Nonlocal observables in gauge theories

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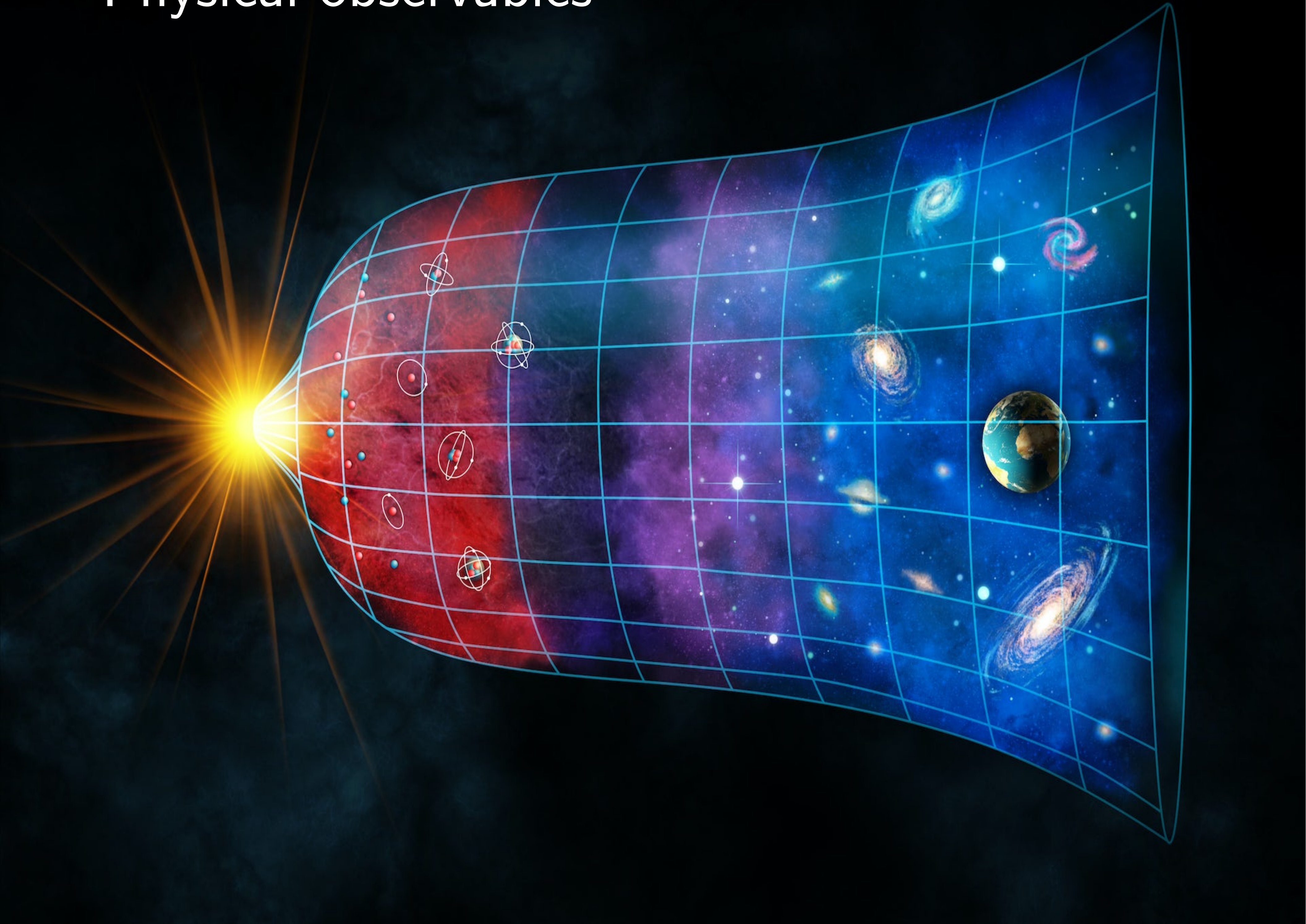
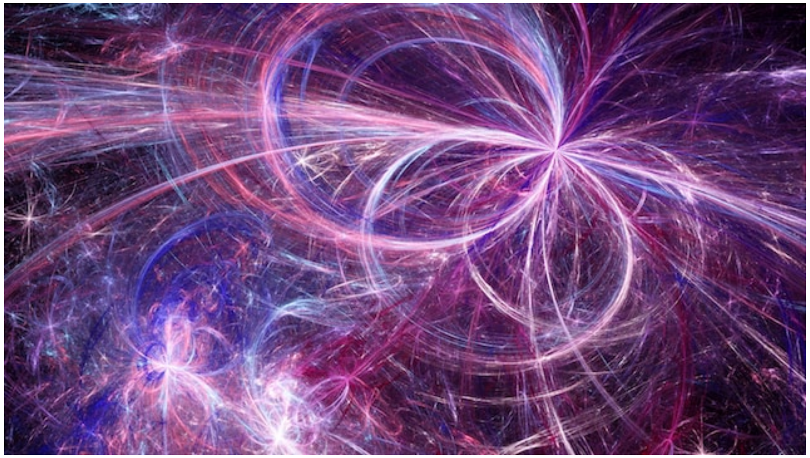
Physical observables

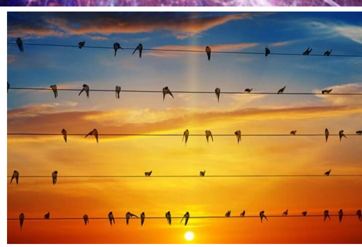


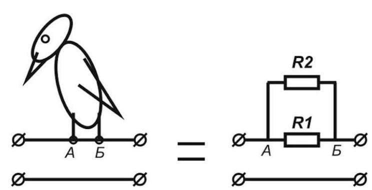
Previous episodes

Exemples des théories de jauge:

Électromagnétisme: $G = U(1)$

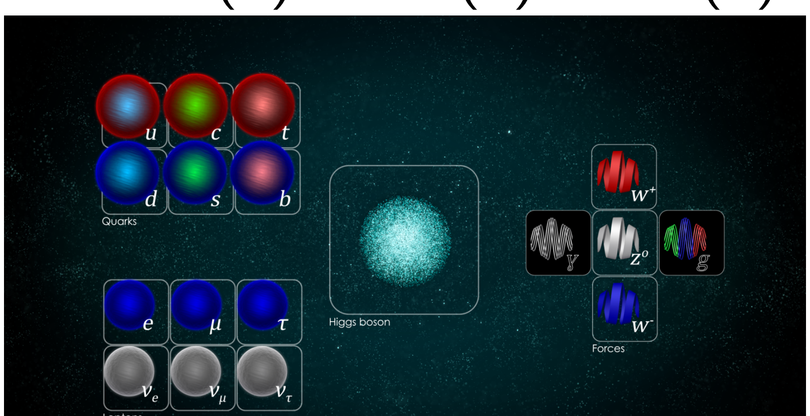






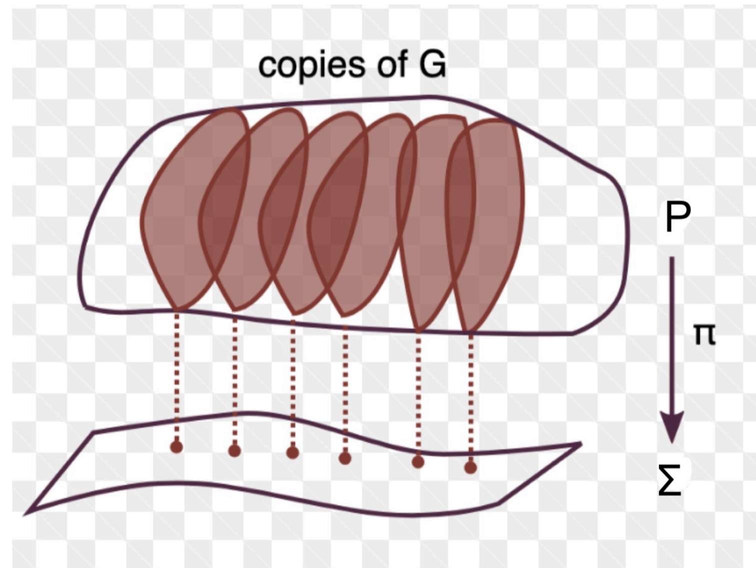
Modèle standard –

$G = U(1) \times SU(2) \times SU(3)$



Géométrie habituelle –

G-fibré principal



Les champs:

connexion $A \in \Omega^1(P, \mathfrak{g})$

force de champ – sa courbure

$F = dA + [A, A] \in \Omega^2(P, \mathfrak{g})$.

Yang–Mills theory

$$S_{YM}(A) = -\frac{1}{2g^2}\int_M Tr\, F_A \wedge *F_A$$

Euler–Lagrange equations:

$$d_A * F_A = d * F_A + [A, *F_A] = 0$$

Bianchi identity:

$$d_A F_A = dF_A + [A, F_A] = 0.$$

Nonlocal observables

- **Wilson loop.**[1]
- **'t Hooft loop** - magnetic analog of a Wilson loop.
- **Wilson surface**
- **Higher dimensional extended objects**

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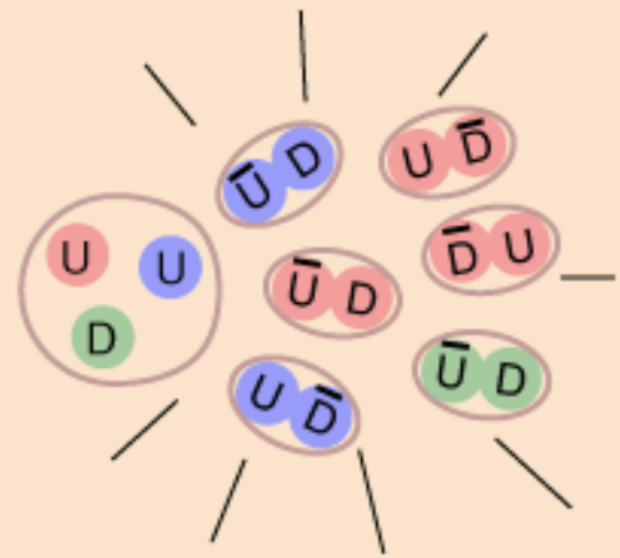
[1] In 1974 Kenneth J. Wilson (1974) argued that quark confinement is equivalent to showing that the relevant lattice gauge theory satisfies what's now known as Wilson's area law.

Confinement of quarks

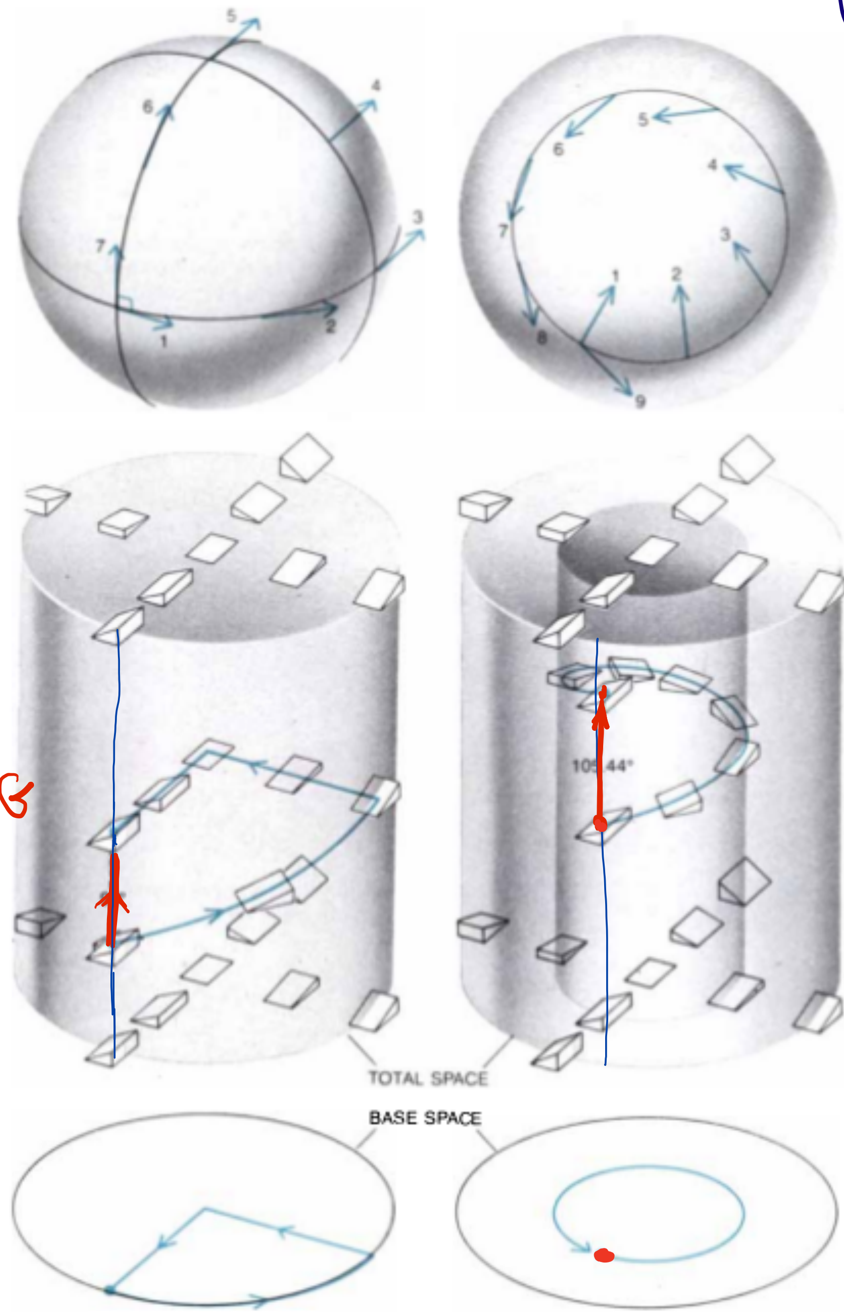
$\langle W_\gamma \rangle \sim e^{-\sigma}$



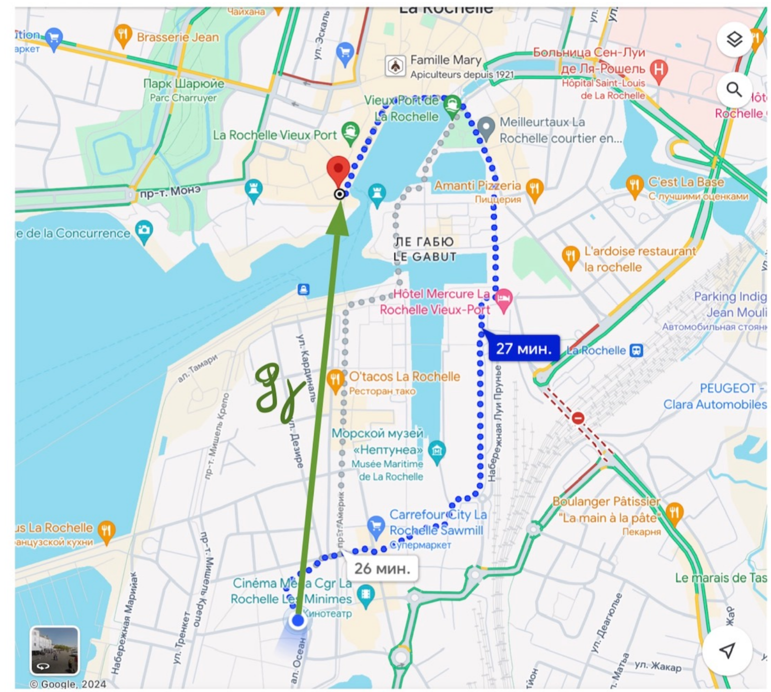
The quarks of a proton are free to move within the proton volume	If you try to pull one of the quarks out, the energy required is on the order of 1 GeV per fermi, like stretching an elastic bag.	The energy required to produce a separation far exceeds the pair production energy of a quark-antiquark pair, so instead of pulling out an isolated quark, you produce mesons as the produced quark-antiquark pairs combine.

Holonomy

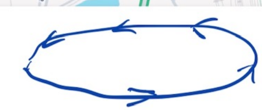
$$W_\gamma^R = \mathrm{Tr}_R \underbrace{P\exp \oint_\gamma A_\mu dx^\mu}_{\text{Holonomy of } A}$$



$g \in H \subset G$



g_γ



γ

$$P \xrightarrow{\pi} M$$

$$P \neq G \times M$$

Holonomy

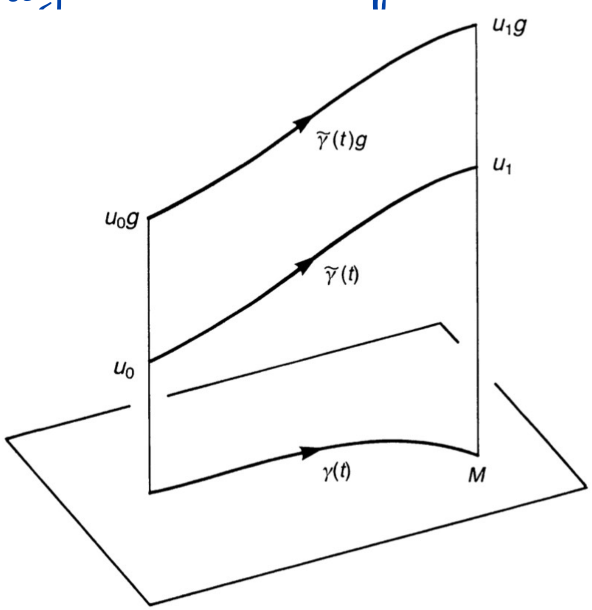
Parallel transport = transport without change

$G \circlearrowright P \longrightarrow M$

$\tilde{\gamma}(t) = \sigma(\gamma(t))\, g(\gamma(t))$

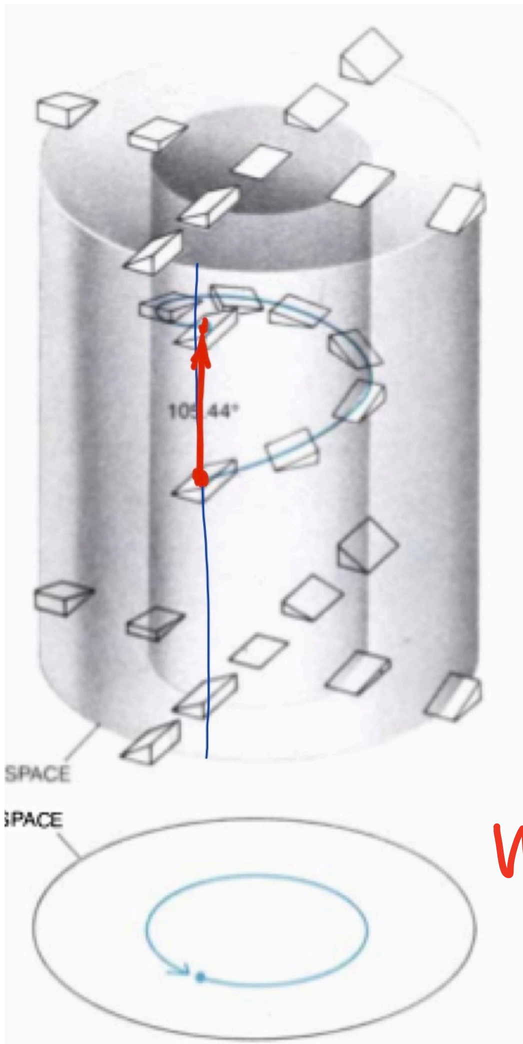
$\gamma : [0,1] \longrightarrow M$

$\tilde{\gamma} : [0,1] \longrightarrow P$ is a horizontal lift of γ if $\pi \circ \tilde{\gamma} = \gamma$ and the tangent vector to $\tilde{\gamma}(t)$ always belongs to $H_{\tilde{\gamma}(t)} P$



Parallel transport from u_0 to u_1

Wilson loop



$$\frac{dg(t)}{dt} = -A(x)\,g(t)$$

Formal solution with $g(0) = \mathrm{Id}$:

$$g(\gamma(t)) = P\exp\left(-\oint_\gamma A_\mu dx^\mu\right)$$

$$W_\gamma^R = \mathrm{Tr}_R \underbrace{P\exp\oint_\gamma A_\mu dx^\mu}_{\text{Holonomy of } A}$$

Wilson loops

1. Quantités invariantes = Variables physiques
2. Correspondent aux representations finies irréductibles de G

3. Donnent toute l'info sur les potentiels de jauge (jusqu'aux transformations de jauges)

$$\underbrace{W_\gamma^R \longrightarrow h(\gamma) \longrightarrow \text{champs de jauge}}_{}$$

Ambrose-Singer thm.

[R. Giles, 1981] ⟵ reconstruction des potentiels de jauge

Wilson Loop

A beautiful observation:

$$R \longleftrightarrow \mathcal{H}$$

$$\mathrm{Tr}_R \, P \exp\left(\int_\Gamma A^R\right) \longleftrightarrow \mathrm{Tr}_{\mathcal{H}} \, P \exp\left(-i \int_\Gamma H\right) \tag{1}$$

Γ - time line with periodic boundary conditions

? Some classical phase space

G acts on it as a symmetry

$$\Big\Downarrow \text{quantize}$$

$$\mathcal{H} \cong R$$

time evolution operator $\longleftrightarrow$ holonomy(A)

Orbits of coadjoint group action

adjoint action: $Ad_g(\xi) = g^{-1}\xi g$,
$g \in G$, $\xi \in \mathfrak{g}$

- An irreducible finite dimensional representation is uniquely determined by its highest weight $\lambda \in \mathfrak{h}^*$, $\mathfrak{h} \subset \mathfrak{g}$ is a Cartan subalgebra of $\mathfrak{g}$.
- Associate to λ the orbit of the coadjoint action in the space of $\mathfrak{g}^*$.
- Denote the coadjoint action by $Ad^*_g(\lambda) = g\lambda g^{-1}$.

The coadjoint orbit:

$$\mathcal{O}_\lambda = \{g\lambda g^{-1} | \lambda \in \mathfrak{g}^*, g \in G\}.$$

Path Integral presentation of a Wilson Line

(due to Alekseev, Faddeev, Shatashvili)

$b : \Gamma \longrightarrow \mathfrak{g}^*$ auxiliary field with values in $\mathcal{O}_\lambda$

$g : \Gamma \longrightarrow G$

such that $b(s) = g(s)\,\lambda\, g(s)^{-1}$ $\quad \lambda \in \mathcal{O}_\lambda$, $\lambda \in \mathfrak{g}^*$

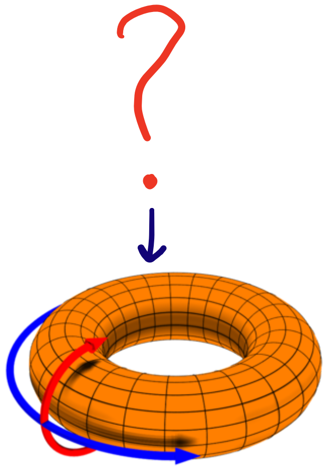
$\alpha = \langle \lambda, g^{-1}dg \rangle$ symplectic potential

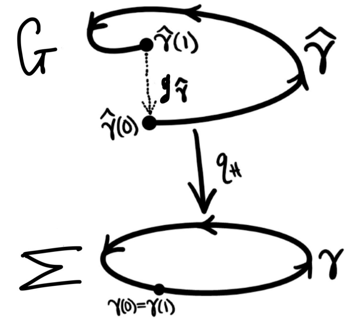
$\omega = \langle \lambda, (g^{-1}dg)^2 \rangle$

$H = -\langle \lambda, g^{-1}Ag \rangle$

$$S = \int_\Gamma \mathrm{Tr}\left(\lambda\left(g^{-1}dg + g^{-1}Ag\right)\right) = \int_\Gamma \mathrm{Tr}\left(b\left(dg\,g^{-1} + A\right)\right)$$

Les observables non-locales. Boucle et surface de Wilson





$$W_\gamma^R = Tr_R Pexp\Big(\int_\gamma A\Big) = \int \mathcal{D}g e^{iS_\lambda(g,A)},$$

où $S_{WL}(A,g,b) = \int_\gamma Trb\Big(dgg^{-1} + A\Big)$ est la fonctionnelle d'action.

$$S_{WS}(A,g) = \int_\Sigma Trb\Big(F_A + (d_A g g^{-1})^2\Big)$$

Résultat important: S_{WS} est définie par une extension équivariante de la forme de Kirillov–Kostant–Souriau sur $\mathcal{O}_\lambda$

Poisson σ-model for a Wilson surface

- View Wilson surface action as a version of the action S_{ϖ_O} interacting with the external gauge field A.

Poisson σ-model version of the action

$$S_\sigma(b, A, \alpha) = \int_\Sigma \mathrm{Tr}\, b \left(F_A + (d_A g g^{-1} + \alpha)^2 - (d_A g g^{-1})^2 \right).$$

Poisson σ-model as a BF theory

$$S_\sigma(b, A, \alpha) = \int_\Sigma \mathrm{Tr}\, b\, F_{A+\alpha},$$

where the field b is constrained and $A + \alpha$ is a new connection on P.

Wilson surface theory

Theory on a closed surface

The formula of the Wilson surface theory for a closed surface for a class $[P]$ of principal bundles $P \to \Sigma$:

$$Z_{WS}^{\Sigma}(C_i, \lambda) = \frac{\chi_\lambda(C_i)}{d_\lambda}. \qquad (10)$$

Result 3: Topological interactions

The presence of a Wilson surface modifies the partition function of the background theory multiplying by a phase $e^{i\varphi_\gamma} = \frac{\chi_\lambda(C_\gamma)}{d_\lambda}$ the individual contributions for each class of principal bundles:

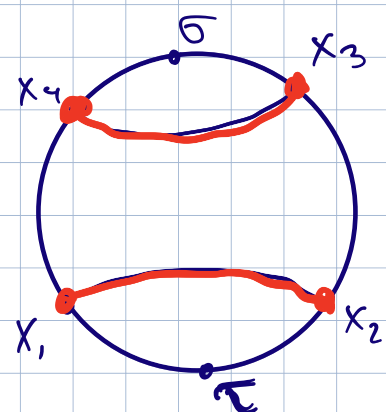
$$Z^{interact} = \sum_{\gamma \in \pi_1(G)} Z^{backgr}(C_\gamma) \cdot e^{i\varphi_\gamma}.$$

Bilocal observables in Schwarzian quantum mechanics

$$\langle \mathcal{O}(x_1,x_2)\,\mathcal{O}(x_3,x_4)\rangle = \frac{1}{Z}\int_{x_3}^{x_4} d\sigma \int_{x_1}^{x_2} d\tau\, e^{-\tilde{S}(u,\tau,\sigma)}$$

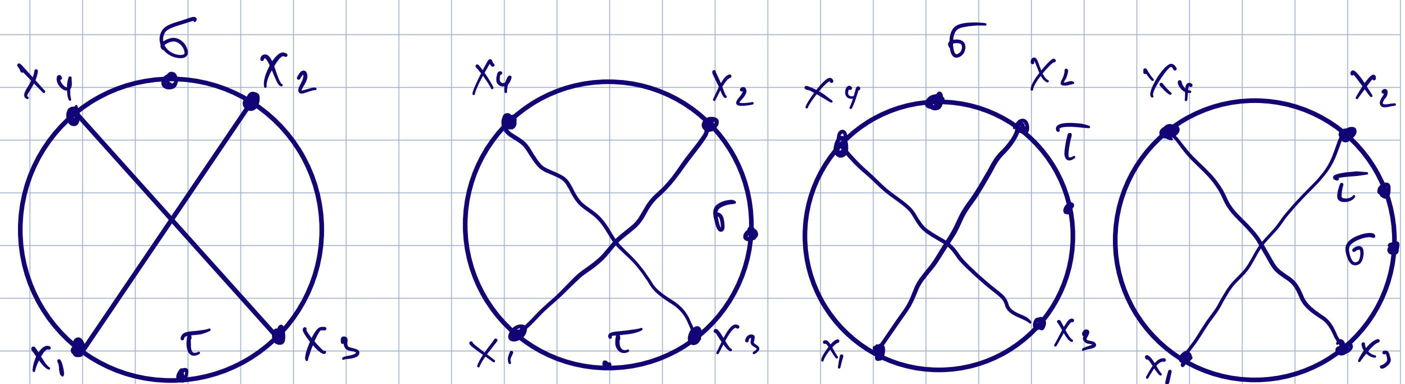
$$\tilde{S}(u,\sigma,\tau) = \int_0^L u'^2\,dx + \frac{1}{2}\sum_{i=1}^{4} u(x_i) - u(\sigma) - u(\tau)$$

TO ;

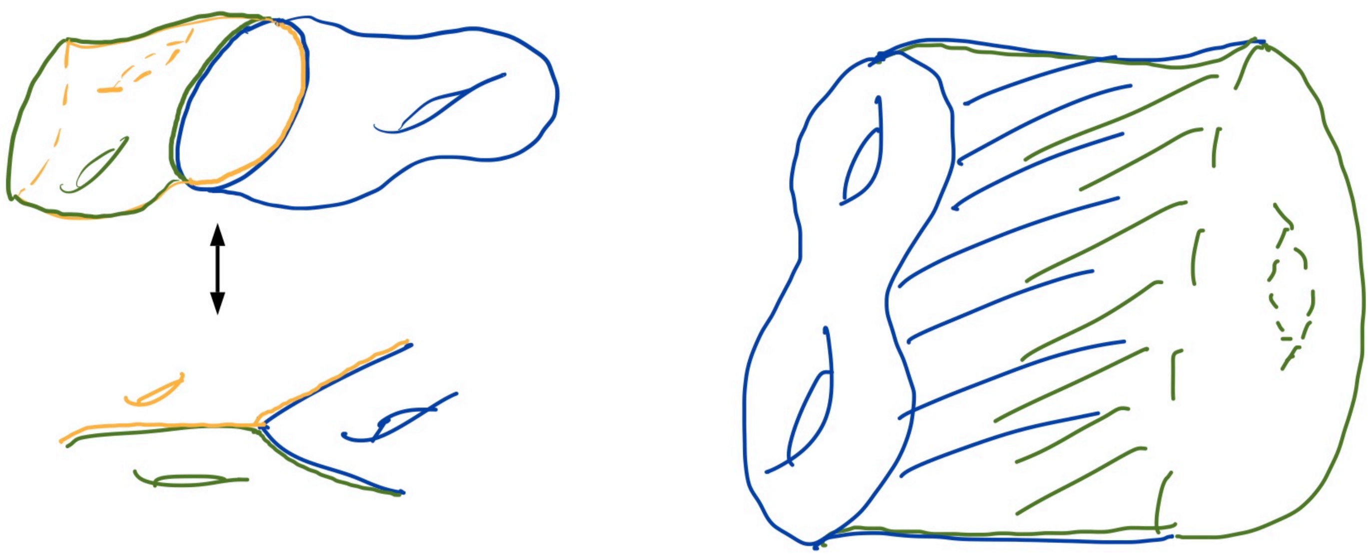


$\langle \mathcal{O}\mathcal{O}\rangle_{OTO} \sim e^{\lambda\Delta}$ · chaos

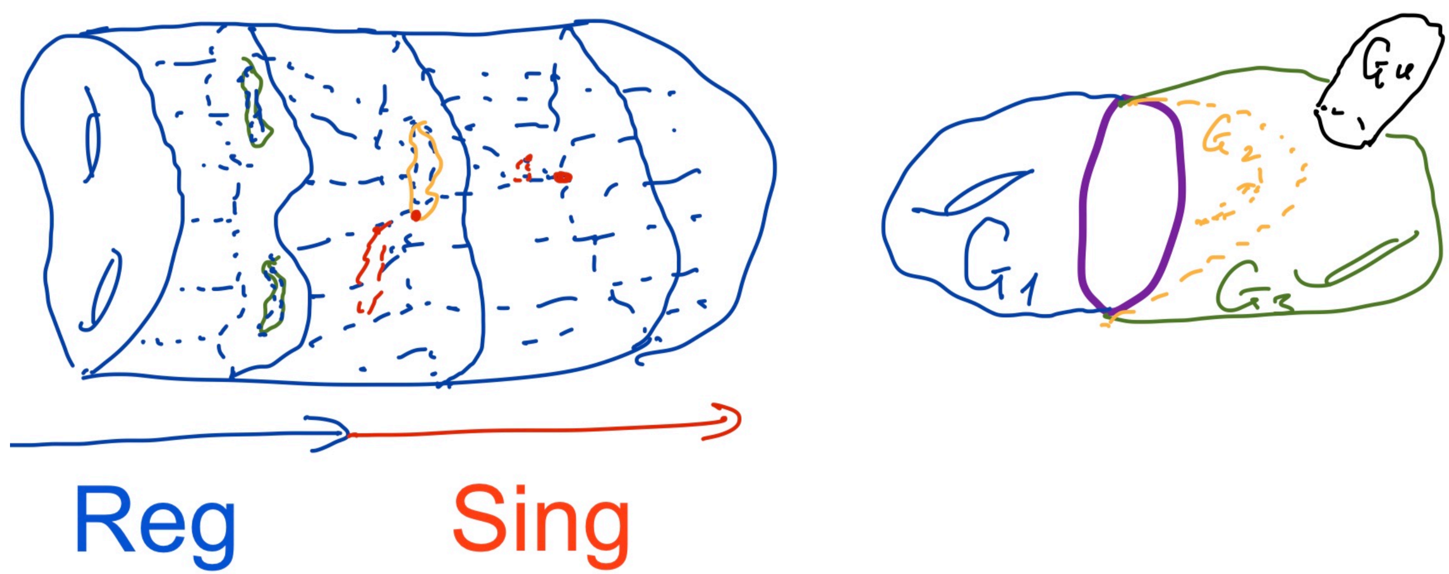
OTO ;



Généralisations de Wilson surfaces en 2d et SUSY



Interaction proche et distante



Interactions continues et systèmes multivers

Généralisations supersymétriques

- Lignes de Wilson supersymétriques. Superholonomie (avec Anton Galaev, Alexei Kotov et Vladimir Salnikov)

$$W_R^\gamma = \mathrm{tr}_R \, P\exp \oint_\gamma \left(A_\mu dx^\mu + \Phi ds \right)$$

gauge field $\longrightarrow$ gauge multiplet $\quad A, \Phi, \Psi$

$\delta A \sim \Psi \;, \; \delta\Phi \sim \Psi$

- n-défauts supersymétriques / observables

Based on

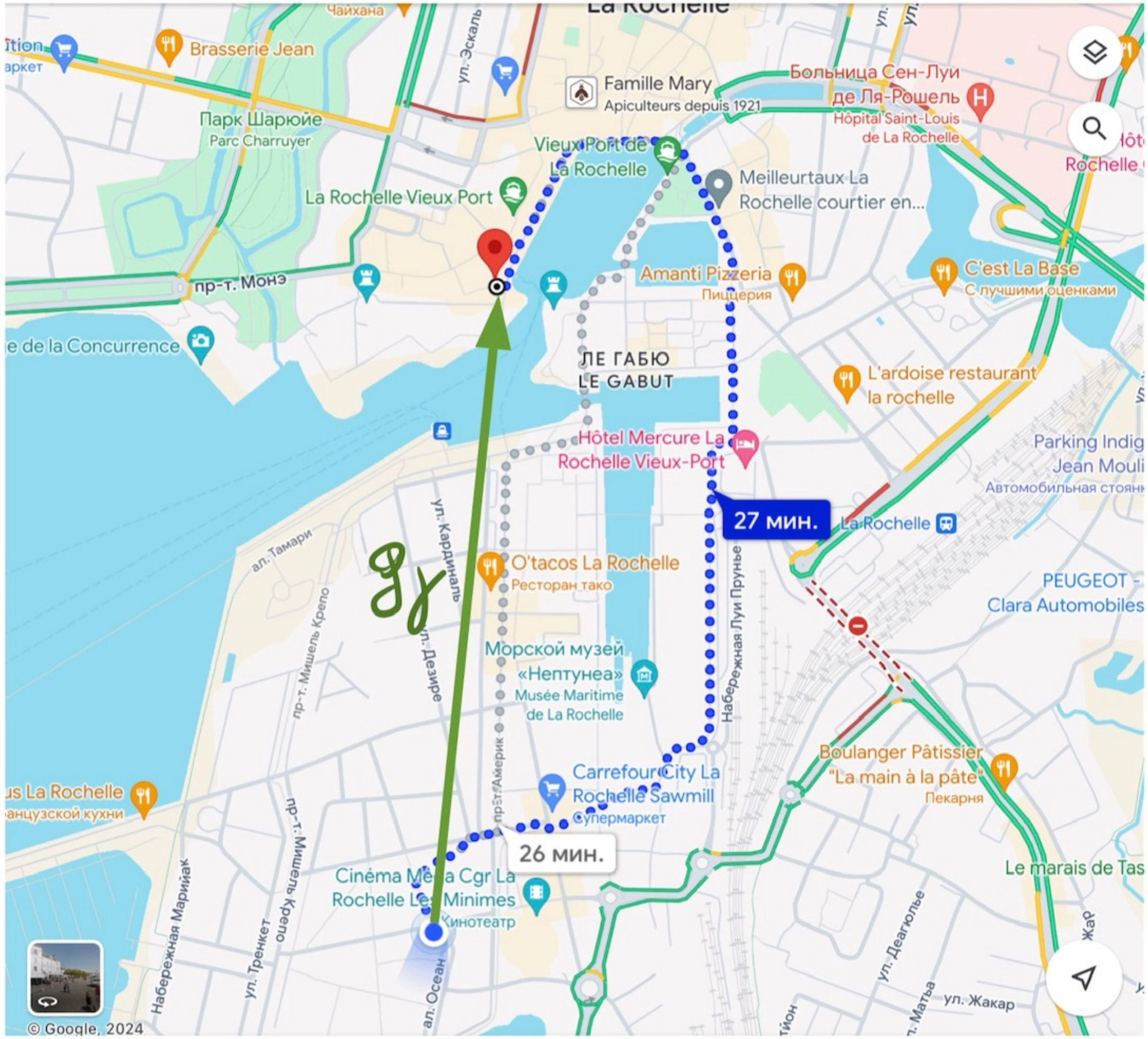
[1] O. Chekeres, **Quantum Wilson surfaces and topological interactions**, Journal of High Energy Physics 02(2019)030

[2] A. Alekseev, O. Chekeres, P. Mnev,
Wilson surface observables from equivariant cohomology,
Journal of High Energy Physics 11(2015)093

[3] O. Chekeres, V. Salnikov,
Odd Wilson surfaces, arXiv:2403.09820 [hep-th]

[2] A. Alekseev, O. Chekeres, D. R. Youmans,
Towards Bosonization of Virasoro Coadjoint Orbits, Annales Henri Poincaré 25 (2024)

Merci pour votre attention!



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Bonne baignade!