Laboratoire de Mécanique
des Contacts et des Structures
UMR 5259

On thermodynamically admissible data driven computational mechanics

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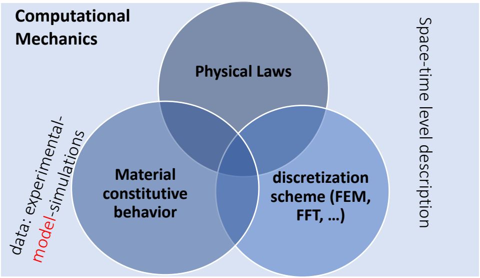
Merging Computational Mechanics (CM) and Data Driven (DD) approaches

Data are ubiquitous in computational mechanics

input parameters:
loading, BCs,
geometry, …



output quantities of interest:

Local fields
Tensor components
Energetic quantities,
…

Multi-dimensionality curse (in/out)
Material Model ignorance

- ✓ ROM based surrogate models: by pass expansive computational process
- ✓ Model Free solvers: by pass the modeling step (data-simulations paradigm)
- ✓ **Data are agnostic ➔ Physically based approaches**

ROM based surrogate model

Real time 10D welding simulations

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Model free computational mechanics

Explicit model computational mechanics

$$\begin{cases} \operatorname{div}\boldsymbol{\sigma} + \mathbf{f} = \mathbf{0} & \mathbf{x} \in \Omega \\ \boldsymbol{\varepsilon} = \boldsymbol{\nabla}^s \mathbf{u} & \mathbf{x} \in \Omega \\ \mathbf{u} = \mathbf{u}_0 \quad \mathbf{x} \in \Gamma^D; & \boldsymbol{\sigma} \cdot \mathbf{n} = \mathbf{t}_0 \quad \mathbf{x} \in \Gamma^N \\ \boldsymbol{\sigma} = \partial_{\boldsymbol{\varepsilon}} \Psi \quad \text{and} & \dot{\boldsymbol{\alpha}} \in \partial_{\boldsymbol{\mathcal{A}}} \varphi \end{cases}$$

Model free computational mechanics

$$\begin{cases} \operatorname{div}\boldsymbol{\sigma} + \mathbf{f} = \mathbf{0} & \mathbf{x} \in \Omega \\ \boldsymbol{\varepsilon} = \boldsymbol{\nabla}^s \mathbf{u} & \mathbf{x} \in \Omega \\ \mathbf{u} = \mathbf{u}_0 \quad \mathbf{x} \in \Gamma^D; & \boldsymbol{\sigma} \cdot \mathbf{n} = \mathbf{t}_0 \quad \mathbf{x} \in \Gamma^N \\ \mathcal{M} = \{\boldsymbol{z}^* \mid \boldsymbol{z}^* = (\boldsymbol{\varepsilon}^*, \boldsymbol{\sigma}^*)\} \end{cases}$$

Double distance problem: the DDCM solution is given by [T.Kirchdoerfer and M.Ortiz, 2016]

$$\boldsymbol{z}^{\text{sol}} = \min_{\boldsymbol{z}^* \in \mathcal{M}} \min_{\boldsymbol{z} \in \mathcal{P}} \|\boldsymbol{z} - \boldsymbol{z}^*\|_\bullet^2 \quad \text{with} \quad \mathcal{P} = \mathcal{C}^{\text{adm}} \cap \mathcal{S}^{\text{adm}}$$

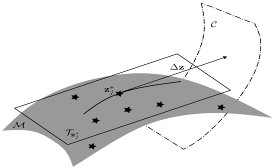
Alternate minimization problem: iterative alternate global-local optimization

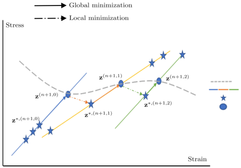
Global step : $$(\boldsymbol{z})^k = \arg\min_{\boldsymbol{z} \in \mathcal{P}} \|\boldsymbol{z} - (\boldsymbol{z}^*)^k\|_\bullet^2 = \frac{1}{2} \int_\Omega (\boldsymbol{\varepsilon} : \mathbb{A} : \boldsymbol{\varepsilon} + \boldsymbol{\sigma} : \mathbb{A}^{-1} : \boldsymbol{\sigma})\, dV$$

Local step : $$(\boldsymbol{z}^*)^{k+1} = \arg\min_{\boldsymbol{z}^* \in \mathcal{M}} \|\boldsymbol{z}^* - (\boldsymbol{z})^k\|_\cdot^2 = \frac{1}{2} (\boldsymbol{\varepsilon} : \mathbb{A} : \boldsymbol{\varepsilon} + \boldsymbol{\sigma} : \mathbb{A}^{-1} : \boldsymbol{\sigma})$$

Tangent distance DDCM for dissipative media[1]

- Thermodynamically consistent model free framework
- Tangent distance minimization
- Observable state variables
- Material data basis: $\boldsymbol{\varepsilon}^*, \boldsymbol{\sigma}^*, \Psi^*, \mathbb{C}_t$





- Variational global problem at iteration $k+1$ consists in:

$$
\left\{
\begin{array}{l}
\text{having the pair } \boldsymbol{z}^* = (\boldsymbol{\varepsilon}^*, \boldsymbol{\sigma}^*)\text{, find the pair } \boldsymbol{z} = (\boldsymbol{\varepsilon}, \boldsymbol{\sigma}) \text{ that minimizes the functional:} \\
\mathcal{L}(\mathbf{u}, \boldsymbol{\sigma}, \Delta\boldsymbol{\varepsilon}, \boldsymbol{\eta}, \bar{\boldsymbol{\eta}}, \boldsymbol{\lambda}) = \| (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^* - \Delta\boldsymbol{\varepsilon}, \boldsymbol{\sigma} - \boldsymbol{\sigma}^* - \mathbb{C}_t : \Delta\boldsymbol{\varepsilon}) \|^2_{\mathbb{A}(\Omega)} \\
+ \int_{\Gamma_D} \boldsymbol{\lambda} \cdot (\mathbf{u} - \mathbf{u}_0)\, dS + \int_{\Omega} \boldsymbol{\eta} \cdot (\boldsymbol{\nabla} \cdot \boldsymbol{\sigma} + \mathbf{f})\, dV + \int_{\Gamma_N} \bar{\boldsymbol{\eta}} \cdot (\boldsymbol{\sigma} \cdot \mathbf{n} - \mathbf{t}_0)\, dV
\end{array}
\right.
$$

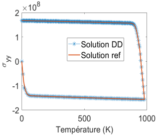
- Plasticity evolution: $(\boldsymbol{z}, \boldsymbol{z}^*) \rightarrow \Psi^*(\boldsymbol{\varepsilon}^*) \rightarrow d = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - \Psi^*(\boldsymbol{\varepsilon}^*)$

[1] Pham, D., Blal, N., Gravouil, A., 2023. Tangent distance data driven computational mechanics for irreversible behaviors, (FEAD)

Tangent distance DDCM for dissipative media

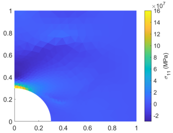
Applications

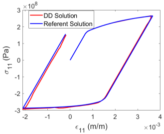
- Mixed Chaboche hardening
  - State laws: $\mathbf{X} = \frac{2}{3} AC\boldsymbol{\alpha}$, $R = bQq$
  - Evolution laws: $\dot{\boldsymbol{\alpha}} = \dot{\boldsymbol{\epsilon}}_p - C\boldsymbol{\alpha}\dot{p}$, $\dot{R} = (1 - bq)\dot{p}$
  - Yielding surface: $f(\boldsymbol{\sigma}, \mathbf{X}, R) = \sigma_{eq}(\boldsymbol{\sigma} - \mathbf{X}) - \sigma_y - R$



- Non-linear isotropic hardening

$$R(p) = \sigma_\infty + (\sigma_y - \sigma_\infty)\exp(-bp)$$





Concluding remarks

- Good agreement compared to standard FEM
- Data greedy strategy
- Difficulty to experimentally obtain all the needed data
- An hybrid approach: $\|z^* - z^{BK} + \delta z\|$

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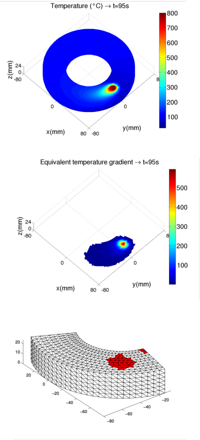
Reduced Order Models

- Reducing the system dimension complexity
- Linear(ized) system size significantly reduced
- Need to reduce the dissipation flow equations → Hyper-reduction
- Multi-parametric simulations: rapid simulations and **real time** simulations

Variational Reduced Order Modeling (VROM)

Propose reduced order models devoted to dissipative continua within a variational framework that preserves the constitutive laws structure



(a) Selected element in control volume

D.Ryckelynck. A priori hyperreduction model : an adaptive approach. International Journal of Computational Physics, 202 :346–366. 2005
Zhang and al. Efficient hyper reduced-order model (HROM) for parametric studies of the 3D thermo-elasto-plastic calculation. FEAD, 2014

Thermodynamically admissible Full Order Model

▶ Generalized Standard Materials (GSM) framework

- State variables: $\boldsymbol{\varepsilon}$ strain field, θ the temperature and $\boldsymbol{\alpha}$ internal variables
- Convex positive semi-continuous potentials: Ψ and φ (or equivalently φ^*)

$$\begin{cases} s = \dfrac{\partial \Psi}{\partial \theta}, \quad \boldsymbol{\sigma}^{\text{rev}} = \dfrac{\partial \Psi}{\partial \boldsymbol{\varepsilon}}, \quad \boldsymbol{X}_{\boldsymbol{\alpha}} = \dfrac{\partial \Psi}{\partial \boldsymbol{\alpha}} \\ \dfrac{\boldsymbol{\nabla}\theta}{\theta} \in \dfrac{\partial \varphi}{\partial \mathbf{q}}, \quad \boldsymbol{\sigma}^{\text{irr}} \in \dfrac{\partial \varphi}{\partial \dot{\boldsymbol{\varepsilon}}}, \quad \boldsymbol{\mathcal{A}} \in \dfrac{\partial \varphi}{\partial \dot{\boldsymbol{\alpha}}} \Leftrightarrow \mathbf{q} \in \dfrac{\partial \varphi^*}{\partial \boldsymbol{\nabla}\theta/\theta}, \quad \dot{\boldsymbol{\varepsilon}} \in \dfrac{\partial \varphi^*}{\partial \boldsymbol{\sigma}^{\text{irr}}}, \quad \dot{\boldsymbol{\alpha}} \in \dfrac{\partial \varphi^*}{\partial \boldsymbol{\mathcal{A}}} \end{cases} \quad ; \quad \boldsymbol{\sigma} = \boldsymbol{\sigma}^{\text{rev}} + \boldsymbol{\sigma}^{\text{irr}}$$

▶ Extended Lagrangian functional for dissipative media

$$\mathcal{A} = \int_I \int_\Omega \underbrace{\rho \|\dot{\mathbf{u}}\|^2}_{2\mathcal{K}} dV\, dt \Rightarrow \boxed{\mathcal{L}(\mathbf{u}, \theta, \boldsymbol{\alpha}) = \int_I \left(\mathcal{K} - \mathcal{E}_p - \int_\Omega \int \left(\rho \dot{\theta} s - \mathbf{q} \cdot \frac{\boldsymbol{\nabla}\theta}{\theta} \right) d\tau dV - \int_\Omega \int \mathcal{D} d\tau dV \right) dt}$$

with : $\mathcal{D} = \rho\theta s^{\text{irr}} = \boldsymbol{\sigma}^{\text{irr}} : \dot{\boldsymbol{\varepsilon}} - \dfrac{\partial \Psi}{\partial \boldsymbol{\alpha}} \cdot \dot{\boldsymbol{\alpha}}$ the irreversible entropy production

- Going variational

$$\begin{cases} \int_\Omega \rho \ddot{\mathbf{u}} \cdot \delta \mathbf{u} dV - \int_\Omega (\boldsymbol{\nabla} \cdot \Psi_{,\boldsymbol{\varepsilon}} + \mathbf{f}) \cdot \delta \mathbf{u} dV = 0 \quad ; \int_{\partial\Omega} (\boldsymbol{\sigma} \cdot \mathbf{n} - \mathbf{t}) \cdot \delta \mathbf{u} dV = 0 \quad \forall \delta \mathbf{u} \in \mathcal{V}_u \\ \int_I \int_\Omega \left(\frac{\partial \Psi}{\partial \boldsymbol{\alpha}} + \frac{\partial \varphi}{\partial \dot{\boldsymbol{\alpha}}} \right) \cdot \delta \boldsymbol{\alpha} dt = 0 \quad \forall \delta \alpha \in \mathcal{V}_\alpha \\ \int_I \int_\Omega \int \left(\rho c_p \dot{\theta} + \operatorname{div} \mathbf{q} - \rho\theta \frac{\partial}{\partial \theta} (\boldsymbol{\sigma}^{\text{rev}} : \dot{\boldsymbol{\varepsilon}} - \boldsymbol{\mathcal{A}} : \dot{\boldsymbol{\alpha}}) - (\boldsymbol{\sigma}^{\text{irr}} : \dot{\boldsymbol{\varepsilon}} + \boldsymbol{\mathcal{A}} \cdot \dot{\boldsymbol{\alpha}}) - r \right) \delta\theta d\tau dV dt = 0 \end{cases}$$

- FOM thermo-mechanical local field equations

$$\Rightarrow \boxed{\begin{cases} \rho \ddot{\mathbf{u}} = \boldsymbol{\nabla} \cdot \boldsymbol{\sigma} + \mathbf{f} \quad \forall \mathbf{x} \in \Omega \quad \text{and} \quad \boldsymbol{\sigma} \cdot \mathbf{n} = \mathbf{t} \quad \forall \mathbf{x} \in \partial\Omega \\ \frac{\partial \Psi}{\partial \boldsymbol{\alpha}} + \frac{\partial \varphi}{\partial \dot{\boldsymbol{\alpha}}} = 0 \quad \forall \mathbf{x} \in \Omega \\ \rho c_p \dot{\theta} + \operatorname{div} \mathbf{q} = \rho\theta \frac{\partial}{\partial \theta} (\boldsymbol{\sigma}^{\text{rev}} : \dot{\boldsymbol{\varepsilon}} - \boldsymbol{\mathcal{A}} : \dot{\boldsymbol{\alpha}}) + (\boldsymbol{\sigma}^{\text{irr}} : \dot{\boldsymbol{\varepsilon}} + \boldsymbol{\mathcal{A}} \cdot \dot{\boldsymbol{\alpha}}) + r \quad \forall \mathbf{x} \in \Omega \end{cases}}$$

Revisiting dissipative ROM within a variational thermodynamically admissible framework

- Reduced state variables bases

$$(\mathbf{u}, \theta, \boldsymbol{\alpha}) \in \text{Span}\left\{\boldsymbol{\Upsilon}_i\right\}_{i=1..N_u} \times \text{Span}\left\{\Theta_i\right\}_{i=1..N_\theta} \times \text{Span}\left\{\boldsymbol{\Phi}_i\right\}_{i=1..N_a}$$

- Variational Reduced Model (VROM) ansatz

$$\mathbf{u}\left(\mathbf{x}, t; \mu\right) = \sum_{i=1}^{N_u} \boldsymbol{\Upsilon}_i\left(\mathbf{x}, \mu\right)\xi_i(t) \quad \boldsymbol{\alpha}\left(\mathbf{x}, t, \mu\right) = \sum_{i=1}^{N_a} \boldsymbol{\Phi}_i\left(\mathbf{x}, \mu\right)\alpha_i(t) \quad \theta\left(\mathbf{x}, t; \mu\right) = \sum_{i=1}^{N_a} \Theta_i\left(\mathbf{x}, \mu\right)_i \theta_i(t)$$

- VROM framework: find the reduced variables solution of the stationary problem

$$\mathcal{L}^{\text{ROM}} \rightarrow \underset{\{\alpha_i\},\{\xi_i\},\{\theta_i\}}{\text{Stat}}$$

keeping a GSM framework:

$$\Psi^{\text{ROM}}(\overline{\boldsymbol{\xi}}, \overline{\boldsymbol{\alpha}}) \quad \text{and} \quad \varphi^{\text{ROM}}(\dot{\overline{\boldsymbol{\alpha}}}) \quad \text{s.t.} \quad \begin{cases} \boldsymbol{\tau} = \partial_{\overline{\boldsymbol{\xi}}}\Psi^{\text{ROM}} \quad \overline{X}_a = \partial_{\overline{\boldsymbol{\alpha}}}\Psi^{\text{ROM}} \\ \overline{\mathbf{a}} \in \partial_{\dot{\overline{\boldsymbol{\alpha}}}}\varphi^{\text{ROM}} \Leftrightarrow \dot{\overline{\boldsymbol{\alpha}}} \in \partial_{\overline{\mathbf{a}}}\left(\varphi^{\text{ROM}}\right)^* \end{cases}$$

Rate dependent materials

- FOM dissipation potential and non-conservative thermodynamical forces

$$\varphi(\dot{\boldsymbol{\alpha}}) = \frac{1}{2}\eta\|\dot{\boldsymbol{\alpha}}\|^2 \quad \text{where} \quad \boldsymbol{\mathcal{A}} = \frac{\partial\varphi}{\partial\dot{\boldsymbol{\alpha}}} = \eta\dot{\boldsymbol{\alpha}}$$

- VROM associated dissipation potential and non-conservative reduced thermodynamical forces

$$\Psi^{\text{ROM}}\left(\overline{\boldsymbol{\xi}}, \overline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\theta}}\right) = \int_{\Omega} \Psi dV \quad \text{and} \quad \varphi^{\text{ROM}}\left(\dot{\overline{\boldsymbol{\alpha}}}\right) = \int_{\Omega} \varphi dV$$

Rate dependent materials: e.g. visco-elasticity

▶ FOM

$$\begin{cases} \mathcal{L}^{\text{vis}}(\mathbf{u}, \boldsymbol{\varepsilon}, \lambda) = \int_I \left(\mathcal{K} - \mathcal{E}_p - \int_\Omega \mathcal{A} : \boldsymbol{\varepsilon}^{\text{vis}} dV \right) dt + \int_I \int_\Omega \lambda \,\text{tr}(\dot{\boldsymbol{\varepsilon}}^{\text{vis}}) dV\, dt \\ \Psi\left(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^{\text{vis}}\right) = \frac{1}{2}\left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{vis}}\right) : \mathbb{C} : \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{vis}}\right) \quad \text{and} \quad \varphi(\dot{\boldsymbol{\varepsilon}}^{\text{vis}}) = \frac{1}{2}\eta\|\dot{\boldsymbol{\varepsilon}}^{\text{vis}}\|^2 \end{cases} \overset{\delta\mathcal{L}^{\text{vis}}}{\Longrightarrow} \boxed{\begin{cases} \rho\ddot{\mathbf{u}} = \text{div}\,\boldsymbol{\sigma} + \mathbf{f} \\ \eta\dot{\boldsymbol{\varepsilon}}^{\text{vis}} = \boldsymbol{\sigma} - \text{tr}\,\boldsymbol{\sigma}\mathbf{I} \end{cases}}$$

▶ VROM

$$\begin{cases} \Psi^{\text{ROM}}\left(\overline{\boldsymbol{\xi}}, \overline{\boldsymbol{\alpha}}\right) = \frac{1}{2}\overline{\boldsymbol{\xi}} \cdot \boldsymbol{K} \cdot \overline{\boldsymbol{\xi}} + \frac{1}{2}\overline{\boldsymbol{\alpha}} \cdot \boldsymbol{K}^{\text{vis}} \cdot \overline{\boldsymbol{\alpha}} - \overline{\boldsymbol{\xi}} \cdot \boldsymbol{K}^{\text{c}} \cdot \overline{\boldsymbol{\alpha}} \\ \varphi^{\text{ROM}}\left(\dot{\overline{\boldsymbol{\alpha}}}\right) = \eta\|\dot{\overline{\boldsymbol{\alpha}}}\|_2^2 \Rightarrow \overline{\mathbf{a}} = \frac{\partial\varphi^{\text{ROM}}}{\partial\overline{\boldsymbol{\alpha}}} = \eta\dot{\overline{\boldsymbol{\alpha}}} \\ \mathcal{L}^{\text{vis}}_{\text{ROM}}\left(\overline{\boldsymbol{\xi}}, \overline{\boldsymbol{\alpha}}\right) = \int_I \left(\mathcal{K}^{\text{ROM}} - \mathcal{E}_p^{\text{ROM}} - \overline{\boldsymbol{\alpha}} \cdot \overline{\mathbf{a}}\right) dt \end{cases} \quad \text{with} \quad \begin{cases} K_{ij} = \int_\Omega \boldsymbol{\nabla}^s\boldsymbol{\Upsilon}_i : \mathbb{C} : \boldsymbol{\nabla}^s\boldsymbol{\Upsilon}_j dV \\ K^{\text{vis}}_{ij} = \int_\Omega \boldsymbol{\Phi}_i : \mathbb{C} : \boldsymbol{\Phi}_j dV \\ K^c_{ij} = \int_\Omega \boldsymbol{\nabla}^s\boldsymbol{\Upsilon}_i : \mathbb{C} : \boldsymbol{\Phi}_j dV \end{cases}$$

▶ VROM Euler-Lagrange field equations

$$\boxed{\begin{cases} \boldsymbol{M} \cdot \ddot{\overline{\boldsymbol{\xi}}} = \boldsymbol{K} \cdot \overline{\boldsymbol{\xi}} - \boldsymbol{K}^{\text{c}} \cdot \overline{\boldsymbol{\alpha}} - \boldsymbol{f} \quad (f_i = \langle\mathbf{f}, \boldsymbol{\Upsilon}_i\rangle_{L_2} \quad \text{reduced external force}) \\ \eta\dot{\overline{\boldsymbol{\alpha}}} = \boldsymbol{K} \cdot \overline{\boldsymbol{\xi}} - \boldsymbol{K}^{\text{c}} \cdot \overline{\boldsymbol{\alpha}} \end{cases}}$$

Rate independent materials

- FOM dissipation potential and non-conservative thermodynamical forces

$$\varphi(\dot{\boldsymbol{\alpha}}) = \sigma\|\dot{\boldsymbol{\alpha}}\| \quad \text{where} \quad \boldsymbol{\mathcal{A}} \in \frac{\partial \varphi}{\partial \dot{\boldsymbol{\alpha}}} = \sigma \frac{\dot{\boldsymbol{\alpha}}}{\|\dot{\boldsymbol{\alpha}}\|}$$

- VROM associated dissipation potential and non-conservative thermodynamical reduced forces

$$\varphi^{\text{ROM}}\left(\dot{\bar{\boldsymbol{\alpha}}}\right) = \int_\Omega \varphi(\dot{\boldsymbol{\alpha}})dV \rightarrow \tilde{\varphi}\left(\dot{\bar{\boldsymbol{\alpha}}}\right) = \sigma\|\dot{\bar{\boldsymbol{\alpha}}}\|_2 \Rightarrow \quad \bar{\mathbf{a}} \in \frac{\partial \tilde{\varphi}}{\partial \dot{\bar{\boldsymbol{\alpha}}}} \Leftrightarrow \bar{\mathbf{a}} \in \sigma \frac{\dot{\bar{\boldsymbol{\alpha}}}}{\|\dot{\bar{\boldsymbol{\alpha}}}\|_2}$$

- Reduced evolution law: $\dot{\bar{\boldsymbol{\alpha}}} = \lambda \dfrac{\partial \tilde{f}}{\partial \bar{\mathbf{a}}}$ with the yielding VROM surface $\tilde{f}\left(\mathbf{a}\right) = \|\bar{\mathbf{a}}\|_2 - \sigma \leq 0$

VROM for rate independent materials

Theorem

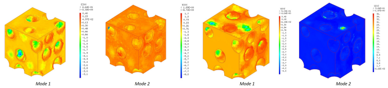
- *The reduced dissipation potential* $\tilde{\varphi} = \sigma\|\dot{\bar{\boldsymbol{\alpha}}}\|_2$ *defines a VROM upper bound:* $\varphi^{ROM} \leq \tilde{\varphi}$

Proof
Consider the concave function $g : \bullet \longmapsto \sigma\sqrt{\bullet}$. The dissipation potential function $\tilde{\varphi}$ can be then written as $\varphi^{\text{ROM}}\left(\dot{\bar{\boldsymbol{\alpha}}}\right) = g\left(\|\dot{\bar{\boldsymbol{\alpha}}}\|_2^2\right)$. The concavity of g and the L2-orthogonality of the reduced modes $\boldsymbol{\Phi}_i$ ensure

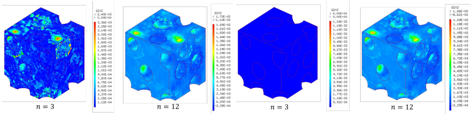
$$\fint_\Omega g(\|\dot{\bar{\boldsymbol{\alpha}}}\|_2^2)dV \leq g\left(\fint_\Omega \|\dot{\bar{\boldsymbol{\alpha}}}\|_2^2 dV\right) \Rightarrow \fint_\Omega \sigma\|\dot{\boldsymbol{\alpha}}\| \leq \sigma\sqrt{\left(\fint_\Omega \|\dot{\bar{\boldsymbol{\alpha}}}\|_2 dV\right)}$$

$$\Rightarrow \varphi^{\text{ROM}} \leq \sigma\sqrt{\int_\Omega (\boldsymbol{\Phi}_i \cdot \boldsymbol{\Phi}_j dV)\,\dot{\alpha}_i\dot{\alpha}_j} = \sigma\sqrt{\dot{\alpha}_i^2} = \sigma\|\dot{\bar{\boldsymbol{\alpha}}}\|_2 = \tilde{\varphi}\left(\dot{\bar{\boldsymbol{\alpha}}}\right)$$

▶ Example of the reduced basis modes for a heterogeneous matrix-inclusion medium within J2 von Mises plasticity (3 modes)



▶ Local plastic field obtained with the VROM



VROM: concluding remarks

- **Clustering** into N_c clusters $\Omega = \bigcup_{c=1}^{N_c} \Omega_c$ with the ROM ansatz

$$\boldsymbol{\alpha}\left(\mathbf{x}, t\right) = \sum_{i=1}^{\tilde{N}_c} \boldsymbol{\Phi}_i^{(c)}\left(\mathbf{x}\right) \alpha_i^{(c)}(t) \quad \forall \mathbf{x} \in \Omega_c$$

The flow evolution law fulfills the normality law $\overline{\dot{\boldsymbol{\alpha}}^{(c)}} = \lambda^{(c)} \frac{\overline{\mathbf{a}^{(c)}}}{\|\overline{\mathbf{a}^{(c)}}\|_2} \quad \forall c = 1..N_c$ where the internal variables flow is orthogonal to the boundary of the reduced domain
$\tilde{f}^{(c)}\left(\overline{\mathbf{a}^{(c)}}\right) = \|\overline{\mathbf{a}^{(c)}}\|_2 - k_c\sigma \quad k_c = |\Omega_c|/|\Omega|$

- Weakly intrusive reduced order model
- Suquet NTFA approach for multi-scale simulations
- The proposed rate independent VROM is equivalent to a **Linear Comparison Reduced Order Model** [2]

[2] N. BLAL. Variational Reduced Order Models for dissipative problems within the formalism of standard generalized materials and the extended Lagrangian functional. Submitted.

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ΦHOPGD: Physically based High Order Proper Generalized Decomposition (ongoing)

- Variables separation representation of the QoI w.r.t the model extra-coordinates

$$\mathbf{u}^{\text{ROM}}(\mathbf{x}, t, \boldsymbol{\mu}) = \sum_{i=1}^{N} \boldsymbol{\Upsilon}_i(\mathbf{x})\xi_i(t)\prod_{j=1}^{d}\zeta_i^j(\mu_j) \quad \text{s.t.} \quad \min \|\mathbf{u}^{\text{ROM}} - \mathbf{u}^*\|$$

- Offline-online approach → Adaptive solutions without any need to re-run FE simulations
- ROM builder complexity: data (primal and dual QoI)
- Respecting the material behavior with only primal data and a small number of ROM modes

$$\mathbf{u}^{\text{ROM}}(\mathbf{x}, t, \boldsymbol{\mu}) = \sum_{i=1}^{N} \boldsymbol{\Upsilon}_i(\mathbf{x})\xi_i(t)\prod_{j=1}^{d}\zeta_i^j(\mu_j) \quad \text{s.t.} \quad \min \|\mathbf{u}^{\text{ROM}} - \mathbf{u}^*\| + \langle \Lambda, F(\mathbf{u}, \boldsymbol{\alpha}; \dot{\boldsymbol{\alpha}})\rangle$$

Conclusions

Data driven approaches are efficient tools to deal with complex multiscale, multi-physics and multi-parametric computational mechanics

- High-dimensional spaces
- Real time simulations
- Uncertainty and variability issues
- Rapid design

Data sciences should not be purely used as data driven process in CM. They need a scientific approach keeping

- Experimental investigations and computational needs
- Physical foundations
- Mathematical considerations