

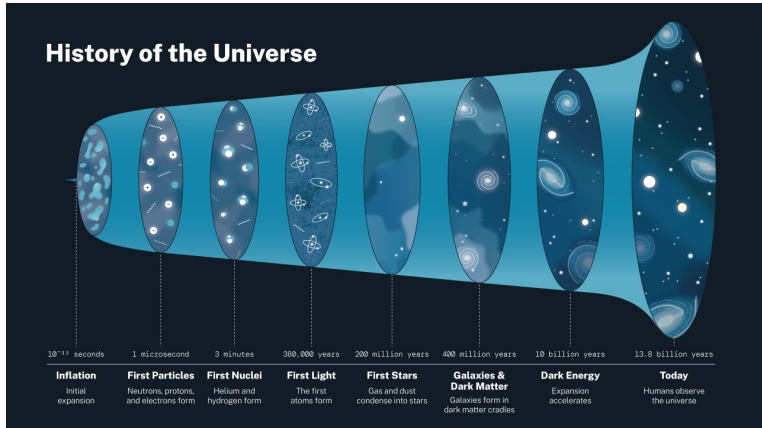
Some Thoughts Concerning Spacetime Models in Cosmology : Role of Gravitation and Electromagnetism

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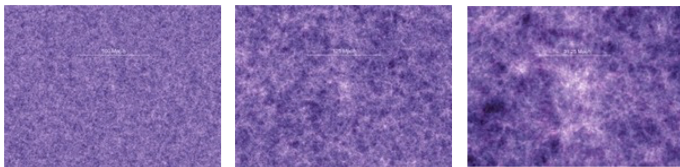
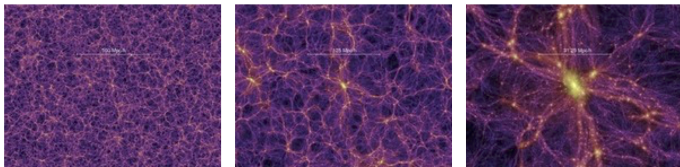
From Bing-bang ...



<https://science.nasa.gov/universe/the-universe-is-expanding-faster-these-days-and-dark-energy-is-responsible-so-what-is-dark-energy/>

Millennium Run $> 10^9$ particles to trace the evolution of matter distribution in a cube of the Universe $\ell \simeq 2 \times 10^9$ light-years on a side. (Max Planck Institute)

(H) : $t = 13,6$ Giga-years



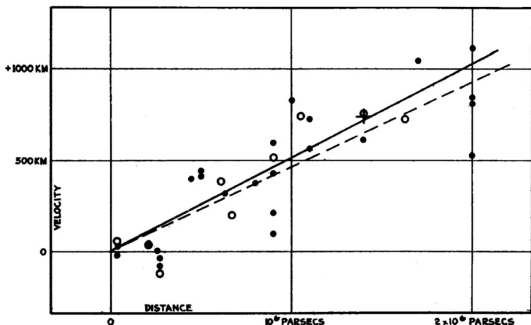
(L) : $t = 0,21$ Giga-years

<https://wwwmpa.mpa-garching.mpg.de/galform/virgo/millennium/>

Expansion of the Universe (1929)

- **Hubble 1929** (Experimental) : The Universe is **expanding** governed by *matter and radiation background*.

Velocity-Distance Relation among Extra-Galactic Nebulae.

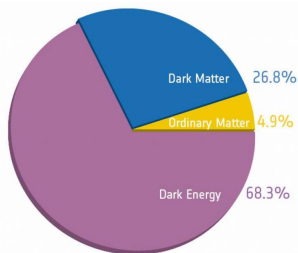
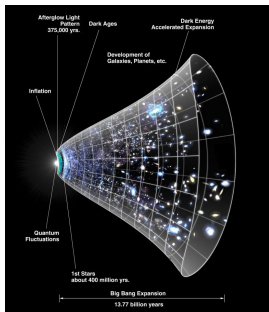


- **Friedman equations 1922** (Theory) models Universe expansion, assumed homogeneous and isotropic, filled of **perfect fluid**. **Introducing Cosmological Constant Λ was necessary !**

<https://www.pnas.org/doi/full/10.1073/pnas.1424299112>

Acceleration of the expansion (1998)

Major finding: Two teams (Perlmutter & al., and Riess & al. 1998) found that the Universe **expansion is accelerating**.



Change of Paradigm: Consensus that $\simeq 70\%$ of **Universe** neither matter nor radiation, but an unknown **Dark Energy** Λ as source of the acceleration.

Goal of the present work

- **CHALLENGE.** Determination of **physical contents** of this hypothetical **Dark Energy** is (*and will remain for a while*) one of the biggest challenge for physicists (<https://www.darkenergysurvey.org>).
- **HYPOTHESIS.** We re-consider the **Electro-vacuum scenario** to understand if the electromagnetic background in an **EC spacetime** would contribute to this **Dark Energy**, and overall evolution.
- **THEORY.** Spacetime is a **Generalized Continuum** (compact and connected e.g. Whyburn 1935) endowed with:
 - **Metric** $g_{\alpha\beta}$,
 - **Connection** $\Gamma_{\alpha\beta}^{\gamma}$, metric-compatible with/out **torsion** $\mathbb{N}_{\alpha\beta}^{\gamma}$,
 - **Volume-form** $\omega_n = \Omega dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$, $\Omega > 0$.¹
 - **Lagrangian density** : $\mathcal{L} = \mathcal{L}(\text{geometry, physical field})$

¹Volume-form ω_n is not considered at its fair value in most papers !

Part I. Einstein Relative Gravitation

1 Einstein Spacetime $(\mathcal{B}, \mathbf{g}, \bar{\nabla}, \bar{\omega}_n)$ endowed with :

$$\begin{cases} \mathbf{g} &= g_{\alpha\beta} dx^\alpha \otimes dx^\beta \\ \bar{\Gamma}_{\alpha\beta}^\gamma &= \frac{1}{2} g^{\gamma\lambda} (\partial_\alpha g_{\lambda\beta} + \partial_\beta g_{\alpha\lambda} - \partial_\lambda g_{\alpha\beta}) \\ \bar{\omega}_n &= \sqrt{|\text{Detg}|} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \end{cases}$$

2 Hilbert-Einstein action + Perfect Fluid with Lagrangian \mathcal{L}_{PF}

$$\mathcal{S}_{HE} := \frac{1}{2\chi} \int_{\mathcal{B}} (\bar{\mathcal{R}} - 2\Lambda) \bar{\omega}_n - \int_{\mathcal{B}} \mathcal{L}_{PF} \bar{\omega}_n, \quad \bar{\mathcal{R}} = g^{\alpha\beta} \bar{\mathcal{R}}_{\alpha\beta}$$

3 Variation equation (Directional derivative along $\delta g^{\alpha\beta}$)

$$\begin{aligned} \delta \mathcal{S}_{HE} &= \int_{\mathcal{B}} \left[\frac{1}{2\chi} \left(\bar{\mathcal{R}}_{\alpha\beta} - \frac{\bar{\mathcal{R}}}{2} g_{\alpha\beta} + \Lambda g_{\alpha\beta} \right) \delta g^{\alpha\beta} - T_{\alpha\beta}^{\text{fl}} \delta g^{\alpha\beta} \right] \bar{\omega}_n \\ &+ \frac{1}{2\chi} \int_{\mathcal{B}} g^{\alpha\beta} \left[\bar{\nabla}_\alpha (\delta \bar{\Gamma}_{\beta\lambda}^\gamma) - \bar{\nabla}_\beta (\delta \bar{\Gamma}_{\alpha\lambda}^\gamma) \right] \bar{\omega}_n = 0 \end{aligned}$$

ERG : Field equations and Friedmann equations (1922)

- ① **Field equations** of Einstein (1915) (e.g. PF : Minguzzi 2023)

$$\boxed{\frac{1}{2\chi} (\bar{G}_{\alpha\beta} + \Lambda g_{\alpha\beta}) = (\rho + p) u_{\alpha} u_{\beta} - p g_{\alpha\beta}}, \quad \bar{G}_{\alpha\beta} := \bar{\mathfrak{R}}_{\alpha\beta} - \frac{\bar{\mathcal{R}}}{2} g_{\alpha\beta}$$

- ② **Isotropy & Homogeneity** → **FLRW metric** (curvature $k = -1, 0, +1$)

$$ds^2 = (dx^0)^2 - R^2(x^0) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right)$$

- ③ **Friedman equations** within Einstein spacetime (Friedman 1922):

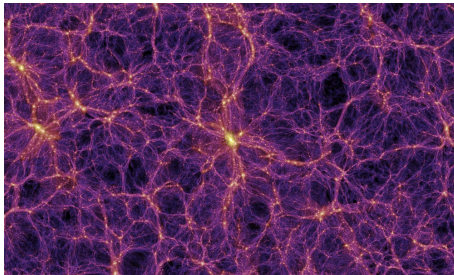
$$\boxed{\begin{cases} \left(\frac{\dot{R}}{R} \right)^2 &= \frac{8\pi G}{3} \rho - \frac{\Lambda}{3} - \frac{k}{R^2} \\ \frac{\ddot{R}}{R} &= -\frac{4\pi G}{3} (\rho + 3p) - \frac{\Lambda}{3} \end{cases}}$$

Standard Model : $\Lambda < 0$ accelerates expansion \simeq Dark Energy (> 1998)

Some remarks on Λ (Peebles & Ratra 2003)

- Deduced Λ phenomenologically from experiments has very **small values**, near zero $\Lambda \simeq 1.09 \times 10^{-52} m^{-2}$. **Why not zero ? Is it constant or variable ?** (e.g. Weinberg 1988)
- Despite its success, description of vacuum **Dark Energy** in terms of Λ suffers from other important problem, since **no unique explanation** from known physics (**Matter, EM and TH Fields, Q-Particles ...**).
- **Various possibilities** exist : Quantum physics (e.g. Zel'dovich 1967, Weinberg 1988, Rugh & Zinkernagel 2002, ...), **Electromagnetic field** (e.g. Jimenez et al. 2009), **Spacetime Defects** (e.g. Ivanov & Wellenzohn 2016, Milton 2022), ... **New physics of Exotic matter ?**
- Namely, **extension** of Friedman equations to Gravitation with **Torsion**, remains an **active research** in Cosmology: e.g. Minkowski 1986, Kranas 2019, Medina et al. 2019, Iosofidis 2020, Pereira et al. 2022, ...

Part II. EC Relative Gravitation and Electromagnetism

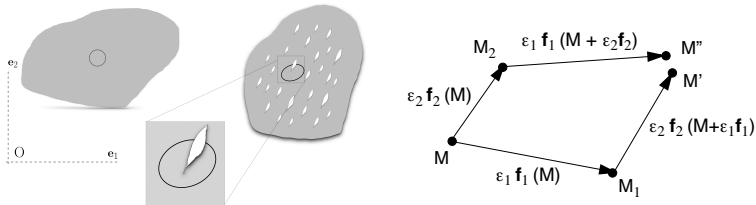


50 million light year-long **GIGANTIC FILAMENTS** in the cosmic web possessing weak **MAGNETIC FIELDS** : Could it help to explain why the Universe is expanding faster than it should (**Vernstrom et al. 2019**).

<https://spaceaustralia.com/news/magnetic-fields-found-cosmic-filaments>

EC spacetime: Loops \rightarrow Torsion, Curvature (e.g. R 1996)

Continuum \mathcal{B} : " Smooth " with **loops** and defects (e.g. R 1996)



Einstein-Cartan Spacetime $(\mathcal{B}, g, \nabla, \omega_n)$ with:

- **Torsion** $\mathfrak{K}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}$
- **Curvature** $\mathfrak{R}^{\gamma}_{\alpha\beta\lambda} = (\partial_{\alpha}\Gamma^{\gamma}_{\beta\lambda} + \Gamma^{\gamma}_{\alpha\mu}\Gamma^{\mu}_{\beta\lambda}) - (\partial_{\beta}\Gamma^{\gamma}_{\alpha\lambda} + \Gamma^{\gamma}_{\beta\mu}\Gamma^{\mu}_{\alpha\lambda})$
- **Volume-form** $\omega_n := \bar{\omega}_n e^{\vartheta(\mathfrak{K})}$ (e.g. Mosna & Saa 1995, 2005)

Warning : Compatibility of g , ∇ and ω_n is obviously desirable !

Gravitation & Electromagnetism (GEM model)

- **Action of GEM model** (Einstein-Palatini + Yang-Mills):²

$$\mathcal{S}_{GEM} := \frac{1}{2\chi} \int_{\mathcal{M}} \mathcal{R} \omega_n - \frac{1}{4} \int_{\mathcal{M}} \mathcal{H}^{\mu\nu} \mathcal{F}_{\mu\nu} \omega_n, \quad \omega_n := e^{\vartheta} \bar{\omega}_n$$

- **Electromagnetic variables** : Faraday strength tensor (primal variables)
& Excitation tensor (dual variables) (e.g. Tonnelat 1959):

$$\mathcal{F}_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B^3 & -B^2 \\ E_2 & -B^3 & 0 & B^1 \\ E_3 & B^2 & -B^1 & 0 \end{pmatrix} \quad \mathcal{H}^{\mu\nu} = \begin{pmatrix} 0 & D^1 & D^2 & D^3 \\ -D^1 & 0 & H_3 & -H_2 \\ -D^2 & -H_3 & 0 & H_1 \\ -D^3 & H_2 & -H_1 & 0 \end{pmatrix}$$

- **Constitutive laws** in 3D description (vacuum spacetime)

$$\begin{cases} \mathbf{D} & = & \epsilon_0 \mathbf{E} \\ \mathbf{H} & = & \mu_0^{-1} \mathbf{B} \end{cases} \quad \longrightarrow \quad \mathcal{H} = \mathcal{H}(\mathcal{F})$$

²MCP-L : We consider exactly the same shape as usual for Lagrangian.

GEM : Variation of variables in EC spacetime

- **Variation equation :**

$$\delta \mathcal{S}_{GEM} = 0, \quad \forall \delta g^{\mu\nu}, \delta \mathcal{N}_{\mu\nu}^{\gamma}, \delta A_{\nu}$$

- **Variation of geometric variables:**³

$$\left\{ \begin{array}{l} \delta g^{\mu\nu} = \delta g^{\mu\nu} \\ \delta \mathcal{N}_{\mu\nu}^{\gamma} = \delta \Gamma_{\mu\nu}^{\gamma} - \delta \Gamma_{\nu\mu}^{\gamma} \\ \delta \mathcal{N}_{\alpha\beta\lambda}^{\gamma} = \nabla_{\alpha}(\delta \Gamma_{\beta\lambda}^{\gamma}) - \nabla_{\beta}(\delta \Gamma_{\alpha\lambda}^{\gamma}) + \mathcal{N}_{\alpha\beta}^{\mu} \delta \Gamma_{\mu\lambda}^{\gamma} \\ \delta \omega_n = \left(-\frac{1}{2} g_{\mu\nu} \delta g^{\mu\nu} + \frac{\partial \theta}{\partial \mathcal{N}_{\mu\nu}^{\lambda}} \delta \mathcal{N}_{\mu\nu}^{\lambda} \right) \omega_n \end{array} \right.$$

- **Variation of the Faraday strength tensor** $\mathcal{F}_{\mu\nu} := \nabla_{\mu} A_{\nu} - \nabla_{\nu} A_{\mu}$:

$$\delta \mathcal{F}_{\mu\nu} = \nabla_{\mu}(\delta A_{\nu}) - \nabla_{\nu}(\delta A_{\mu}) - A_{\rho} \delta \mathcal{N}_{\mu\nu}^{\rho}$$

Blue Terms and Red Terms make the difference !

³If ∇ and $\nabla + \delta\nabla$ have the same geodesics, then the variation $\delta \Gamma_{\mu\nu}^{\gamma}$ is necessarily **skew-symmetric (Hint : $(\nabla + \delta\nabla)_{\mathbf{u}} \mathbf{u} = 0$) !**

GEM : Field equations

- Variation equation**

$$\delta \mathcal{S}_{GEM} = \int_{\mathcal{M}} (\mathbb{M}^\nu \delta A_\nu + \mathbb{E}_{\mu\nu} \delta g^{\mu\nu} + \mathbb{L}_\gamma^{\mu\nu} \delta \mathcal{N}_{\mu\nu}^\gamma) \omega_n + \underbrace{\text{B. Term}}_{=0} = 0$$

(RQ : Compatibility of ∇ , g and $\omega_n \implies$ B. Term = Divergence)

- Field equations** (Maxwell, Einstein-Maxwell-Cartan, Linkage)

$$\left\{ \begin{array}{l} \mathbb{M}^\nu := \nabla_\mu \mathcal{H}^{\mu\nu} = 0 \\ \mathbb{E}_{\mu\nu} := \frac{1}{2\chi} G_{\mu\nu} + T_{\mu\nu}^{\text{em}} = 0 \\ \mathbb{L}_\gamma^{\mu\nu} := \frac{1}{2\chi} \mathcal{D}_\gamma^{\mu\nu} + \mathcal{H}^{\mu\nu} A_\gamma - \frac{1}{4} \mathcal{H}^{\alpha\beta} \mathcal{F}_{\alpha\beta} \frac{\partial \vartheta}{\partial \mathcal{N}_{\mu\nu}^\gamma} = 0 \end{array} \right.$$

with **Einstein tensor** and spacetime **Distortion Tensor** $\mathcal{D}_\gamma^{\mu\nu}$:

$$\left\{ \begin{array}{l} G_{\mu\nu} := \mathcal{R}_{\mu\nu} - \frac{\mathcal{R}}{2} g_{\mu\nu} \quad (\neq \bar{G}_{\mu\nu}) \\ \mathcal{D}_\gamma^{\mu\nu} := g^{\beta\nu} \mathcal{N}_{\gamma\beta}^\mu + \mathcal{R} \frac{\partial \vartheta}{\partial \mathcal{N}_{\mu\nu}^\gamma}, \quad \mathcal{N}_{\mu\nu}^\nu = \nabla_\mu \vartheta \end{array} \right.$$

1. GEM : Maxwell equations $M^\nu = 0$

- **Covariant Maxwell equations** (nb = 4) (respecting the Minimal Coupling Principle)

$$M^\nu := \nabla_\mu \mathcal{H}^{\mu\nu} = 0$$

with (EM radiation in vacuum spacetime):

$$\mathcal{F}_{\alpha\beta} := \nabla_\alpha A_\beta - \nabla_\beta A_\alpha, \quad \mathcal{H}^{\mu\nu} = \epsilon_0 g^{\mu\alpha} g^{\nu\beta} \mathcal{F}_{\alpha\beta}$$

- **Wave equation** (influence of EC geometry to EM, R 2018)

$$-g^{\alpha\beta} \nabla_\alpha \nabla_\beta A^\mu + g^{\mu\alpha} \nabla_\alpha \nabla_\beta A^\beta - g^{\mu\alpha} \mathfrak{R}^\gamma_{\beta\alpha} \nabla_\gamma A^\beta + g^{\mu\alpha} \mathfrak{R}_{\alpha\beta} A^\beta = 0$$

- **EC influences on EM wave** : **Twist due to torsion** and bending due to curvature.

(Some details presented during GDR-GDM - R 2019)

2. GEM : Einstein-Maxwell-Cartan equations $\mathbb{E}_{\mu\nu} = 0$

- **Bending equation** of the EC spacetime (nb = 16)

$$\mathbb{E}_{\mu\nu} := \frac{1}{2\chi} G_{\mu\nu} + T_{\mu\nu}^{\text{em}} = 0$$

with the EM energy-momentum (Minkowski 1908, Abraham 1909):

$$T_{\mu\nu}^{\text{em}} = \frac{1}{8} \left(\mathcal{H}^{\lambda\rho} \mathcal{F}_{\lambda\rho} \right) g_{\mu\nu} + \frac{1}{4} \mathcal{H}^{\lambda\rho} \left(g_{\mu\lambda} \mathcal{F}_{\rho\mu} + \mathcal{F}_{\mu\lambda} g_{\rho\nu} \right)$$

$$T_{\text{em}-M}^{\mu\nu} = \begin{bmatrix} (1/2)(\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H}) & \mathbf{E} \times \mathbf{H} \\ \mathbf{D} \times \mathbf{B} & \mathbf{T}^M \end{bmatrix}$$

- **Influences of EM**

- 1 **Electromagnetic field** influences the bending (and twisting) of the EC spacetime.
- 2 In general, EM breaks the **isotropic symmetry** which is one of the two fundamental Cosmological Principles.

3. GEM : Linkage equations (distorsion) : $\mathbb{L}_\gamma^{\mu\nu} = 0$

Distorsion equation due to EM field (nb = 64)

$$\mathbb{L}_\gamma^{\mu\nu} := \frac{1}{2\chi} \mathcal{D}_\gamma^{\mu\nu} + \mathcal{H}^{\mu\nu} A_\gamma - \frac{1}{4} \mathcal{H}^{\alpha\beta} \mathcal{F}_{\alpha\beta} \frac{\partial \vartheta}{\partial \mathbb{N}_{\mu\nu}^\gamma} = 0$$

Physical Sources for the spacetime distortion $\mathcal{D}_\gamma^{\mu\nu}$ are:

- **Specific contribution** : **Chern-Simons current** (e.g. Grasso & Rubinstein 2001, Jackiw & Pi 2003, R 2018)

$$\mathcal{H}^{\mu\nu} A_\nu \longrightarrow \begin{pmatrix} \mathbf{B} \cdot \mathbf{A} \\ \mathbf{E} \times \mathbf{A} \end{pmatrix} \quad \begin{array}{l} \rightarrow \text{Magnetic Helicity} \\ \rightarrow \text{Spin Angular Momentum} \end{array}$$

- **Specific contribution** : **Yang-Mills Lagrangian** (mandatory to volume-form ω_n)

$$-\frac{1}{4} \mathcal{H}^{\alpha\beta} \mathcal{F}_{\alpha\beta} \longrightarrow \mathcal{L}_{EM} := \frac{1}{2} (\mathbf{D} \cdot \mathbf{E} - \mathbf{B} \cdot \mathbf{H})$$

Highlights 1. Torsion-Vector model $\aleph_\gamma \delta_\beta^\mu - \aleph_\beta \delta_\gamma^\mu$

- **Field equations** with $\vartheta = \vartheta(\aleph_\mu)$ (same shape for Maxwell equation):

$$\begin{cases} \frac{1}{2\chi} \left(\overline{G}_{\mu\nu} - \aleph_\mu \aleph_\nu - \frac{1}{2} (g^{\alpha\beta} \aleph_\alpha \aleph_\beta) g_{\mu\nu} \right) + T_{\mu\nu}^{\text{em}} = 0 \\ \frac{1}{2\chi} \mathcal{D}_\gamma^{\mu\nu} + \mathcal{H}^{\mu\nu} A_\gamma - \frac{1}{12} \mathcal{H}^{\alpha\beta} \mathcal{F}_{\alpha\beta} \delta_\gamma^\nu \frac{\partial \vartheta}{\partial \aleph_\mu} = 0 \end{cases}$$
$$\mathcal{D}_\gamma^{\mu\nu} := g^{\beta\nu} (\aleph_\gamma \delta_\beta^\mu - \aleph_\beta \delta_\gamma^\mu) + \frac{\mathcal{R}}{3} \delta_\gamma^\nu \frac{\partial \vartheta}{\partial \aleph_\mu}$$

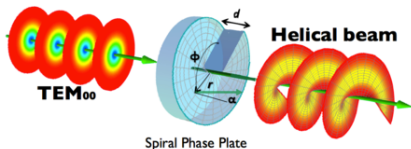
- 1 Tensor product $\aleph_\mu \aleph_\nu$ breaks **isotropic symmetry** (Cosmology Principle);
- 2 Term $\Lambda_\aleph := -\frac{1}{2} g^{\alpha\beta} \aleph_\alpha \aleph_\beta$ acts a variable "**Cosmological Constant**".
- 3 **Torsion vs. EM** For nearly flat Universe $\mathcal{R} \simeq 0$ and (R 2018, 2024):

$$\aleph_\mu = -\frac{4}{3} \chi g_{\mu\alpha} \mathcal{H}^{\alpha\beta} A_\beta - \frac{\mathcal{L}}{2} \frac{\partial \vartheta}{\partial \aleph_\mu}$$

Integral curves of \aleph_μ (\simeq **gigantic filaments** of large scale Universe ?), are directly linked to **Chern-Simmons current** (Magnetic Helicity & Spin Angular Momentum).

Highlights 2. Illustration of Chern-Simons Current

Physical interpretation : $\mathcal{H}^{\alpha\beta} A_\beta$ is the 4-dim expression of **Spin Angular Momentum** \mathbf{L}_{spin} (optics, ...): **T**ransverse **E**lectro**M**agnetic wave (green axis)



Moment of 3D-Poynting vector $\mathbf{E} \times \mathbf{B}$ (e.g. Allen et al. 1992)

$$\begin{aligned} \mathbf{J} &:= \int_{\mathcal{M}} \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dv \\ &= \sum_{i=1}^3 \int_{\mathcal{M}} E_i (\mathbf{r} \times \nabla) A_i dv + \int_{\mathcal{M}} \mathbf{E} \times \mathbf{A} dv := \mathbf{L}_{\text{orbital}} + \mathbf{L}_{\text{spin}} \end{aligned}$$

for paraxial and non-paraxial EM field (Arrayas & Trueba 2018).

Highlights 3. GEM vs. Brans-Dicke vs. Standard

- 1 **GEM** model (Geometric Approach):

$$\mathcal{S}_{GEM} = \int_{\mathcal{B}} \left[\frac{1}{2\chi} \left(\bar{\mathcal{R}} - 3 g^{\beta\lambda} \aleph_{\beta} \aleph_{\lambda} \right) - \frac{1}{4} \mathcal{H}^{\mu\nu} \mathcal{F}_{\mu\nu} \right] e^{\vartheta} \bar{\omega}_n$$

with $\aleph_{\mu} = \nabla_{\mu} \vartheta$ and $\Gamma_{\mu\nu}^{\gamma} = \bar{\Gamma}_{\mu\nu}^{\gamma} + \mathcal{T}_{\mu\nu}^{\gamma}$ with **contortion tensor**.

- 2 **Brans-Dicke** theory (Brans & Dicke 1961) (Physical Approach):

$$\mathcal{S}_{JBD-ST} = \int_{\mathcal{B}} \left[\frac{1}{2\chi} \left(\bar{\mathcal{R}} - \frac{\varpi_{BD}}{\phi^2} \eta^{\beta\lambda} \nabla_{\beta} \phi \nabla_{\lambda} \phi \right) - \frac{1}{4} \bar{\mathcal{H}}^{\mu\nu} \bar{\mathcal{F}}_{\mu\nu} \right] \phi \bar{\omega}_n$$

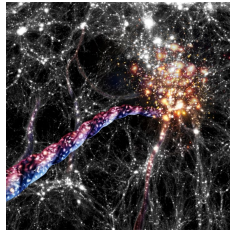
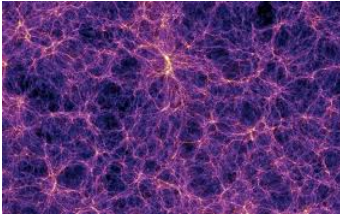
Dilaton ϕ : scalar function (e.g. Saa 1995, Peracaula et al. 2019),

- 3 **Einstein-Maxwell** model (Standard Model):

$$\mathcal{S}_{HE-YM} = \frac{1}{2\chi} \int_{\mathcal{B}} \left[(\bar{\mathcal{R}} - 2\Lambda) - \frac{1}{4} \bar{\mathcal{H}}^{\mu\nu} \bar{\mathcal{F}}_{\mu\nu} \right] \bar{\omega}_n$$

Standard Model \subset Physical Approach \subset Geometric Approach

Part III. Final Remarks and Outlook



(L) **Cosmic web** : Clusters of galaxies via filaments. (R) **Spinning** filaments. **Magnetic fields & Gravitation** are suggested as the forces shaping out the large-structure of the cosmos.) ⁴

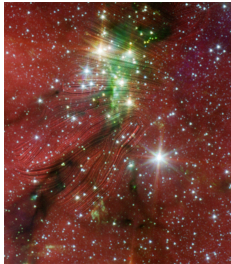
⁴e.g. **Vernstrom et al. 2019** "Discovery of **Magnetic Fields** Along Stacked Cosmic Filaments as Revealed by Radio and X-Ray Emission". e.g. **Wang et al. 2021** "Possible observational evidence for cosmic **Filament Spin**", (..)

Concluding Remarks

- **Generalized Continuum** with **action \mathcal{S}_{GEM}** is a valuable candidate for spacetime models $\mathcal{B}(\mathbf{g}, \nabla, \omega_n)$ for Gravitation & Electromagnetism.
- **Torsion** defect may **accelerate the universe expansion**, but breaks isotropy Principle. Torsion-Vector is suggested as a candidate for investigating **gigantic filaments**.
- **Electromagnetic field (Cosmic Microwave background)** is directly **linked of torsion field** of the Universe.
- Knowledge of **Magnetic fields** remains **active research**: their strength, their evolution, and their role of the **Cosmic Web** formation (*i.e.* filaments, galaxies and clusters) are worth.
- **GEM** model constitutes sound basis for **extending Standard Model** of Cosmology, and hopefully might open the path to more sophisticated **Scalar Tensor Theory of Gravitation** (\rightarrow Quantum Dilaton Gravity).

Merci pour votre attention !

Electromagnetics \longleftrightarrow **Torsion** \longleftrightarrow "Dark Energy"



<https://skyandtelescope.org/astronomy-news/gallery-magnetic-fields/>
Photo centre : **Sciences et Avenir, Mars 2024**



J Radofilao (... – 2021[†]): mathématicien, spéléologue (cf. *Babakotia radofilai*, cartographie des grottes de l'*Ankarana*), compositeur de 32 cantiques, enseignant en théologie ...