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Structures de Dirac en Physique et Mécanique

Structures de Dirac en Physique et Mécanique

- Géométrie différentielle
  - symplectique, Poisson, Dirac
  - (un peu de) différentielle graduée

- Mécanique
  - dissipation / interaction ($\supset$ SHP)
  - contraintes
  - approche variationnelle

- Physique
  - symmetries des fonctionnelles
  - jaugeage (et Q-cohomologie equivariante)
  - sigma-modèles

Géométrie généralisée (et graduée)

Poisson manifold $\to (T^*[1]M, Q_\pi)$

Consider a Poisson manifold M,
$\{\cdot,\cdot\}\colon C^\infty(M) \times C^\infty(M) \to C^\infty(M)$.

A Poisson bracket can be written as $\{f, g\} = \pi(\mathrm{d}f, \mathrm{d}g)$, where $\pi \in \Gamma(\Lambda^2 TM)$ is a bivector field. $\pi^{ij}(x) = \{x^i, x^j\}$.

Consider $T^*[1]M$ (coords. $x^i(0), p_i(1)$), with a degree 1 vector field

$$Q_\pi = \left\{\frac{1}{2}\pi^{ij} p_i p_j, \cdot\right\}_{T^*M} = \pi^{ij}(x) p_j \frac{\partial}{\partial x^i} - \frac{1}{2}\frac{\partial \pi^{jk}}{\partial x^i} p_j p_k \frac{\partial}{\partial p_i}$$

Jacobi identity:

$$\{f, \{g, h\}\} + \{g, \{h, f\}\} + \{h, \{f, g\}\} = 0$$
$$\Leftrightarrow [\pi, \pi]_{SN} = 0 \quad \Leftrightarrow Q_\pi^2 = 0$$

Twisted Poisson structure

Consider a bivector field $\pi \in \Gamma(\Lambda^2 TM)$.
It defines an antisymmetric bracket $\{f, g\} = \pi(\mathrm{d}f, \mathrm{d}g)$.
Let $H \in \Omega^3_{cl}(M)$.
A couple (π, H) defines a <u>twisted Poisson structure</u> if it satisfies the twisted Jacobi identity:

$$[\pi, \pi]_{SN} = < H, \pi \otimes \pi \otimes \pi >$$

Consider $T^*[1]M$ (coords. $x^i(0), p_i(1)$), with a degree 1 vector field

$$Q_{\pi,H} = \pi^{ij}(x) p_j \frac{\partial}{\partial x^i} - \frac{1}{2} C_i^{jk}(x) p_j p_k \frac{\partial}{\partial p_i},$$

where $C_i^{jk} = \frac{\partial \pi^{jk}}{\partial x^i} + H_{ij'k'} \pi^{jj'} \pi^{kk'}$.

Twisted Jacobi identity $\Leftrightarrow Q^2_{\pi,H} = 0$

Courant algebroids, Dirac structures

Let us construct on $E = TM \oplus T^*M$ a twisted exact Courant algebroid structure, governed by a closed 3-form H on M.
The symmetric pairing: $< v \oplus \eta, v' \oplus \eta' >= \eta(v') + \eta'(v)$,
the anchor: $\rho(v \oplus \eta) = v$
the H-twisted bracket (Dorfman):

$$[v \oplus \eta, v' \oplus \eta'] = [v, v']_{\text{Lie}} \oplus (\mathcal{L}_v \eta' - \iota_{v'} \mathrm{d}\eta + \iota_v \iota_{v'} H).$$

Courant algebroids, Dirac structures

A Courant algebroid (Liu, Weinstein, and Xu; Courant) is a vector bundle $E \to M$ equipped with the following operations: a symmetric non-degenerate pairing $< \cdot, \cdot >$ on E, an $\mathbb{R}$-bilinear bracket $[\cdot, \cdot] : \Gamma(E) \otimes \Gamma(E) \to \Gamma(E)$ on sections of E, and an anchor ρ which is a bundle map $\rho : E \to TM$, satisfying the axioms:

$$\begin{aligned} \rho(\varphi) < \psi, \psi >= 2 < [\varphi, \psi], \psi >, \\ [\varphi, [\psi_1, \psi_2]] = [[\varphi, \psi_1], \psi_2] + [\psi_1, [\varphi, \psi_2]], \\ 2\,[\varphi, \varphi] = \rho^*(d < \varphi, \varphi >), \end{aligned}$$

where $\rho^* : T^*M \to E$ (identifying E and E^* by $< \cdot, \cdot >$).

Theorem. (D. Roytenberg) Courant algebroids $\leftrightarrow$ degree 2 symplectic manifolds with compatible Q-structures.

Theorem. (P. Ševera) Exact (ρ surjective, $rkE = 2\,dimM$) Courant algebroids are classified by $H^3_{dR}(M)$.

Courant algebroids, Dirac structures

Let us construct on $E = TM \oplus T^*M$ a twisted exact Courant algebroid structure, governed by a closed 3-form H on M.
The symmetric pairing: $< v \oplus \eta, v' \oplus \eta' >= \eta(v') + \eta'(v)$,
the anchor: $\rho(v \oplus \eta) = v$
the H-twisted bracket (Courant – Dorfman):

$$[v \oplus \eta, v' \oplus \eta'] = [v, v']_{\text{Lie}} \oplus (\mathcal{L}_v \eta' - \iota_{v'} \mathrm{d}\eta + \iota_v \iota_{v'} H). \qquad (1)$$

A Dirac structure $\mathcal{D}$ is a maximally isotropic (Lagrangian) subbundle of an exact Courant algebroid E closed with respect to the bracket (1).

Trivial example: $\mathcal{D} = TM$ for $H = 0$.

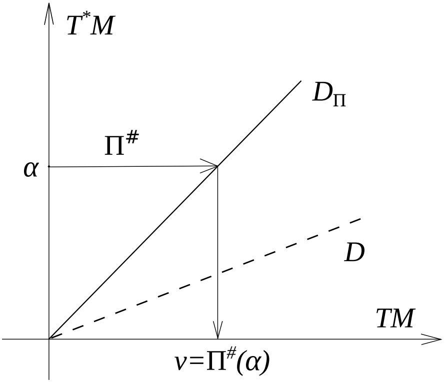
if not – almost Dirac

Dirac structures: Poisson example.

Example. $\mathcal{D} = \textit{graph}(\Pi^\sharp)$

Isotropy $\Leftrightarrow$
π^{ij} antisymmetric.

Involutivity $\Leftrightarrow$
Π (twisted) Poisson.



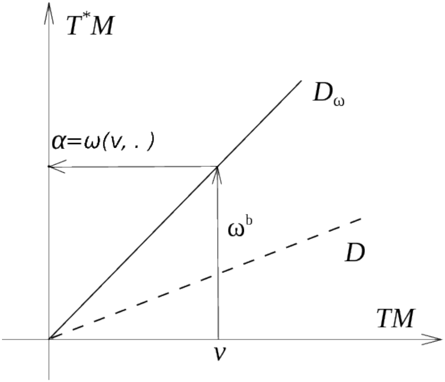
$$\mathcal{D}_\Pi = \{(\Pi^\sharp\alpha, \alpha)\}$$

Dirac structures: (pre)symplectic example.

Example. $\mathcal{D} = graph(\omega)$

Isotropy $\Leftrightarrow$
ω_{ij} antisymmetric.

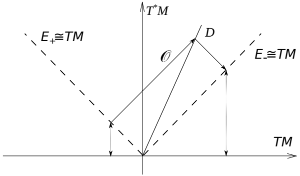
Involutivity $\Leftrightarrow$
ω closed.



$$\mathcal{D}_\omega = \{(v, \iota_v \omega)\}$$

Dirac structures: general

Choose a metric on $M \Rightarrow TM \oplus T^*M \cong TM \oplus TM$,
Introduce the eigenvalue subbundles $E_\pm = \{v \oplus \pm v\}$
of the involution $(v, \alpha) \mapsto (\alpha, v)$. Clearly, $E_+ \cong E_- \cong TM$.



(Almost) Dirac structure – graph of an orthogonal operator $\mathcal{O} \in \Gamma(\mathrm{End}(TM))$:
$(v, \alpha) = ((\mathrm{id} - \mathcal{O})w, g((\mathrm{id} + \mathcal{O})w, \cdot))$
Dirac structure = almost Dirac + (Jacobi-type) integrability condition:

$$g\left(\mathcal{O}^{-1}\nabla_{(\mathrm{id}-\mathcal{O})\xi_1}(\mathcal{O})\xi_2, \xi_3\right) + cycl(1,2,3) = \frac{1}{2}H((\mathrm{id}-\mathcal{O})\xi_1, (\mathrm{id}-\mathcal{O})\xi_2, (\mathrm{id}-\mathcal{O})\xi_3).$$

Remark (!) Any $\mathcal{D}[1]$ can be equipped with a Q-structure

$$Q = (1-O)^i_{i'} a^{i'} \frac{\partial}{\partial x^i} + \frac{1}{2} C^i_{jk} a^j a^k \frac{\partial}{\partial a^i}, \text{ with}$$

$$C^i_{jk} = (1-O)^m_j \Gamma^i_{mk} - (j \leftrightarrow k) + O^{m;i}_k O_{mj} + \tfrac{1}{2} H^i_{j'k'} (1-O)^{j'}_j (1-O)^{k'}_k.$$

and/or Lie algebroid structure inherited from Courant algebroid.

(Histoire de la)

Mécanique

Préservation des structures de la géométrie généralisée en calcul numérique 2/3

Vladimir Salnikov

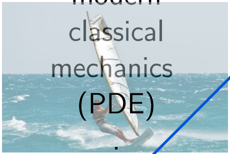


"Paris", 6 Novembre 2020) Year 0 A.C.

Groupement de Recherche
Géométrie Différentielle et Mécanique

Philosophy:

Geometry encodes the physics of the system

	Mechanical property	Geometric description
 classical mechanics (ODE)	conservation of energy	Poisson / symplectic
	symmetries	Lie groups/algebras, Cartan moving frames
	dissipation / interaction power balance; constraints	(almost) Dirac
	control	(singular) foliations
 modern classical mechanics (PDE)	conservation of energy	multisymplectic
	symmetries	Cartan moving frames
	dissipation / interaction	Stokes–Dirac
	$rot(grad) = 0$, $div(rot) = 0$	$d^2 = 0$ – DEC
	control	foliations

Razafindralandy

de Saxcé, Carré

Falaize Hélie, Roze

Hubert

Lozienko

Ryvkin, Laurent-G.

F G M

I G

1

Ryvkin

Ayoub

2 DG/Q variétés

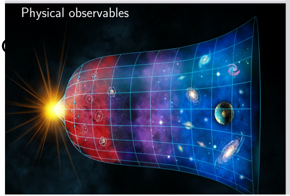
Open questions

Near at hand:

- Poisson integrators → Cosserat
- Dynamics for general Dirac structures → CLKRS
- Application to control theory

Slightly further away

- Field theories / sigma models for mechanics and beyond. E.g. Poisson Sigma Model via n-plectic geometry.
- PDEs in general
- Discretization and symbolic cal… d



Contraintes
(liaisons)

Dynamique de Dirac pour les problemes mecaniques

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Pseudo-geometric integrators

(Daria Loziienko)

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CNRS & La Rochelle University

Other remarks / work in progress

1. We understood why Marsden inspired method was not really geometric.
1.bis it was pseudo-geometric of order (1,2)

holonomic
vs
non-holonomic

2. Dirac-2 was not much better: something like order (1,2 ; 2,3)

3. TODO: I still want it to be (honestly) variational

Approche variationnelle de la dynamique de Dirac

not almost

Vladimir Salnikov

GDR Groupement de recherche
GDR-GDM n°2043
Géométrie Différentielle et Mécanique

ENS-PS, 25 Novembre 2022

cnrs LaSIE Université La Rochelle
La Rochelle Université

Dissipation/
interaction

Systèmes Hamiltoniens à ports apprentissage et un peu de Dirac

PHS

Benjamin

Vladimir Salnikov

CNRS & La Rochelle University

Journée GdR GDM,
IRCAM, 7 mai 2024

Learning the PHS structure – some remarks

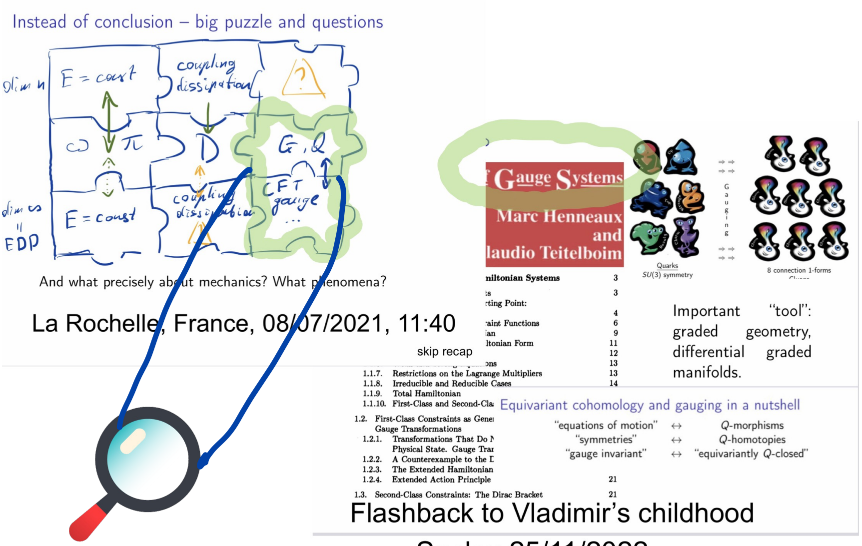
- First step – Machine Learning methods.
  Proof of concept – OK. — Need bounds ⚠
  Hamiltonian VS generic training ? Success criteria ?

- Second step – "catalog" of symplectic / Poisson structures, computation of cohomologies and compatible vector fields.

- Maybe a new way of defining **canonical forms of systems of differential equations**.

Physique

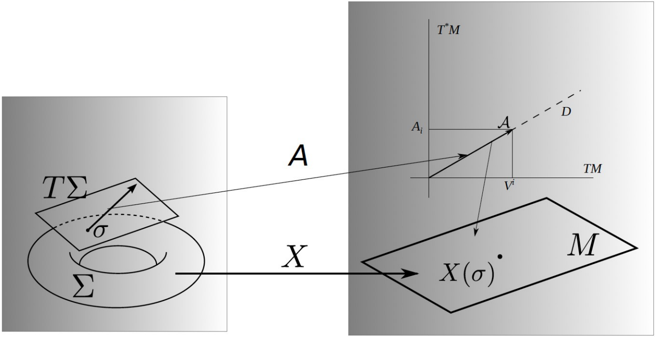
Previous episodes (Paris '23)



Saclay 25/11/2022

Jaugeage des sigma modèles

Sigma models, gauging



World-sheet (space-time) $\mapsto$ Target

Simple examples

$$B \in \Omega^2(M) \quad \text{or} \quad H \in \Omega^3(M)$$

$$S = \int_\Sigma X^* B \qquad S = \int_{\Sigma_3} X^* H$$

Sigma model example – gauging problem

$$S[X] := \int_\Sigma X^* B$$

$B \in \Omega(M)$, $dim(\Sigma) = d = deg(B)$.
Assume that there is a Lie group G acting on M that leaves B invariant. It induces a G-action on M^Σ, which leaves S invariant.

The functional S is called (locally) gauge invariant, if it is invariant even with respect to the group $G^\Sigma \equiv C^\infty(\Sigma, G)$;
the invariance w.r.t. G is called a rigid (global) invariance.

Extending the functional S to a functional $\widetilde{S}$ defined on $(X, A) \in M^\Sigma \times \Omega^1(\Sigma, \mathfrak{g})$ by means of so-called minimal coupling,

$$\widetilde{S}_{2D}[X, A] := \int_\Sigma \left(X^* B - A^a X^* \iota_{v_a} B + \frac{1}{2} A^a A^b X^* \iota_{v_a} \iota_{v_b} B \right) .$$

Example: (part of) the Standard Model

Quarks

$SU(3)$ symmetry

$\Rightarrow \Rightarrow$
$\Rightarrow \Rightarrow$

Gauging

$\Rightarrow \Rightarrow$
$\Rightarrow \Rightarrow$

8 connection 1-forms

Gluons

Equivariant cohomology for Q-manifolds

Let $(\mathcal{M}, Q)$ be a Q-manifold, and let $\mathcal{G}$ be a subalgebra of the algebra of degree -1 commuting vector fields ε on $\mathcal{M}$, which is closed w.r.t. the Q-derived bracket: $[\varepsilon, \varepsilon']_Q = [\varepsilon, [Q, \varepsilon']]$.

Definition. Call a differential form (superfunction) ω on $\mathcal{M}$ <u>$\mathcal{G}$-horizontal</u> if $\varepsilon\omega = 0$, for any $\varepsilon \in \mathcal{G}$.

Definition. Call a differential form (superfunction) ω on $\mathcal{M}$ <u>$\mathcal{G}$-equivariant</u> if $(ad_Q\varepsilon)\omega := [Q, \varepsilon]\omega = 0$, for any $\varepsilon \in \mathcal{G}$.

Definition. Call a differential form (superfunction) ω on $\mathcal{M}$ <u>$\mathcal{G}$-basic</u> if it is $\mathcal{G}$-horizontal and $\mathcal{G}$-equivariant.

Remark. For Q-closed superfunctions $\mathcal{G}$-horizontal $\Leftrightarrow$ $\mathcal{G}$-basic

Remark. Usual equivariant cohomology can be recovered.

Key idea to apply to gauge theories:
Replace "gauge invariant" by "equivariantly Q-closed"

Poisson sigma model

World-sheet: Σ (closed, orientable, with no boundary, $dim = 2$).
Target: Poisson manifold (M, π). The functional is defined over the space of vector bundle morphisms $T\Sigma \to T^*M$

Field content: scalar fields $X^i : \Sigma \to M$ and 1-form valued ("vector") fields: $A_i \in \Omega^1(\Sigma, X^*T^*M)$.

The action functional: $S = \int_\Sigma A_i \wedge \mathrm{d}X^i + \frac{1}{2}\pi^{ij} A_i \wedge A_j$,

Equations of motion:

$$\begin{aligned} \mathrm{d}X^i &+ \pi^{ij} A_j = 0, \\ \mathrm{d}A_i &+ \pi^{jk}_{,i} A_j A_k = 0. \end{aligned}$$

Gauge transformations (complicated !):

$$\begin{aligned} \delta_\varepsilon X^i &= \pi^{ji} \varepsilon_j, \\ \delta_\varepsilon A_i &= \mathrm{d}\varepsilon_i + \pi^{jk}_{,i} A_j \varepsilon_k \end{aligned}$$

where $\varepsilon = \varepsilon_i \mathrm{d}X^i \in \Gamma(X^*T^*M)$ a 1-form.

Gauging the Wess–Zumino term

WZ-term: $H \in \Omega^{dim\Sigma+1}(M), \quad \mathrm{d}H = 0, \quad \partial\tilde{\Sigma} = \Sigma,$

$$S[X] := \int_{\tilde{\Sigma}} X^* H$$

Obstructions to gauging:
B. de Wit, C. Hull, M. Rocek "New topological terms in gauge invariant action" ('87)
C. M. Hull and B. J. Spence, "The Gauged Nonlinear σ Model With Wess-Zumino Term" ('89)
C. M. Hull and B. J. Spence, "The Geometry of the gauged sigma model with Wess-Zumino term" ('91)

Upshot: Gauging of such a WZ-term is possible, if and only if H permits an equivariantly closed extension. (J.M. Figueroa-O'Farrill and S. Stanciu, '94)

Limitation: number of gauge fields $= dimG$.

Equivariant cohomology

A Lie group G acting on a smooth manifold M. $\Omega^\bullet(M/G)$ – ?

First assume that G acts freely on M, i.e. M/G is a topological space. Consider $p\colon M \to M/G$, $\omega_0 \in \Omega^\bullet(M/G)$. $\omega = p^*(\omega_0)$ is well defined, ω is called basic.

Property (defining) of a basic form: $\iota_v \omega = 0$, $\mathcal{L}_v \omega = 0$, $v \in \mathcal{G}$.

Equivariant differential(s): $\tilde{d} = (d + \iota_v)$. $\tilde{d}^2|_{basic} = 0$

If the group does not act freely one can still perform a similar construction but modifying the manifold $M \to M \times EG \to$ huge space of differential forms, but not in cohomology. Instead one considers the Weil model or the Cartan model of equivariant cohomology, by defining the action on the Lie algebra valued connections (of degree 1) and curvatures (of degree 2) with some compatibility conditions.

Remark. d increases the form degree, ι_v decreases.

Equivariant cohomology for Q-manifolds

Let $(\mathcal{M}, Q)$ be a Q-manifold, and let $\mathcal{G}$ be a subalgebra of the algebra of degree -1 commuting vector fields ε on $\mathcal{M}$, which is closed w.r.t. the Q-derived bracket: $[\varepsilon, \varepsilon']_Q = [\varepsilon, [Q, \varepsilon']]$.

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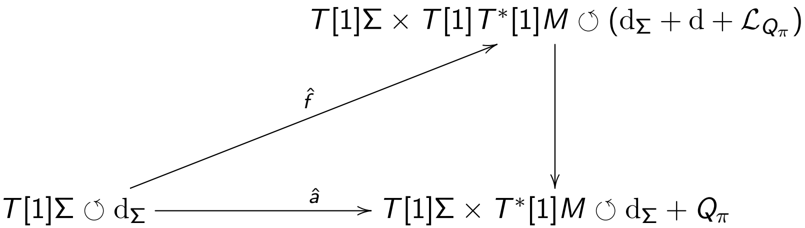
Remark. For Q-closed superfunctions $\mathcal{G}$-horizontal $\Leftrightarrow$ $\mathcal{G}$-basic

Key idea to apply to gauge theories:
Replace gauge invariant by equivariantly Q-closed

Gauge transformations of the PSM



Functional: $S_{PSM} = \int_{\Sigma^3} \hat{f}^*(\mathrm{d}p_i \mathrm{d}x^i)$.

E.o.m.: $\hat{f}^*(\mathrm{d}x^i) = 0$, $\hat{f}^*(\mathrm{d}p_i) = 0$.

Degree -1 vector field on $T^*[1]M$: $\varepsilon = \varepsilon_i \frac{\partial}{\partial p_i}$
$\to$ 1-form on M: $\varepsilon = \varepsilon_i \mathrm{d}x^i$
Canonical lift to $T[1]T^*[1]M$: $\tilde{\varepsilon} = \mathcal{L}_\varepsilon$.

Gauge transformations: $\hat{f}^*([\hat{Q}, \hat{\varepsilon}]\cdot)$. Invariance $\Leftrightarrow$ $\boxed{\mathrm{d}\varepsilon = 0}$

(cf. M.Bojowald, A.Kotov, T.Strobl '04; A.Kotov, T.Strobl '07).

(Twisted) Poisson sigma model

PSM (P.Schaller and T.Strobl, N.Ikeda – 1994)
Functional on vector bundle morphisms from $T\Sigma$ to T^*M, where (M, π) Poisson.

Twisted PSM, PSM with background
(C.Klimcik and T.Strobl, J.-S.Park – 2002)
Functional on vector bundle morphisms from $T\Sigma$ to T^*M, where $(M, (\pi, H))$ twisted Poisson.

$$S_{HPSM} = S_{PSM} + \int_{\Sigma^{(3)}} X^*(H)$$

where $\partial\Sigma^{(3)} = \Sigma$, $H \neq 0 \Rightarrow$ Wess-Zumino term.

Gauge transformations of the twisted PSM

Theorem (V.S., T.Strobl) Any smooth map from Σ to the space $\Gamma(T^*M)$ of sections of the cotangent bundle to a twisted Poisson manifold M defines an infinitesimal gauge transformation of the twisted PSM governed by (π, H) in the above sense, if and only if for any point $\sigma \in \Sigma$ the section $\varepsilon \in \Gamma(T^*M)$ satisfies

$$\boxed{\mathrm{d}\varepsilon - \iota_{\pi^\sharp\varepsilon} H = 0}$$

where d is the de Rham differential on M.

Dirac sigma model

Dirac sigma model (A.Kotov, P.Schaller, T.Strobl – 2005)
Functional on vector bundle morphisms from $T\Sigma$ to $\mathcal{D}$.
(Generalizes twisted PSM and G/G WZW model).

$$S^0_{DSM} = \int_\Sigma g(\mathrm{d}X,^\wedge (1+\mathcal{O})A) + g(A,^\wedge \mathcal{O}A) + \int_{\Sigma_3} H.$$

Important remark: Vector bundle morphisms $\to$ degree preserving maps between graded (Q-) manifolds

Gauge transformations of the DSM

Theorem (V.S., T.Strobl) Any smooth map from Σ to the space $\Gamma(\mathcal{D})$ of sections of the Dirac structure $\mathcal{D} \subset TM \oplus T^*M$ defines an infinitesimal gauge transformation of the (metric independent part of the) Dirac sigma model governed by $\mathcal{D}$ in the above sense, if and only if for any point $\sigma \in \Sigma$ the section $v \oplus \eta \in \Gamma(\mathcal{D})$ satisfies

$$\boxed{\mathrm{d}\eta - \iota_v H = 0}$$

where d is the de Rham differential on M.

Remark 1. H non-degenerate – 2-plectic geometry.

Remark 2. Hydrodynamics (stationary Lamb equation).

Extension of the gauge algebra

Proposition. The Lie algebra $(\tilde{\mathcal{G}}, [,]_{\tilde{Q}})$ of degree -1 commuting vector fields, generalizing the $\mathcal{L}$. lift, is isomorphic to the semi-direct product of Lie algebras $\mathcal{G} \Subset \mathcal{A}$, where
$\mathcal{G}$ is a Lie algebra of 1-forms $T^*[1]M$ with the bracket

$$[\varepsilon^1, \varepsilon^2] = \mathcal{L}_{\pi^{\#}\varepsilon^1}\varepsilon^2 - \mathcal{L}_{\pi^{\#}\varepsilon^2}\varepsilon^1 - \mathrm{d}(\pi(\varepsilon^1, \varepsilon^2)) + \iota_{\pi^{\#}\varepsilon^1}\iota_{\pi^{\#}\varepsilon^2}H,$$

obtained from the (twisted) Lie algebroid of T^*M (anchor $= \pi^{\#}$);
$\mathcal{A}$ is a Lie algebra of covariant 2-tensors on M with a bracket

$$[\bar{\alpha}, \bar{\beta}] =< \pi^{23}, \bar{\alpha} \otimes \bar{\beta} - \bar{\beta} \otimes \bar{\alpha} >,$$

(the upper indeces "23" of π stand for the contraction on the 2d and 3rd entry of the tensor product);
$\mathcal{G}$ acts on $\mathcal{A}$ by

$$\rho(\varepsilon)(\bar{\alpha}) = \mathcal{L}_{\pi^{\#}\varepsilon}(\bar{\alpha}) - < \pi^{23}, (\mathrm{d}\varepsilon - \iota_{\pi^{\#}\varepsilon}H) \otimes \bar{\alpha} > .$$

Gauging $\rightarrow$ twisted Poisson sigma model

Consider a subalgebra $\mathcal{GT} \subset \tilde{\mathcal{G}}$ defined by

$$\mathrm{d}\varepsilon - \iota_{\pi^{\#}\varepsilon} H = 0$$
$$\bar{\alpha}^A = 0.$$

Theorem (V.S., T.Strobl). Consider the graded manifold $\mathcal{M} = T[1]T^*[1]M$, equipped with the Q-structure $\tilde{Q} = \tilde{Q}_\pi$, governed by an H-twisted Poisson bivector Π, such that the pull-back of H to a dense set of orbits of Π is non-vanishing. The $\mathcal{GT}$ equivariantly closed extension of the given 3-form H defines uniquely the functional of the twisted Poisson sigma model.

Remark. Non-degeneracy of the pull-back of H is a sufficient condition; PSM is obtained by gauging a vanishing 3-form.

Gauging $\to$ Dirac sigma model

Consider a subalgebra $\mathcal{GT} \subset \tilde{\mathcal{G}}$ defined by

$$\mathrm{d}\eta - \iota_v H = 0$$
$$\tilde{\alpha}^A = 0.$$

Theorem (V.S., T.Strobl).

Let H be a closed 3-form on M and D a Dirac structure on $(TM \oplus T^*M)_H$ such that the pullback of H to a dense set of orbits of D is non-zero. Then the $\mathcal{GT}$-equivariantly closed extension $\tilde{H}$ of H is unique and $\int_{\Sigma^3} f^*(\tilde{H})$ yields the (metric-independent part of) the Dirac sigma model on $\Sigma = \partial\Sigma^3$.

V.S., T.Strobl, "Dirac Sigma Models from Gauging", Journal of High Energy Physics, 11(2013)110.
V.S. "Graded geometry in gauge theories and beyond", Journal of Geometry and Physics, Volume 87, 2015.

Generality of the DSM in dim = 2.

Standard gauging: introduce Lie algebra valued 1-forms $A^a e_a \in \Omega^1(\Sigma, \mathfrak{g})$.

Lie algebra acting on the target $\Rightarrow$ $e_a \mapsto v_a \in \Gamma(TM)$.
Rigid invariance $\Rightarrow$ $e_a \mapsto \alpha_a \in \Gamma(T^*M)$

Composite gauge fields:
$(V, A) = (v_a A^a, \alpha_a A^a) \in \Omega^1(\Sigma, X^*(TM \oplus T^*M))$.

V and A are dependent $\Rightarrow$ isotropy condition.

Gauge transformations $\Rightarrow$ integrability condition.

Generality of the DSM in dim $= 2$, details

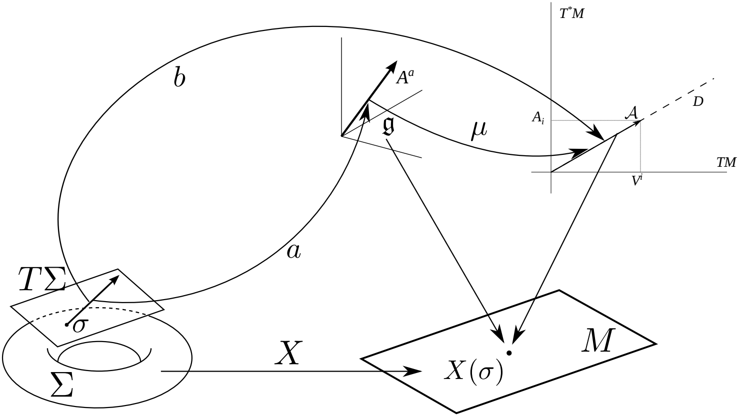
Courant algebroid: $E = TM \oplus T^*M$
Pairing: $< v \oplus \eta, v' \oplus \eta' >= \eta(v') + \eta'(v)$,
Anchor: $\rho(v \oplus \eta) = v$
$[v \oplus \eta, v' \oplus \eta'] = [v, v']_{\text{Lie}} \oplus (\mathcal{L}_v \eta' - \iota_{v'} \mathrm{d}\eta + \iota_v \iota_{v'} H)$. $\quad (*)$

Q–symplectic realization: $T^*[2]T[1]M$ $(p_i(1), \psi_i(2), \theta^i(1), x^i(0))$.
(cf. Roytenberg)
$Q = \{\mathcal{Q}, \cdot\}$, where $\mathcal{Q} = \theta^i \psi_i + \frac{1}{6} H_{ijk} \theta^i \theta^j \theta^k$ and
$\epsilon = \eta_i \theta^i + v^i p_i$, $\varepsilon = \{\epsilon, \cdot\}$. $[\varepsilon, \varepsilon']_Q \Leftrightarrow (*)$

To recover (A, V) gauge transformations, consider
$T[1]E, \tilde{Q} = \mathrm{d} + \mathcal{L}_Q, \tilde{\varepsilon} = \mathcal{L}_\varepsilon$.

$A_i = f^*(p_i), V^i = f^*(\theta^i) \Rightarrow \cdots \Rightarrow \cdots \Rightarrow$ DSM

Generality of the DSM in dim = 2.



A.Kotov, V.S., T.Strobl, "2d Gauge theories and generalized geometry", Journal of High Energy Physics, 08(2014)021.

Et bien
on
observe !

Alekseev - Chekeres

Poisson σ-model for a ~~Wilson~~ surface

- View Wilson surface action as a version of the action S_{ϖ_O} interacting with the external gauge field A.

Poisson σ-model version of the action

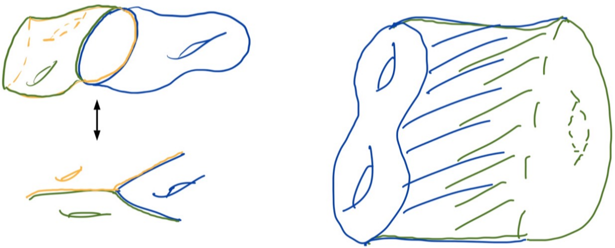
$$S_\sigma(b, A, \alpha) = \int_\Sigma \mathrm{Tr}\, b \left(F_A + (d_A g g^{-1} + \alpha)^2 - (d_A g g^{-1})^2 \right).$$

Poisson σ-model as a BF theory

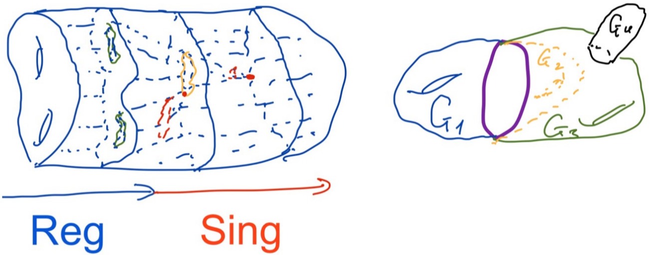
$$S_\sigma(b, A, \alpha) = \int_\Sigma \mathrm{Tr}\, b\, F_{A+\alpha},$$

where the field b is constrained and $A + \alpha$ is a new connection on P.

Généralisations de Wilson surfaces en 2d et SUSY



Interaction proche et distante



Interactions continues et systèmes multivers

Merci

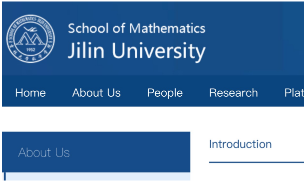
pour votre attention !

Publicité :

1)

2) Postdoc 3 ans

+



et encore merci !