torsion as defect of deformation

$$\boxed{\Delta e^a = \int (\text{strain} + \text{rotation})\, dt}$$

Given the permanent coframe variation Δe^a, define:

1-forme :

$$\boxed{\Delta e^a \sim \int (E + H)\, dt}$$

spatial exterior derivative $d(\Delta e^a)$

$$\boxed{T^a_{\text{eff}} := d(\Delta e^a)}$$

$$e^a = e^a{}_\mu\, dx^\mu$$

Properties:

$$d(\Delta e^a) = \partial_{[\mu}(\Delta e^a{}_{\nu]})\, dx^\mu \wedge dx^\nu$$

$$\Delta e^a = e^a_{\text{final}} - e^a_{\text{initial}}$$

- non-propagating,
- topological (holonomy),
- not an Einstein–Cartan torsion,
- measures **incompatibility of accumulated deformation**.

$$\Delta e^a \sim \int_{-\infty}^{+\infty} (\text{strain} + \text{rotation})\, dt$$

Plastic defect = non-exact coframe variation.

MMC	Gravitation
u^a	histoire dynamique
du^a	Δe^a (déformation mémorisée)
$d(du^a)$	$d(\Delta e^a) = T^a_{\text{eff}}$

So similar to Cartan Torsion but without additional field

$$\boxed{T^a = de^a + \omega^a{}_b \wedge e^b}$$

Effective Torsion from Memory: residual rotation when integration on a loop : holonomy

- Memory induces permanent coframe change Δe^a

$$\Delta e^a \sim \int (\text{distortion} + \text{rotation})\, dt.$$

$$\boxed{\Delta e^a \;=\; \int_{-\infty}^{+\infty} (\varepsilon^a{}_b + \omega^a{}_b)\, \bar{e}^b\, dt}$$

Effective

  - Effective torsion: $T^a_{\text{eff}} = d(\Delta e^a)$
  - Topological, non-propagating torsion, not Einstein Cartan torsion $T^a = de^a + \omega^a{}_b \wedge e^b.$

$$\varepsilon_{ij} \;\sim\; \int_{-\infty}^{+\infty} E_{ij}\, dt \qquad (\text{displacement memory})$$

$$\boxed{\text{coframe} \;\equiv\; \text{integrated deformation (strain + rotation)}}$$

$$\omega_{ij} \;\sim\; \int_{-\infty}^{+\infty} H_{ij}\, dt \qquad (\text{spin memory})$$

$$\boxed{e^a = \text{primitive géométrique de la déformation}}$$

$$\boxed{\Delta e^a \;\sim\; \int_{-\infty}^{+\infty} (E^a{}_b + H^a{}_b)\, dt}$$

6) Difference between Torsion of Einstein Cartan and effective torsion in Weyl Cosserat

Physical Picture

- Gravitational waves: elastic response
  - Memory effects: plastic scars of spacetime
  - Edge = elongation, Screw = spin

Einstein-Cartan describes a new fundamental spacetime, already "filled" with torsion/defects.

$$T_{\mu\nu}{}^{a} \sim S_{\mu\nu}{}^{a}$$

New field added at GR

I start with classical general relativity, without torsion, and after the passage of a gravitational wave through an effective elastoplastic medium,I end up with something resembling an Einstein-Cartan type torsion field, but without having modified general relativity itself.

$$\Delta e^{a} \neq d(\text{quelque chose})$$

GW creates local residual permanent defects that recreates torsion

$$T^{a}_{\text{eff}} = d(\Delta e^{a}) \neq 0$$

Fundamental Einstein-Cartan

The Einstein Cartan Torsion is a fundamental field.

- It is present before any wave.
- It is the direct source of quantum spin.
- It is part of the field equations.

$$\boxed{T_{\mu\nu}{}^{a} \sim S_{\mu\nu}{}^{a}}$$

Effective Torsion link with spin memory

The Effective torsion:

- appears after the fact
- only exists if a wave has passed
- is calculated from the integrated Weyl

It:

- does not propagate
- is not independent
- is not an Einstein source

It is: effective, emergent, topological

$$\Delta e^a \neq d(\text{quelque chose})$$

So we can have equivalent plasticity without Cartan torsion without additional spin field with classical GR

$$T^a_{\text{eff}} = d(\Delta e^a) \neq 0$$

7) Weyl–Cosserat Elasticity and Gravitational Memory:Potential consequences on fundamental physics

Open questions?

Are we sure that there are only 2 memory effects at the fundamental level?

What is the message of the effective level?

A third defect longitudinal not studied in physic at this day

In defect theory, there are 3 effects: edge and screw dislocations in planes, but also longitudinal dislocations with the Franck vector

Effective

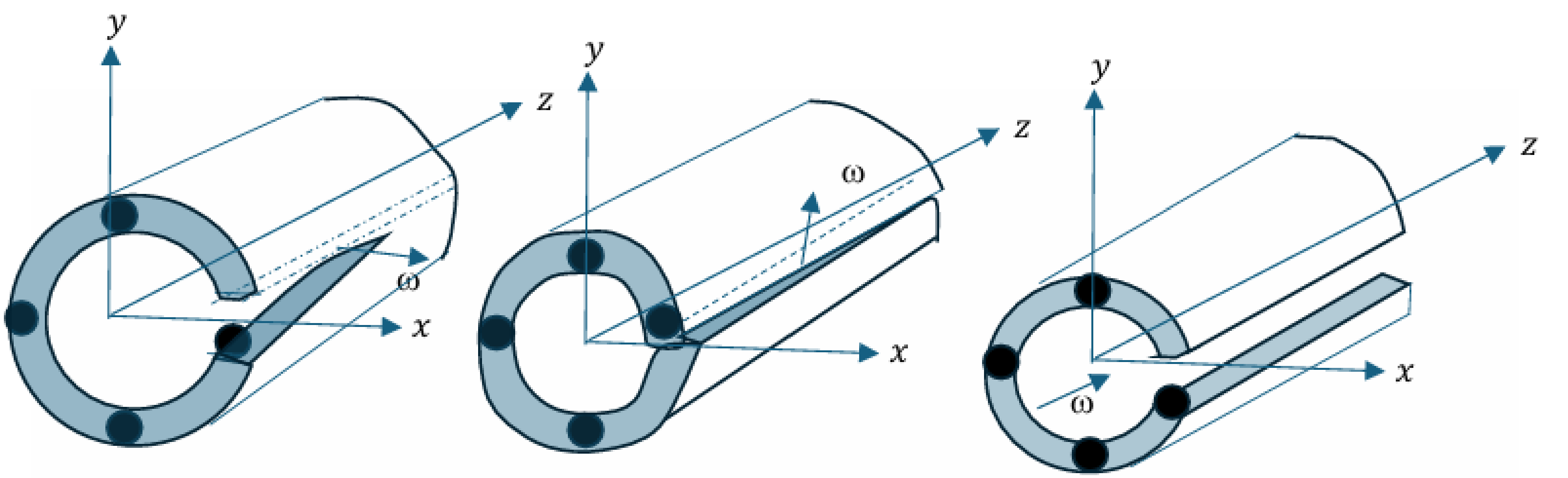


Figure 5: Longitudinal Disclination defect with Franck vector

The Franck vector and speculative longitudinal memory effect

The Frank vector for longitudinal disclination is:

Effective

$$\boxed{\Omega_z = \oint_C \omega_{z\mu}\, dx^\mu = \int_\Sigma R_{z\mu\nu}\, dA^{\mu\nu}}$$

This vector is perpendicular to the transverse planes. Its projections are:

- Transverse projection: $\Omega_z \cdot \hat{e}_\phi$ contributes to spin memory
- Longitudinal component: $\Omega_z \cdot \hat{e}_z$ suggests new memory effect

6.4 Conjectural Longitudinal Memory Effect

We propose a longitudinal memory effect:

Fondamental

$$\boxed{\Delta\Phi_{\text{long}} = \oint_C \Delta\,\Gamma_{z\mu} dx^\mu = \int_\Sigma \Delta\, R_{z\mu\nu} dA^{\mu\nu}}$$

The derivation of this postulate is given in appendix C.

This would manifest as permanent rotation of the normal direction to Bondi surfaces—a "twist" along the propagation direction.

Point of view of the Physicist: possible longitudinal memory effect

General Relativity and Quantum Cosmology

[Submitted on 28 Jun 2024 (v1), last revised 14 Jan 2025 (this version, v3)]

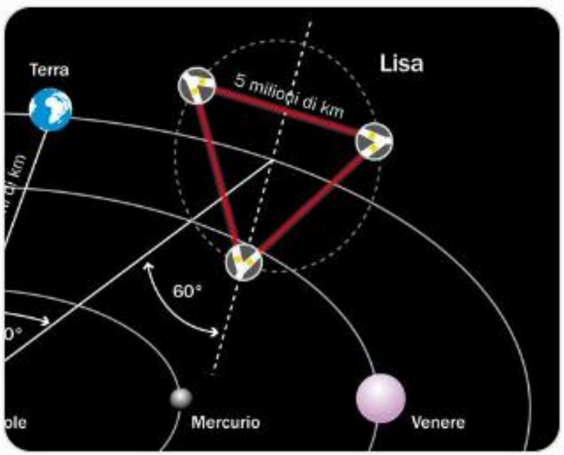
Gravitational wave memory and its effects on particles and fields

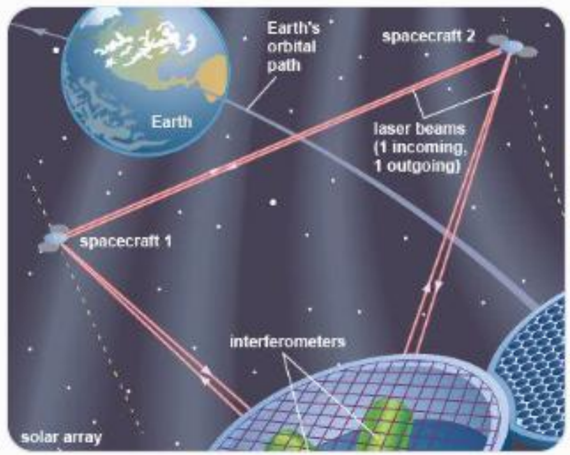
Abraham I. Harte, Thomas B. Mieling, Marius A. Oancea, Elisabeth Steininger

Gravitational wave memory is said to arise when a gravitational wave burst produces changes in a physical system that persist even after that wave has passed. This paper analyzes gravitational wave bursts in plane wave spacetimes, deriving memory effects for timelike and null geodesics, massless scalar fields, and massless spinning particles whose motion is described by the spin Hall equations. We find that all such effects are characterized by four "memory tensors," three of which are independent. We also show that memory effects for null geodesics can have strong longitudinal components, even in vacuum general relativity. When considering massless particles with spin, we solve the spin Hall equations analytically by showing that there exists a conservation law associated with each conformal Killing vector. For the scattering of fields by gravitational waves, we show that given any solution to the massless scalar field equation in flat spacetime, a weak-field solution in a plane wave spacetime can be generated just by applying an appropriate differential operator -- an operator that is constructed from the aforementioned memory tensors. Memory effects for scalar fields are illustrated for both incoming plane waves and higher-order Gaussian beams. We also present a numerical comparison between the spin Hall equations and the full evolution of localized wave packets with angular momentum. Although we work in plane wave spacetimes, which are physically idealized, similar results are also expected to apply for sufficiently small systems affected by distantly generated gravitational waves. Using the Penrose limit, our results may also apply to ultrarelativistic systems at arbitrary locations in arbitrary (even nonradiating) spacetimes.

LISA

Laser Interferometer Space Antenna





How Lisa work

- Each satellite send a laser beam and measure phase shift (Wave length known) => extract ΔL

$$\Delta\phi = \frac{2\pi}{\lambda}\,\Delta L$$

$$\lambda \approx 1\,\mu\mathrm{m} = 10^{-6}\,\mathrm{m}$$

$$\Delta L = 10^{-12}\,\mathrm{m}$$

$$\Delta\phi \approx 2\pi \times 10^{-6}$$

- We extract the strains in 3 directions as a rosette gauge

$$\varepsilon_1 = \frac{\Delta L_1}{L_1}, \quad \varepsilon_2 = \frac{\Delta L_2}{L_2}, \quad \varepsilon_3 = \frac{\Delta L_3}{L_3}$$

Reconstruction of the strain tensor

$$\varepsilon_1 = \frac{\Delta L_1}{L_1}, \quad \varepsilon_2 = \frac{\Delta L_2}{L_2}, \quad \varepsilon_3 = \frac{\Delta L_3}{L_3}$$

$$\gamma_{xy} = 2\varepsilon_{xy}.$$

- bras 1 : $\theta = 0^\circ$
- bras 2 : $\theta = 60^\circ$
- bras 3 : $\theta = 120^\circ$

$$\varepsilon(\theta) = \varepsilon_{xx} \cos^2\theta + \varepsilon_{yy} \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$\varepsilon_1 = \varepsilon_{xx}$$

$$\varepsilon_2 = \frac{1}{4}\varepsilon_{xx} + \frac{3}{4}\varepsilon_{yy} + \frac{\sqrt{3}}{4}\gamma_{xy}$$

$$\varepsilon_3 = \frac{1}{4}\varepsilon_{xx} + \frac{3}{4}\varepsilon_{yy} - \frac{\sqrt{3}}{4}\gamma_{xy}$$

$$\varepsilon_2 + \varepsilon_3 = \frac{1}{2}\varepsilon_{xx} + \frac{3}{2}\varepsilon_{yy}$$

$$\boxed{\varepsilon_{xx} = \varepsilon_1}$$

$$\varepsilon_2 - \varepsilon_3 = \frac{\sqrt{3}}{2}\gamma_{xy}$$

$$\boxed{\gamma_{xy} = \frac{2}{\sqrt{3}}(\varepsilon_2 - \varepsilon_3)}$$

$$\boxed{\varepsilon_{xy} = \frac{1}{\sqrt{3}}(\varepsilon_2 - \varepsilon_3)}$$

$$\boxed{\varepsilon = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{pmatrix}}$$

$$\varepsilon_{xx} = \varepsilon_1$$

$$\varepsilon_{yy} = \frac{2}{3}\left(\varepsilon_2 + \varepsilon_3 - \frac{1}{2}\varepsilon_1\right)$$

$$\varepsilon_{xy} = \frac{1}{\sqrt{3}}(\varepsilon_2 - \varepsilon_3)$$

Results of classical GR linearized

Ligo /LISA Arm measure

$$h_{ij}(t) = h_+(t)\, e^+_{ij} + h_\times(t)\, e^\times_{ij}$$

$$\varepsilon_{ij} = \frac{1}{2} h_{ij}$$

$$\varepsilon_{xy} = \frac{1}{2} h_\times$$

$$e^+ = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad e^\times = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\varepsilon_{ij} = \frac{1}{2} \begin{pmatrix} h_+ & h_\times \\ h_\times & -h_+ \end{pmatrix}$$

$$\gamma_{xy} = 2\varepsilon_{xy}$$

$$\boxed{\gamma_{xy} \approx h_\times}$$

In transversal planes

LISA (3 values of teta)

$$h_{ij} = \begin{pmatrix} h_+ & h_\times \\ h_\times & -h_+ \end{pmatrix}$$

Pure shear

$$\boxed{\varepsilon = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{pmatrix}}$$

$$\frac{\Delta L}{L} = \frac{1}{2} n^i n^j h_{ij}$$

$$\hat{n} = (\cos\theta, \sin\theta)$$

$$\frac{\Delta L_i}{L_i} = \frac{1}{2}\left[h_+ \cos(2\theta_i) + h_\times \sin(2\theta_i)\right]$$

$$\frac{\Delta L}{L} = \frac{1}{2}\left[h_+ \cos(2\theta) + h_\times \sin(2\theta)\right]$$

Pure lengthening and shortening

$$\varepsilon_{xx} = \varepsilon_1$$

$$\varepsilon_{yy} = \frac{2}{3}\left(\varepsilon_2 + \varepsilon_3 - \frac{1}{2}\varepsilon_1\right)$$

$$\varepsilon_{xy} = \frac{1}{\sqrt{3}}(\varepsilon_2 - \varepsilon_3)$$

If GR is complete: no residual effect

$$h(t) = F_+(\theta, \phi, \psi)\, h_+(t) + F_\times(\theta, \phi, \psi)\, h_\times(t)$$

où :

- $F_+, F_\times$ = facteurs de réponse (géométrie du détecteur)
- ψ = angle de polarisation
- (θ, ϕ) = direction de l'onde

Même si l'onde est perpendiculaire, toujours une **combinaison des deux**.

New physics if:

Relativité générale	Mécanique des milieux continus
h_+	$\varepsilon_{xx} - \varepsilon_{yy}$
$h_\times$	γ_{xy}
tenseur h_{ij}	tenseur des déformations ε_{ij}

$$h_{ij}^{\text{mesuré}} \neq h_+ e_{ij}^+ + h_\times e_{ij}^\times$$

$$h_{ij}^{\text{résidu}} \neq 0$$

If medium 3D with longitudinal strains and associated polarisation

- If it is not possible to reconnect the displacements observed with only h+ and hx so h_L should be necessary

The interferometric strain measured in each arm i of LISA is then generalized as:

$$\boxed{\Delta L_i(t) = F_i^{+}(\theta,\phi)\,h_{+}(t) + F_i^{\times}(\theta,\phi)\,h_{\times}(t) + F_i^{L}(\theta,\phi)\,h_L(t)}$$

where F_i^L represents the longitudinal response function.

We introduce a phenomenological coupling:

$$\boxed{h_L(t) = \varepsilon\sqrt{h_+^2(t) + h_\times^2(t)}}$$

where:

- $\varepsilon \ll 1$ is a dimensionless parameter encoding the effective rigidity or microstructural coupling of spacetime,
- $h_T = \sqrt{h_+^2 + h_\times^2}$ is the transverse amplitude.

Answer with LISA

$$h_{ij} = h_{ij}^{GR} + \delta h_{ij}^{(microstructure)}$$

4. Observable signature

The standard GR reconstruction yields:

$$\boxed{\Delta L_i^{GR}(t) = F_i^{+} h_{+}(t) + F_i^{\times} h_{\times}(t)}$$

We define the residual signal:

$$\boxed{R_i(t) = \Delta L_i(t) - \Delta L_i^{GR}(t)}$$

which gives:

$$\boxed{R_i(t) = \varepsilon F_i^{L} h_T(t)}$$

$$\frac{\Delta L}{L} = \frac{1}{2} n^i n^j h_{ij}$$

$$\frac{\Delta L}{L} = \frac{1}{2} [F_{+} h_{+} + F_{\times} h_{\times} + \Phi]$$

Range of measurement with LISA of the memory effect

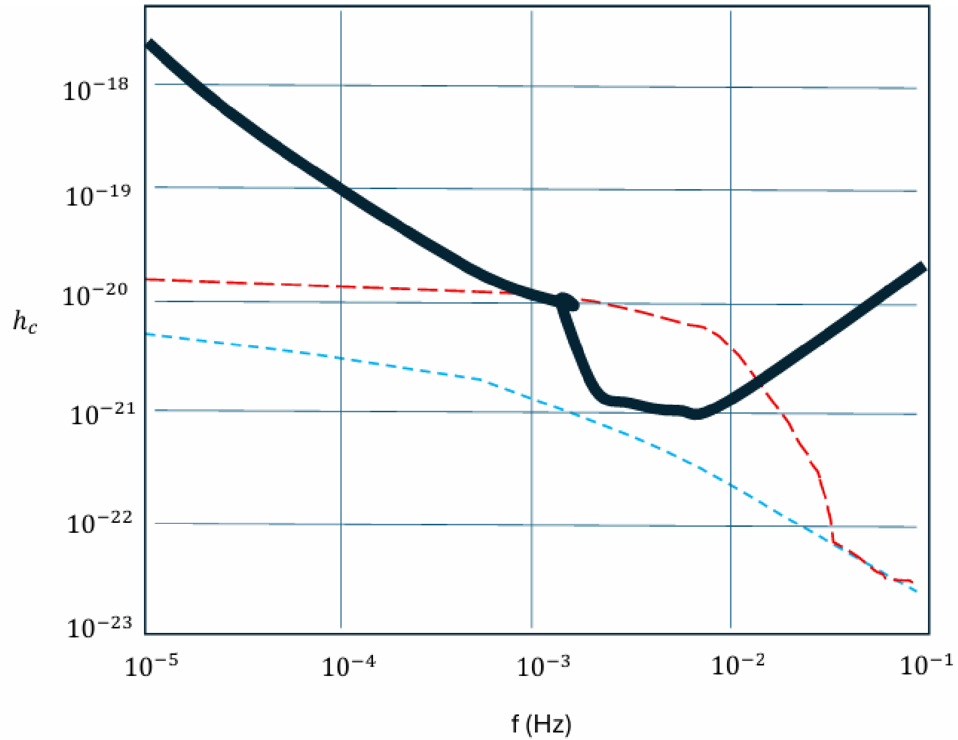


Figure 8: Order of magnitude of the memory effect [57] The continuous black line is the LISA noise amplitude. The long-dashed/red curve uses the MWM to compute the memory; the short-dashed blue curve uses a truncated-inspiral model (see [57] for details)

The spin memory effect is extremely small and is so actually verry difficult to measure [65].

8) Conclusion

$$\boxed{c = \sqrt{\frac{Y}{\rho}}} \qquad \boxed{c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}} \quad \Rightarrow \quad \boxed{\frac{Y}{\rho} = \frac{1}{\varepsilon_0 \mu_0}}$$

Gravitation *carries within itself a hidden electromagnetic–like structure*

Defects *(Burgers vector, torsion) are the integrated traces of this structure*

Mécanique	Électromagnétisme
rigidité Y	$1/\mu_0$
inertie ρ	ε_0

Conclusion

1

Gravitational waves do not produce only elastic deformations. While gravitational waves are usually described as transverse elastic shear waves, the existence of gravitational memory demonstrates that they also leave **permanent residual deformations**. These non-relaxing deformations cannot be described within a purely elastic framework and are naturally interpreted as **plastic deformations of spacetime at an effective level**.

Space time is yielded

2

Within classical Riemannian geometry, Weyl curvature encodes edge- and screw-type defects. In vacuum general relativity, all radiative and memory effects are carried by the Weyl tensor. Its decomposition into electric and magnetic parts encodes, respectively, permanent elongation/shortening (displacement memory) and permanent rotation (spin memory). These two memory effects correspond precisely at the effective level to **edge-type and screw-type defects**, analogous to dislocations in an elastic medium that has undergone local plasticity.

We can have yield in classical GR: No need addition torsion field

Conclusion

3

This naturally leads at the effective level to a Cosserat (micropolar) mechanical description.
Classical Cauchy elasticity cannot account for independent rotational degrees of freedom. A Cosserat continuum, which includes both translations and micro-rotations, provides the natural mechanical framework to describe edge and screw dislocations and the associated plastic deformation of the medium on an effective point of view. Spacetime equipped with gravitational memory behaves effectively as such a Cosserat-type medium.

4

The corresponding medium is a Cosserat medium

The accumulation of local spin memory induces an effective or equivalent Einstein–Cartan–type torsion, without introducing new fields.
The time-integrated spin memory corresponds to accumulated microscopic rotations of Cosserat type. Their non-integrability generates a **geometric torsion with exactly the mathematical structure of Einstein–Cartan torsion**. However, this torsion is **effective, emergent, and post-radiative**: it is not a fundamental field, does not propagate, does not couple to quantum spin, and does not modify Einstein's equations. General relativity remains strictly classical.

Accumulation of local spin create an equivalent torsion

Conclusion

5

The analogy further suggests on an effective point of view the possibility of longitudinal plastic defects.
In real plastic media, transverse dislocations are generically accompanied by longitudinal reconstruction effects. By analogy, the edge and screw memory defects in transverse planes naturally motivate the consideration of **longitudinal plastic deformations of spacetime**, providing a consistent three-dimensional completion of the defect picture. This extension is physically motivated and compatible with recent studies of longitudinal memory effects.

If we push the analogy, possible longitudinal defect too => new memory effect?

6

All such defects must be extremely small and non-propagating, in agreement with observations.
Current interferometric observations (LIGO/Virgo/KAGRA) show no detectable deviation from classical general relativity in wave propagation. This imposes that the effective torsion and associated plastic defects generated by gravitational memory are necessarily **very weak, localized, and topological**, leaving GR phenomenology unchanged at observable scales.

Defects remain small

Discussion with NASA

De : Thorpe, Ira (GSFC-6630) <james.i.thorpe@nasa.gov>
Envoyé : mardi 22 septembre 2020 21:50
À : David IZABEL <d.izabel@enveloppe-metallique.fr>
Cc : Kazanas, Demos (GSFC-6630) <demos.kazanas-1@nasa.gov>
Objet : Re: [EXTERNAL] Possible new measurements on Lisa

Dear Pr. Izabel,

Thank you for your message and your article. My personal focus is more on the instrumentation and measurement aspects of LISA so I'm not sure how well I followed the details of your argument but I think I understand some of the basics. As Rai pointed out in the discussion you shared, LISA's triangular configuration provides an opportunity to measure multiple components of the strain tensor. We typically describe this as a capability to simultaneously measure the two polarization modes of standard GR but there has also been some work to extend this to measurements (or constraints) of the four additional basis tensors possible in non-GR metric theories. I suspect a similar framework could be applied to consider LISA measurements in your "elastic gravity" approach. There is also the possibility that LISA will operate simultaneously with a planned Chinese observatory called Taiji, which has similar geometry and performance characteristics. Others have pointed out that a LISA-Taiji network could be particularly powerful for constraining modified gravity as well as for precision astrometry of GW signals. I'm afraid that is about the limit of my capabilities on the subject, but fortunately I am surrounded by colleagues with more knowledge and experience in gravity theory. One of these is Dr. Demos Kazanas, with whom I shared your note and who expressed some interest in further discussions. I've copied him here in case either of you wish to initiate a conversation. Please let me know if I can provide any additional information regarding LISA.
Kind regards,

-Ira Thorpe, NASA/GSFC

Synthesis

The granularity of the vacuum is not postulated: it is revealed by gravitational memory as an effective topological defect of the Weyl field.

Cosserat quantity	Gravitational analogue	Interpretation
β_{ij} (distortion)	$\Delta E_{ij} = \int E_{ij}\, dt$	Accumulated tidal shear (integrated electric Weyl tensor)
κ_{ij} (wryness)	$\Delta H_{ij} = \int H_{ij}\, dt$	Accumulated frame-dragging (integrated magnetic Weyl tensor)
α_{ij} = curl β_{ij} (dislocation density)	curl(ΔE_{ij})	Memory-induced defect density
$b_i = \oint \beta_{ij}\, dx^j$	$\oint \Delta h_{ij}^{TT}\, dx^j$	Displacement memory (edge dislocation analogue)
φ_i (micro-rotation)	$\tau\_H \int H_{ij}\, x^j\, dt$	Spin memory (screw-type rotational mismatch)
$T_eff^a = d(\Delta e^a)$	Topological effective torsion	Non-propagating torsion remnant produced by memory
Plastic strain	Δh_{ij}^{TT}	Permanent metric change after the wave

$$c_{\text{eff}}^2 = \frac{\text{Current Weyl rigidity } (C_{abcd})}{\text{Effective energy density } (\rho)}$$

Or, in a more formal tensor notation:

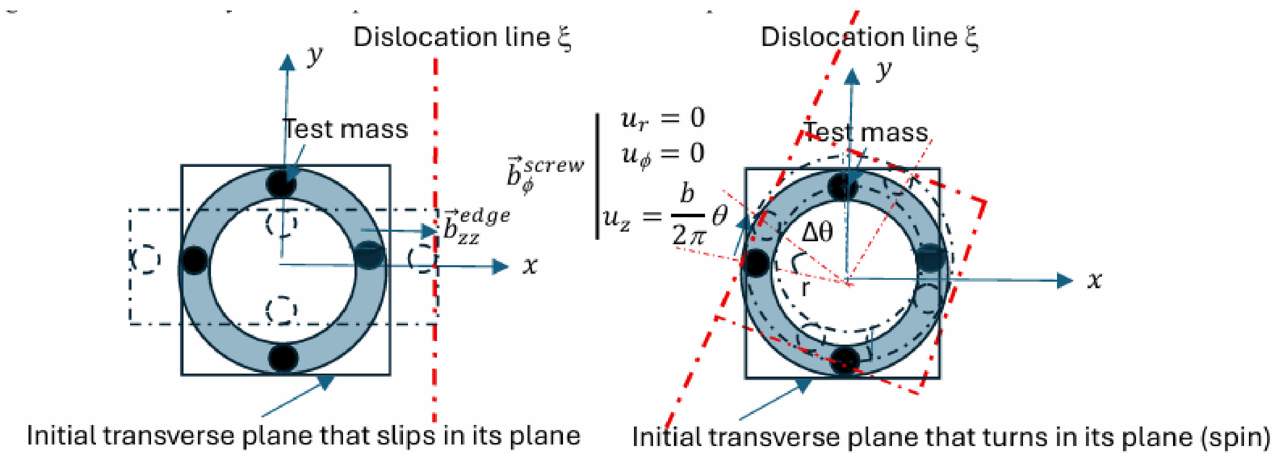
$$c_{\text{eff}}^2 = \frac{||C_{abcd}||}{\rho_{\text{eff}}}$$

where $||C_{abcd}||$ represents an appropriate norm or scalar measure of the Weyl tensor's stiffness (e.g., the Kretschmann scalar or a contracted product), and ρ_{eff} is the effective mass-energy density of the spacetime continuum.

Publications

D.Izabel (2026) Weyl–Cosserat Elasticity and Gravitational Memory: An Effective Microstructured Model of Spacetime, **European Physical journal plus Volume 141 article number, (2026)** DOI.org/10.1140/epjp/s13360-026-07691-9

Effective Defect edge and screw are already included in Riemann tensor in vacuum via Weyl tensor. It's an image of the displacement and spin memory



Weyl–Cosserat Correspondence

Weyl–Cosserat dictionary

Gravity	Mechanics
E_{ij}	translational distortion (edge)
H_{ij}	rotational distortion (screw)
memory	dislocation
holonomy	Burgers vector

Reconstructed 3D Spacetime

- Transverse planes plastically deformed
  - Micro-rotations reconnect planes
  - Effective 3D spacetime with defects

- ✅ **Cosserat micro-rotations** act as *continuous internal spins* of the medium
- ✅ Their **accumulation / non-integrability** generates **geometric torsion**
- ✅ This torsion has **exactly the Einstein–Cartan mathematical structure**
- ❌ It is **not** a fundamental EC theory:
  - no propagating torsion,
  - no coupling to quantum spin,
  - no modification of Einstein's equations.

This is an **effective, emergent, topological torsion**,
the exact analogue of torsion generated by dislocations in a Cosserat crystal.

Weyl/Cosserat/defect theory

Physical / Conceptual Description	Geometric Interpretation	Associated Equations (key forms)
Local Cosserat micro-rotations	Internal (non-quantum) spin degrees of freedom	$\phi_i(x)$
		Cosserat distortion:
		$\beta_{ij} = \partial_j u_i + \varepsilon_{ijk}\, \phi_k$
Spatial accumulation of micro-rotations	Rotational curvature (wryness)	$\kappa_{ij} = \partial_j \phi_i$
Non-integrability of the distortion field	Emergence of topological defects	$\alpha_{ij} = \varepsilon_{jkl}\, \partial_k \beta_{il}$
Dislocation density	Geometric torsion (Cartan sense)	$T^a \;\longleftrightarrow\; \alpha_{ij}$

Weyl/Cosserat/defect theory

Summation / coarse-graining of Cosserat micro-spins	Emergent effective torsion	$T^a_{\text{eff}} = d(\Delta e^a)$
Global flux of the defect	Burgers vector	$b^a = \int_\Sigma T^a_{\text{eff}}$
Gravitational displacement memory (edge)	Edge dislocation (translational defect)	$\Delta h^{TT}_{ij} = 2 \int E_{ij}\, dt$
		$b_i = \oint \Delta h^{TT}_{ij}\, dx^j$
Gravitational spin memory (screw)	Screw dislocation (rotational defect)	$\phi^{\text{eff}}_i \sim \int H_{ij}(t)\, x^j\, dt$
Global geometric description	Einstein–Cartan–type torsion (effective, emergent)	$T^a{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu$
Difference with fundamental Einstein–Cartan theory	Non-dynamical, topological torsion	*No field equation of the form* $T \sim S_{\text{quantum}}$