Dislocations et disclinaisons: de la géométrie aux équations

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Université de Rennes – IRMAR

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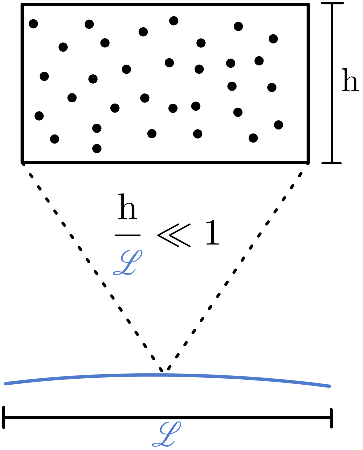
GDR GDM La Rochelle

Classical continuum mechanics



Continuous approximation of a macroscopic material

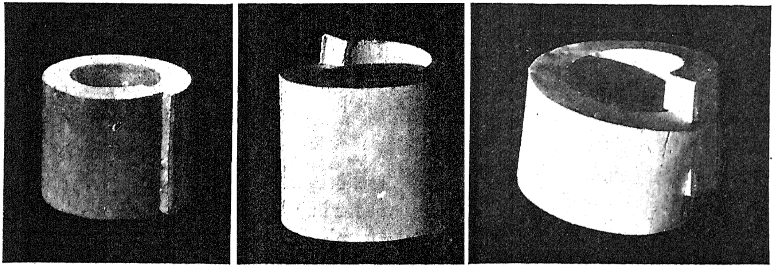
Classical macroscopic map:

$$\varphi : \mathbb{B} \longrightarrow \mathbb{E}$$

which linearises to:

$$\varphi = \mathrm{Id} + u$$

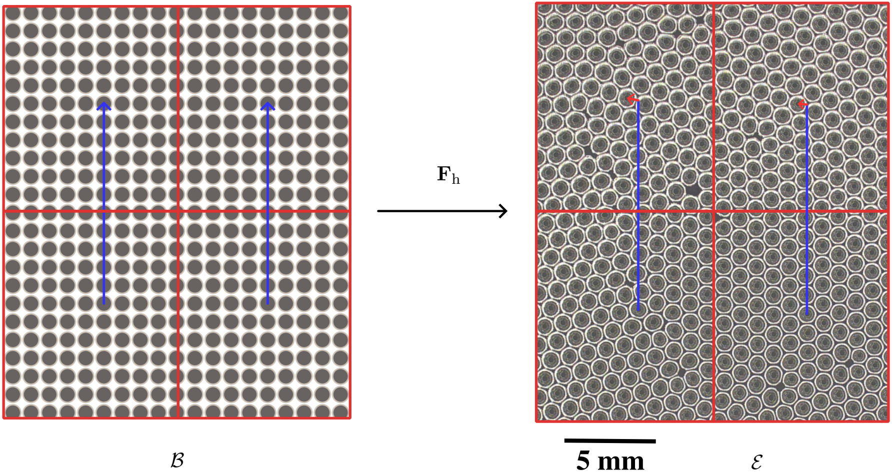
The Volterra process



"Distortions" in the three axes using the Volterra process (before trimming).
V. Volterra, *Sur l'équilibre des corps élastiques multiplement connexes*, 1907

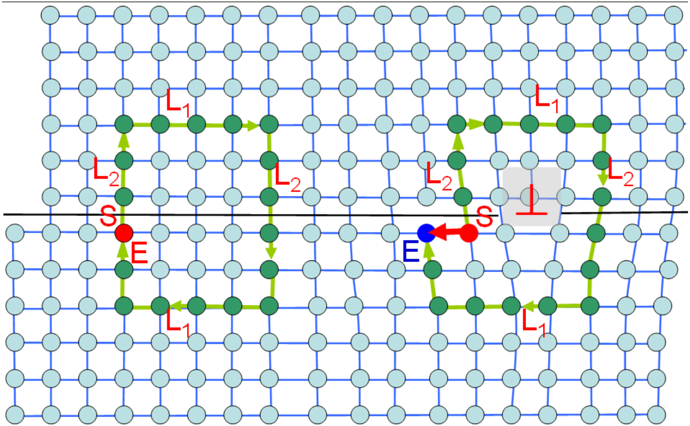
J.M. & W.G. Burgers, *Dislocations in crystal lattices*, 1956
F.C Frank, *On the theory of liquid crystals*, 1958

Placement of macroscopic vectors



M. J. Bowick & L. Giomi, *Two-dimensional matter: order, curvature and defects*, 2009

Burgers circuit



$$\overrightarrow{SE} = \mathrm{Curl}\,(\nabla u) \text{ at } \perp \qquad \text{so } \nabla u \longrightarrow P$$

Micromorphic media

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1. an ambient macroscopic space $\mathbb{E} = \mathbb{R}^3$.
2. a macroscopic body $\mathbb{B} \subset \mathbb{E}$ subset of the ambient space.
3. a macroscopic (smooth) placement map $u : \mathbb{B} \longrightarrow \mathbb{R}^3$
4. a microscopic (smooth) linear placement map $P : \mathbb{B} \longrightarrow \mathbb{R}^{3\times 3}$

E. & F. Cosserat, *Théorie des Corps Déformables*, 1909

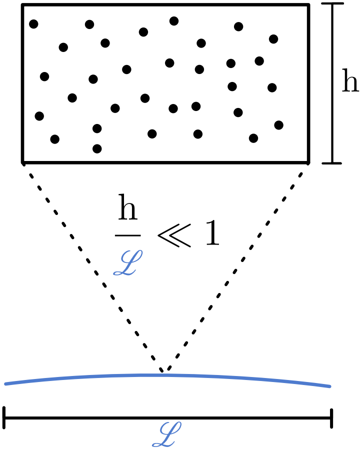
E. Kröner, *Allgemeine kontinuumstheorie der versetzungen und eigenspannungen*, 1959

R.A. Toupin, *Elastic materials with couple-stresses*, 1962

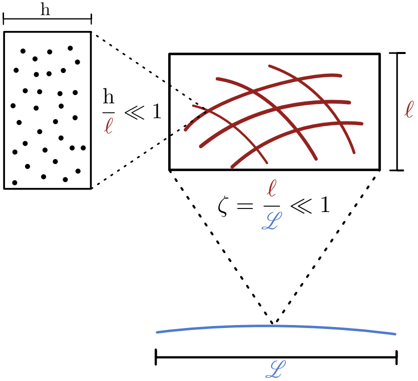
A.C. Eringen & ES Suhubi, *Nonlinear theory of simple micro-elastic solids – I*, 1964

R.D. Mindlin, *Micro-structure in linear elasticity*, 1964

The two-scale interpretation



Continuous approximation of a macroscopic material



Continuous approximation of a micro-structured material

Disclinaisons

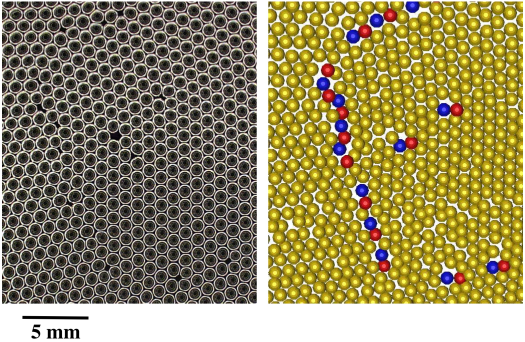


Figure: Photography of soap bubbles (left) and the corresponding computer reconstruction (right). The color scheme highlights the 5-fold (red) and 7-fold (blue) disclinations over 6-fold coordinated bubbles (yellow)

M. J. Bowick & L. Giomi, *Two-dimensional matter: order, curvature and defects*, 2009

$$\overrightarrow{\text{Frank}} = \text{Curl}\,(\nabla P) \qquad \text{so } \nabla P \longrightarrow N$$

Two-scale non-holonomic model

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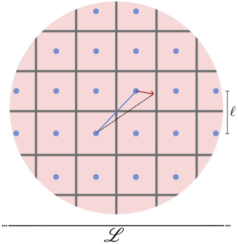
1. an ambient macroscopic space $\mathbb{E} = \mathbb{R}^3$
2. a macroscopic body $\mathbb{B} \subset \mathbb{E}$ subset of the ambient space.
3. a macroscopic (smooth) placement map $u : \mathbb{B} \longrightarrow \mathbb{R}^3$
4. a microscopic (smooth) linear placement map $P : \mathbb{B} \longrightarrow \mathbb{R}^{3\times 3}$
5. a (smooth) first-order placement map $N : \mathbb{B} \times \mathbb{R}^3 \longrightarrow \mathbb{R}^{3\times \dim(\mathbb{B})}$

$$\mathbf{F} := \begin{bmatrix} \mathrm{Id} + \nabla u & \mathbf{0} \\ N & \mathrm{Id} + P \end{bmatrix}$$

$$\mathbf{F}_{\begin{bmatrix} X \\ Y \end{bmatrix}} : \begin{bmatrix} \delta X \\ \delta Y \end{bmatrix} \mapsto \begin{bmatrix} \delta X + \nabla_X u \cdot \delta X \\ \delta Y + P\big|_X \cdot \delta Y + N\big|_{\begin{bmatrix} X \\ Y \end{bmatrix}} \cdot \delta X \\ \end{bmatrix}$$

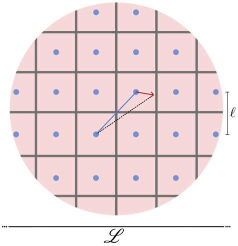
The Cauchy-Green



$$\mathbf{G}\left(\nabla u\right) := \left(\mathrm{Id} + \nabla u\right)^{\mathrm{T}} \cdot \left(\mathrm{Id} + \nabla u\right)$$

$$\mathfrak{G}\left(\mathbf{F}\right) :=?$$

The pseudo-metric



$$\left\|\dashrightarrow\right\|_{\mathfrak{G}} := \left\|\mathbf{F}\cdot \dashrightarrow\right\|_{\mathfrak{g}}$$

$$:= \left\|\text{interpretation}\left(\mathbf{F}\cdot \dashrightarrow\right)\right\|_{\mathbf{g}}$$

$$\text{interpretation}\left(\dashrightarrow\right) = \text{interpretation}\left(\begin{bmatrix} {\color{blue}\rightsquigarrow} \\ {\color{red}\rightsquigarrow} \end{bmatrix}\right)$$

$$:= {\color{blue}\rightsquigarrow} + \varepsilon \cdot {\color{red}\rightsquigarrow}$$

Kernel of the interpretation

$$\mathfrak{G} = \mathbf{F}^{\mathrm{T}} \cdot \begin{bmatrix} \mathrm{Id} & \mathrm{Id} \\ \mathrm{Id} & \mathrm{Id} \end{bmatrix} \cdot \mathbf{F}$$

$$\begin{aligned} \ker \begin{bmatrix} \mathrm{Id} & \mathrm{Id} \\ \mathrm{Id} & \mathrm{Id} \end{bmatrix} &= \ker\left(\text{interpretation}\right) \\ &= \left\{ \begin{bmatrix} \rightsquigarrow \\ \rightsquigarrow \end{bmatrix} \;\middle|\; 0 = \rightsquigarrow + \varepsilon \cdot \rightsquigarrow \right\} \\ &= \left\{ \begin{bmatrix} \varepsilon \cdot \mathbf{v} \\ -\mathbf{v} \end{bmatrix} \;\middle|\; \mathbf{v} \in \mathbb{R}^3 \right\} \end{aligned}$$

$$\begin{aligned} \ker \mathfrak{G} &=: \mathrm{Im}\left(\boldsymbol{\Gamma} \cdot Y + \boldsymbol{\Theta}\right) \\ &\simeq \mathrm{Im}\left(-N \cdot Y + (P - \nabla u)\right) \end{aligned}$$

Generalised frame indifference

Théorème 1 – Orbital invariants

$\mathbf{F} \mapsto \alpha(\mathbf{F})$ is frame indifferent iff there exists $\tilde{\alpha}$ such that

$$\alpha(\mathbf{F}) = \tilde{\alpha}\left(\mathfrak{G}\left(\mathbf{F}\right)\right) \qquad (\textit{if } N \equiv \nabla P)$$

$$\alpha(\mathbf{F}) = \tilde{\alpha}\left(\mathfrak{G}\left(\mathbf{F}\right), \mathfrak{G}\left(\begin{bmatrix} \mathrm{Id} + \nabla u & \mathbf{0} \\ \nabla P & P \end{bmatrix}\right)\right) \qquad (\textit{in general})$$

or, equivalently:

$$\alpha(\mathbf{F}) = \tilde{\alpha}\left(\mathrm{sym}\left(P\right), \nabla u - P, N\right) \qquad (\textit{if } N \equiv \nabla P)$$

$$\alpha(\mathbf{F}) = \tilde{\alpha}\left(\mathrm{sym}\left(P\right), \nabla u - P, N, \nabla P\right) \qquad (\textit{in general})$$

A "simple" quadratic energy

$$\mathbb{B} := [0, L] \qquad\qquad \mathbb{E} := \mathbb{R}$$

$$\begin{aligned}\boldsymbol{\Psi} := \int_0^L & a \left\|P\right\|^2 + b \left\|N\right\|^2 + c \left\|\nabla N\right\|^2 + d \left\|\nabla u - P\right\|^2 \\ & + e \left\|\nabla P - N\right\|^2 + f_0 \cdot u + f_1 \cdot P + f_2 \cdot N \;\mathrm{d}X\end{aligned}$$

minimised only if:

$\forall \delta u, \delta P, \delta N,$

$$\begin{aligned}0 = \int_0^L & a\, P^i_j \,\delta P^i_j + b\, N^i_{jk} \,\delta N^i_{jk} + c\, N^i_{jk,\ell} \,\delta N^i_{jk,\ell} \\ & + d \left(u^i_{,j} - P^i_j\right) \cdot \left(\delta u^i_{,j} - \delta P^i_j\right) + [f_0]_i \, u^i + [f_1]_i^j \, P^i_j + [f_2]_i^{jk} \, P^i_{jk} \\ & + e \left(P^i_{j,k} - N^i_{jk}\right) \cdot \left(\delta P^i_{j,k} - \delta N^i_{jk}\right) \;\mathrm{d}X\end{aligned}$$

The 2D in 2D equations

In the non-holonomic case:

$$[f_0]^i = d\left(u^i_{,jj} - P^i_{j,j}\right) \qquad \text{on } \mathbb{B}$$

$$[f_1]^j_i = a\,P^i_j - d\left(u^i_{,j} - P^i_j\right) - e\left(P^i_{j,kk} - N^i_{jk,k}\right) \qquad \text{on } \mathbb{B}$$

$$[f_2]^{jk}_i = b\,N^i_{jk} - c\,N^i_{jk,\ell\ell} - e\left(P^i_{j,k} - N^i_{jk}\right) \qquad \text{on } \mathbb{B}$$

$$0 = d\left(u^i_{,j} - P^i_j\right)\mathbf{n}^j\,\delta u^i \qquad \text{on } \partial\mathbb{B}$$

$$0 = e\left(P^i_{j,k} - N^i_{jk}\right)\mathbf{n}^k\,\delta P^i_j \qquad \text{on } \partial\mathbb{B}$$

$$0 = c\,N^i_{jk,\ell}\,\mathbf{n}^\ell\,\delta N^i_{jk} \qquad \text{on } \partial\mathbb{B}$$

Formulae for defects

$$[f_2]_i^{jk} = b\, N^i_{jk} - c\, N^i_{jk,\ell\ell} - e\left(P^i_{j,k} - N^i_{jk}\right)$$

$$\mathbf{T}^i_{jk} := N^i_{jk} - N^i_{kj} =: \mathrm{Skew}\, N$$

$$\mathbf{R}^i_{jk\ell} \simeq N^i_{jk,\ell} - N^i_{j\ell,k} =: \mathrm{Curl}\, N$$

Formulae for defects

$$[f_2]_i^{jk} = b\,N^i_{jk} - c\,N^i_{jk,\ell\ell} - e\left(P^i_{j,k} - N^i_{jk}\right)$$

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$$0 \simeq c\,\Delta\mathbf{R} - (b+e)\,\mathbf{R} + \mathrm{Curl}\,f_2$$

Formulae for defects

$$[f_2]_i^{jk} = b\, N^i_{jk} - c\, N^i_{jk,\ell\ell} - e\left(P^i_{j,k} - N^i_{jk}\right)$$

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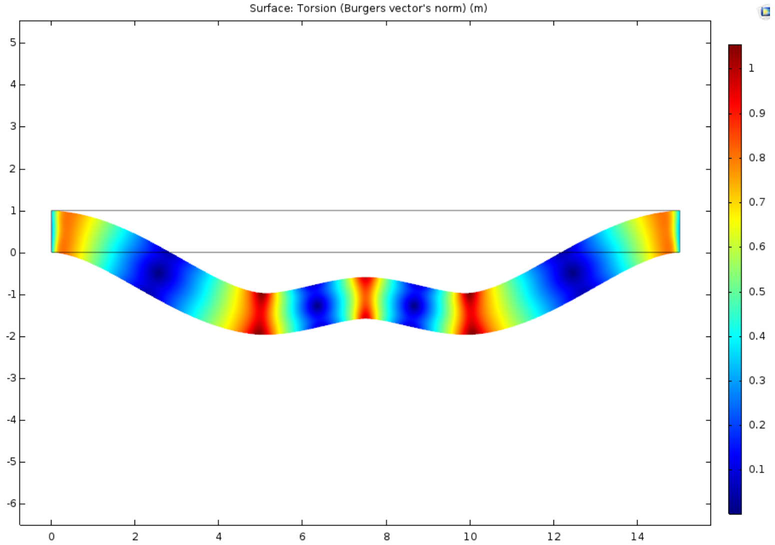
$$\mathbf{R}^i_{jk\ell} \simeq N^i_{jk,\ell} - N^i_{j\ell,k} =: \mathrm{Curl}\, N$$

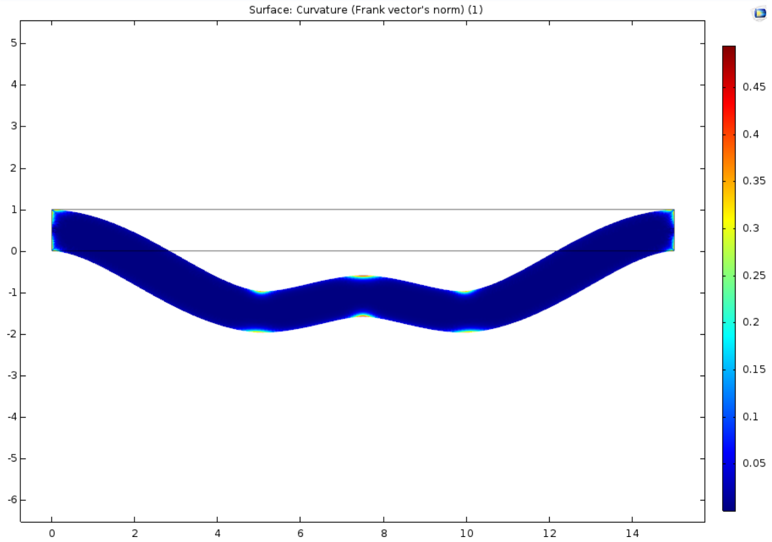
$$0 \simeq c\,\Delta\mathbf{R} - (b+e)\ \mathbf{R} + \mathrm{Curl}\, f_2$$

$$0 = c\,\Delta\mathbf{T} - (b+e)\,\mathbf{T} - \mathrm{Curl}\, P$$

Simulation with Comsol



Simulation with Comsol



A "relaxed" quadratic energy

$$\mathbb{B} := [0, L] \qquad\qquad \mathbb{E} := \mathbb{R}$$

$$\begin{aligned}\mathbf{\Psi} := \int_0^L & a \left\| P \right\|^2 + b \left\| \mathbf{T} \right\|^2 + c \left\| \nabla N \right\|^2 + d \left\| \nabla u - P \right\|^2 \\ & + e \left\| \nabla P - N \right\|^2 + f_0 \cdot u + f_1 \cdot P + f_2 \cdot N \ \mathrm{d}X\end{aligned}$$

P. Neff, *A unifying perspective: the relaxed linear micromorphic continuum*, 2014

minimised only if:

$$\begin{aligned} & \forall \delta u, \delta P, \delta N, \\ & 0 = \int_0^L a\, P^i_j\, \delta P^i_j + b \left(N^i_{jk} - N^i_{kj} \right) \delta N^i_{jk} + c\, N^i_{jk,\ell}\, \delta N^i_{jk,\ell} \\ & \quad + d \left(u^i_{,j} - P^i_j \right) \cdot \left(\delta u^i_{,j} - \delta P^i_j \right) + [f_0]_i\, u^i + [f_1]^j_i\, P^i_j + [f_2]^{jk}_i\, P^i_{jk} \\ & \quad + e \left(P^i_{j,k} - N^i_{jk} \right) \cdot \left(\delta P^i_{j,k} - \delta N^i_{jk} \right) \ \mathrm{d}X \end{aligned}$$

Formulae for defects

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$$[f_2]_i^{jk} = b\left(N^i_{jk} - N^i_{kj}\right) - c\,N^i_{jk,\ell\ell} - e\left(P^i_{j,k} - N^i_{jk}\right)$$

$$\left[\overset{\circlearrowright}{\mathbf{R}}\right]^i_{jk\ell} \coloneqq R^i_{jk\ell} + R^i_{k\ell j} + R^i_{\ell jk}$$

$$0 \simeq c\,\Delta\mathbf{R} - e\,\mathbf{R} - \frac{b}{2}\,\overset{\circlearrowright}{\mathbf{R}} + b\,\nabla\mathbf{T} + \operatorname{Curl} f_2$$

Formulae for defects

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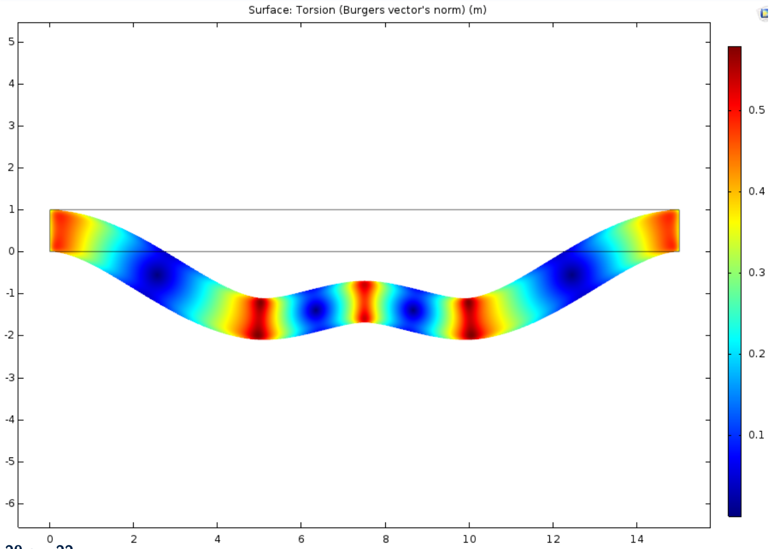
$$[f_2]_i^{jk} = b\left(N^i_{jk} - N^i_{kj}\right) - c\,N^i_{jk,\ell\ell} - e\left(P^i_{j,k} - N^i_{jk}\right)$$

$$\left[\overset{\circlearrowleft}{\mathbf{R}}\right]^i_{jk\ell} \coloneqq R^i_{jk\ell} + R^i_{k\ell j} + R^i_{\ell jk}$$

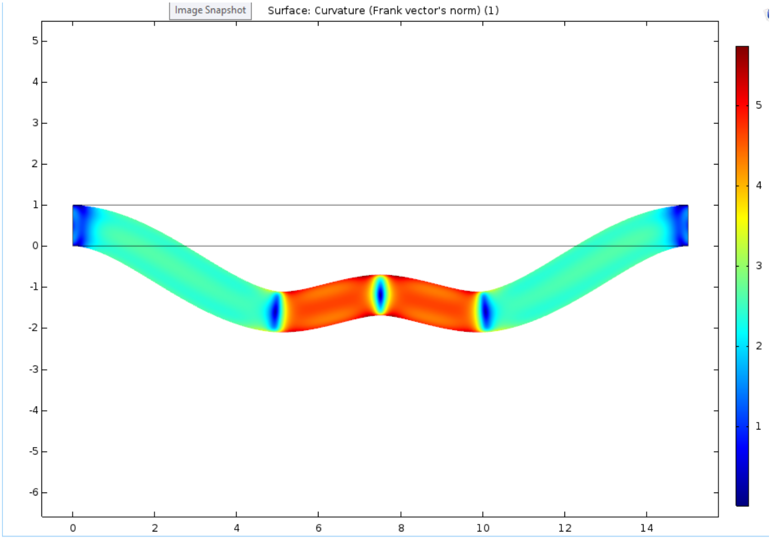
$$0 \simeq c\,\Delta\mathbf{R} - e\,\mathbf{R} - \frac{b}{2}\,\overset{\circlearrowleft}{\mathbf{R}} + b\,\nabla\mathbf{T} + \operatorname{Curl} f_2$$

$$0 = c\,\Delta\mathbf{T} - \frac{b-2e}{2}\,\mathbf{T} + e\,\operatorname{Curl} P - \operatorname{Skew} f_2$$

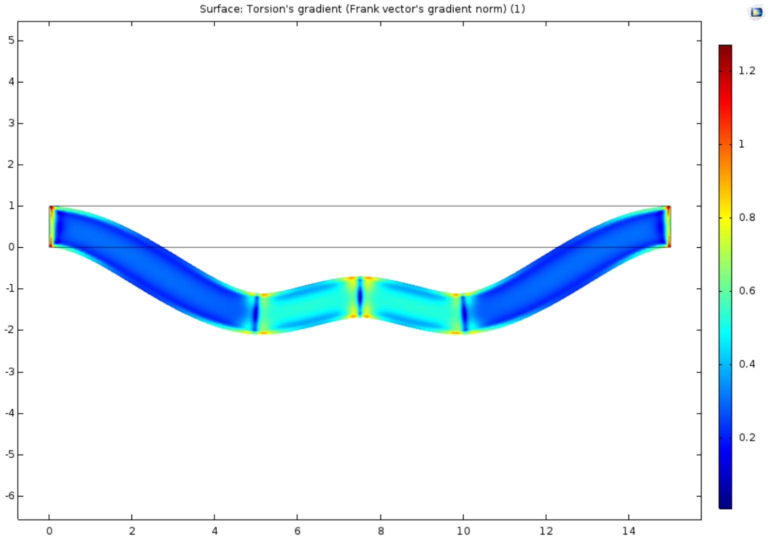
Simulation with Comsol



Simulation with Comsol



Simulation with Comsol



The 1D in 3D model

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transversal affine dependency:

$$u\left(X^1, X^2, X^3\right) = u\left(X^1, 0, 0\right) + P \cdot \begin{bmatrix} 0 \\ X^2 \\ X^3 \end{bmatrix}$$

$$P\left(X^1, X^2, X^3\right) = P\left(X^1, 0, 0\right) + N \cdot \begin{bmatrix} 0 \\ X^2 \\ X^3 \end{bmatrix}$$

$$N\left(X^1, X^2, X^3\right) = N\left(X^1, 0, 0\right)$$

a partial integration over X^2 and X^3 yields:

$$[\cdots] + \frac{a}{4}\left(\overline{N}^i_{j\alpha} + \overline{N}^j_{i\alpha}\right)\left(\overline{N}^i_{j\beta} + \overline{N}^j_{i\beta}\right)\mathbf{I}^{\alpha\beta}$$
$$+ d\left(\overline{P}^i_{\alpha,1} - \overline{N}^i_{1\alpha}\right)\left(\overline{P}^i_{\beta,1} - \overline{N}^i_{1\beta}\right)\mathbf{I}^{\alpha\beta} + e\,\overline{N}^i_{j\alpha,1}\,\overline{N}^i_{j\beta,1}\,\mathbf{I}^{\alpha\beta}$$

Conclusion

A geometric framework for describing defects and the media that host them — where the generalised placement explicitly permits their appearance — provides a rigorous foundation for computing measures of deformation, even in cases involving large deformations.

By applying variational calculus along with certain simplifying assumptions, one can, in the linear regime, derive explicit partial differential equations (PDEs) that describe these defects. These reveal interactions between dislocations and disclinations, which depend on the chosen energy.

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By applying variational calculus along with certain simplifying assumptions, one can, in the linear regime, derive explicit partial differential equations (PDEs) that describe these defects. These reveal interactions between dislocations and disclinations, which depend on the chosen energy.

Thank You

The 1D in 1D energy

$$\mathbb{B} := [0, L] \qquad\qquad \mathbb{E} := \mathbb{R}$$

$$\mathbf{\Psi} := \int_0^L a\,|p|^2 + b\,|n|^2 + c\,\left|n'\right|^2 + d\,\left|u' - p\right|^2 + e\,\left|p' - n\right|^2 \;\mathrm{d}X$$

minimised only if:

$$\forall \delta u, \delta p, \delta n,$$

$$0 = \int_0^L a\,p\,\delta p + b\,n\,\delta n + c\,n'\,\delta n' + d\left(u' - p\right) \cdot \left(\delta u' - \delta p\right) + e\left(p' - n\right) \cdot \left(\delta p' - \delta n\right) \;\mathrm{d}X$$

The 1D in 1D solution

The solution, when $p = u'$ and $n = p'$, is:

$$
\begin{aligned}
0 &= a\ u^{(2)} - b\ u^{(4)} + c\ u^{(6)} && \text{on } \mathbb{B} \\
0 &= \left[a\ u^{(1)} - b\ u^{(3)} + c\ u^{(5)}\right] \delta u && \text{on } \partial\mathbb{B} \\
&= \left[b\ u^{(2)} - c\ u^{(4)}\right] \delta u^{(1)} && \text{on } \partial\mathbb{B} \\
&= \left[c\ u^{(3)}\right] \delta u^{(2)} && \text{on } \partial\mathbb{B}
\end{aligned}
$$

The 1D in 1D solution

The solution, when $n = p'$, is:

$$
\begin{aligned}
0 &= a\ u^{(2)} - b\ u^{(4)} + c\ u^{(6)} && \text{on } \mathbb{B} \\
0 &= \left[a\ u^{(1)} - b\ u^{(3)} + c\ u^{(5)}\right] \delta u && \text{on } \partial\mathbb{B} \\
&= \left[b\ u^{(2)} - c\ u^{(4)}\right] \delta p && \text{on } \partial\mathbb{B} \\
&= \left[c\ u^{(3)}\right] \delta p^{(1)} && \text{on } \partial\mathbb{B}
\end{aligned}
$$

$$
\begin{aligned}
p &= u^{(1)} - \frac{\kappa}{d} \\
\kappa &= \left(1 + \frac{a}{d}\right)^{-1} \left(a\ u^{(1)}(0) - b\ u^{(3)}(0) + c\ u^{(5)}(0)\right)
\end{aligned}
$$

The 1D in 1D solution

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The general solution is:

$$
\begin{aligned}
0 &= \widetilde{a}\, u^{(2)} - \widetilde{b}\, u^{(4)} + c\, u^{(6)} && \text{on } \mathbb{B} \\
0 &= \left[\widetilde{a}\, u^{(1)} - \widetilde{b}\, u^{(3)} + c\, u^{(5)}\right] \delta u && \text{on } \partial\mathbb{B} \\
&= \left[\widetilde{b}\, u^{(2)} - c\, u^{(4)}\right] \delta p && \text{on } \partial\mathbb{B} \\
&= \left[c\, u^{(3)}\right] \delta n && \text{on } \partial\mathbb{B} \\
p &= u^{(1)} - \frac{\kappa}{d} \\
\kappa &= \left(1 + \frac{a}{d}\right)^{-1} \left(a\, u^{(1)}(0) - b\, u^{(3)}(0) + c\, u^{(5)}(0)\right) \\
n &= \left(1 - \frac{b + \frac{ac}{e}}{e + b}\right) p^{(1)} + \frac{c}{b + e}\, p^{(3)}
\end{aligned}
$$

From 2D in 2D to 1D in 2D

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In the 2D in 2D case, $u \in \mathbb{R}^2$, $P \in \mathbb{R}^{2\times 2}$ and $N \in \mathbb{R}^{2\times 2\times 2}$.
Beam hypothesis:

$$u(X_1, X_2) = u(X_1) + P \cdot \begin{bmatrix} 0 \\ X_2 \end{bmatrix}$$

$$P(X_1, X_2) = P(X_1) + N \cdot \begin{bmatrix} 0 \\ X_2 \end{bmatrix}$$

$$N^i_{jk}(X_1, X_2) = N^i_j(X_1)\delta^1_k$$

From 1D in 2D to 1D in 1D

Inextensibility and rigidity:

$$\left\| \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \nabla u \right\| = 1$$

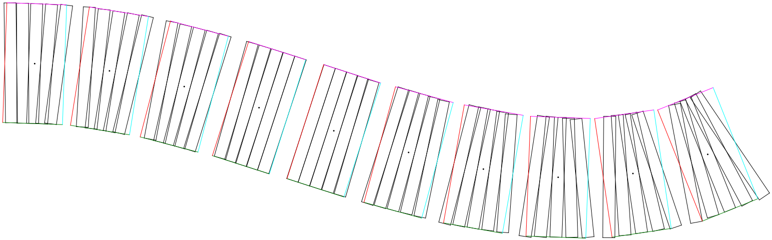
$$\mathrm{Id} + P \in \mathcal{O}(\mathbb{R}^3)$$

$$N = N^{\mathrm{T}}$$

The 1D in 1D plots

$p = u'$ and $n = p'$ case with:

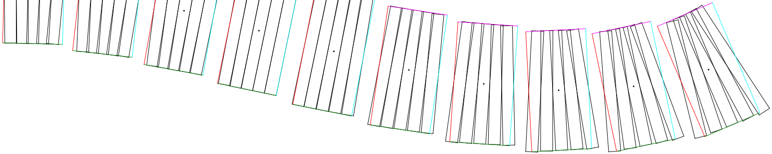
- $(L, a, b, c, d, e) = (5, 0, 1, \frac{1}{25}, 2, 1)$
- $0 = u(0) = u'(0) = u''(0)$
- $u(L) = -0.5$ and $u'(L) = 0.5$



The 1D in 1D plots

$n = p'$ case with:

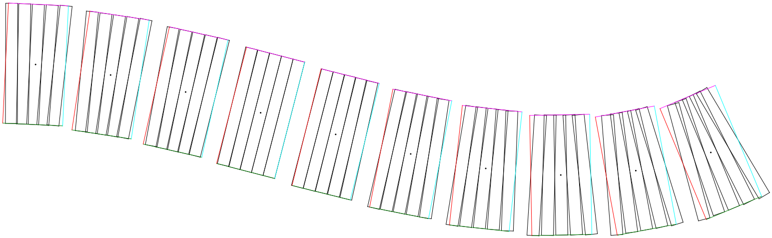
- $(L, a, b, c, d, e) = (5, 0, 1, \frac{1}{25}, 2, 1)$
- $0 = u(0) = p(0) = p'(0)$
- $u(L) = -0.5$ and $p(L) = 0.5$



The 1D in 1D plots

General case with:

- $(L, a, b, c, d, e) = (5, 0, 1, \frac{1}{25}, 2, 1)$
- $0 = u(0) = p(0) = n(0)$
- $u(L) = -0.5$ and $p(L) = 0.5$



Energy of the 1D in 2D model

In the 1D in 2D case, $u \in \mathbb{R}^2$, $P \in \mathbb{R}^{2\times 2}$ and $N \in \mathbb{R}^{2\times 2\times 1}$.

$$\mathbb{B} := [0, L] \qquad \mathbb{E} := \mathbb{R}^2$$

$$\begin{aligned}\boldsymbol{\Psi} := \int_0^L & a\,\|\mathrm{sym}\,(P)\|^2 + b\,\|N\|^2 + c\,\left\|N'\right\|^2 + d\,\left\|u' - P\,_{\big|_1}\right\|^2 \\ & + e\,\left\|P' - N\right\|^2\;\mathrm{d}X\end{aligned}$$

Solution of the 1D in 2D model

$$P_1^1 \in \mathfrak{H}_0\left(a\left(1+\frac{b}{e}\right),\left(b+\frac{ca}{e}\right),c;\kappa^1\left(1+\frac{b}{e}\right)\right)$$

$$P_2^2 \in \mathfrak{H}_0\left(a\left(1+\frac{b}{e}\right),\left(b+\frac{ca}{e}\right),c;0\right)$$

$$P_1^2+P_2^1 \in \mathfrak{H}_0\left(a\left(1+\frac{b}{e}\right),\left(b+\frac{ca}{e}\right),c;\left(1+\frac{b}{e}\right)\kappa^2\right)$$

$$P_1^2-P_2^1 \in \mathfrak{H}_0\left(0,b,c;\left(1+\frac{b}{e}\right)\kappa^2\right)$$

$$\left(u^i\right)^{(1)} = P_1^i - \frac{\kappa^i}{d} \qquad \text{on } \mathbb{B}$$

$$-e\left(N_1^i\right)^{(1)} = \frac{a}{2}\left(P_1^i+P_i^1\right)+\kappa^i - e\left(P_1^i\right)^{(2)} \qquad \text{on } \mathbb{B}$$

$$-e\left(N_2^i\right)^{(1)} = \frac{a}{2}\left(P_2^i+P_i^2\right) - e\left(P_2^i\right)^{(2)} \qquad \text{on } \mathbb{B}$$

where $y \in \mathfrak{H}_0(a,b,c,d;g) \iff -g = a\,y - b\,y^{(2)} + c\,y^{(4)}$

Solution of the 1D in 2D model

$$0 = d\left((u)^{(1)} - P\big|_1\right)\delta u \qquad \text{on } \partial\mathbb{B}$$

$$0 = e\left(P^{(1)} - N\right)\delta P \qquad \text{on } \partial\mathbb{B}$$

$$0 = N^{(1)}\,\delta N^{(1)} \qquad \text{on } \partial\mathbb{B}$$

Dislocations

Université de Rennes

$\mathbf{T}^i_{12} = -N^i_{21}$	for all $n = 1$, $k = 2$ beams
$\mathbf{T}^i_{12} = -\left(P^i_2\right)^{(1)}$	for semi-holonomic beams
$\mathbf{T}^i_{12} = \delta^i_1 \left(P^2_1\right)^{(1)}$	for Timoschenko beams
$\mathbf{T}^i_{12} = \delta^i_1 \left(u^2\right)^{(2)}$	for Euler-Bernouilli beams

Disclinations

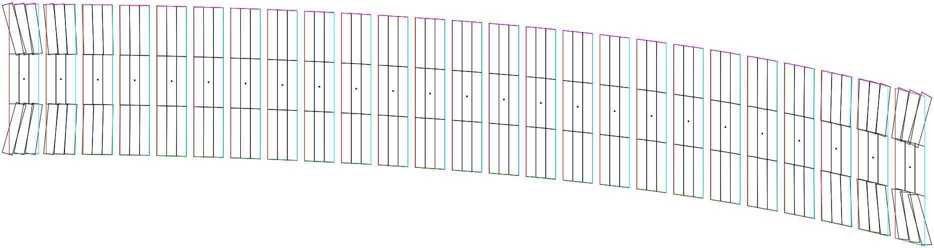
$$\mathbf{R}^i_{jkl} = -N^i_{jl,k} + N^i_{jk,l}$$

$$\begin{aligned}\mathbf{R}^i_j &= \mathrm{Curl}\left(N^i_j\right) \\ &= \begin{bmatrix} 0 & -\mathbf{R}^i_{j12} \\ \mathbf{R}^i_{j12} & 0 \end{bmatrix}\end{aligned}$$

$$\mathbf{R}^i_{j12} \simeq -\left(N^i_{j2}\right)^{(1)}$$

Freeing the second order

- $(L, a, b, c, d, e) = (5, 0, 1, \frac{1}{25}, 1, 3)$
- $u(0) = 0$, $P(0) = 0$ and $N(0) = \left[\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\right]$
- $u(L) = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}$ and $N(L) = \left[\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\right]$



Tank you

Results

- Mechanical construction: no thermodynamics, viscosity, etc.
- Generic placement map: dislocations, disclinations, etc.
- Computation of frame invariants
- Reduction to beam models
- Manifestation of dislocation and disclinations in (generalised) beam models

Future goals

- Numerical analysis of the 2D in 2D case
- Exploration of the generalised beam (u, P, N with an affine transversial dependency)