

Continuum mechanics and weak field general relativity: Study of several mechanical models that reproduce the measured strains of the space time - analyze and consequences

by David Izabel

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Summary

- 1) 6 Principles of equivalence between the elastic analogy of the space medium and general relativity in weak field
- 2) Consequence of these 6-equivalence principles on the characteristics of the equivalent elastic medium associated at the space-time
 - 2.1) Based on classical general relativity of gravitational waves - Basic polarizations and strains in transverse planes – isotropic transverse medium in sheets not connected to gather
 - 2.2) Based on modified general relativity of gravitational waves - complementary polarizations and strains in the propagation direction –isotropic transverse medium in sheets connected to gather
- 3) Consequences about the potential models that can be used to reproduce the forecast and measured strains of the space-time
 - 3.1) Practical characteristic of the elastic medium isotropic transverse
 - 3.2) Study of several mechanical S. Timoshenko models of the space time that can reproduce the order of magnitude of the strains forecast and measured in general relativity
- 4) Numerical applications of the different models
 - 4.1) Models in plane with spatial component of strains (h for space associated at h_{ij} for Gravitational wave GW or space part of gravity prob B experiment)
 - 4.2) Models perpendicular at the plane with temporal component of the strains
 - 4.3) Spatial models
- 5) Possible unification of all the different models in plane and perpendicular at the plane via the intervalle ds^2 in quasi flat metric
 - 5.1) Interval of special relativity
 - 5.2) Mechanic transposition
 - 5.3) Case of the light type interval and associated equation
 - 5.4) Test of the equation basing on the previous model unifying the different Young's modulus obtained
- 6) Come back of the analogy in direction of physics – potential consequences
 - 6.1) Didactic explanations proposed by the elastic analogy of the general relativity in weak field
 - 6.2) Predictive consequences of the elastic analogy of the general relativity in weak field
- 7) How to check all that?
 - 7.1) Measure complementary polarisations and study of their shapes
 - 7.2) Measure lateral motions of the interferometer in 3D
 - 7.3) Measure of the Casimir's strains and forces to have a realistic value of the space Young's modulus and strain elastic energy
- 8) Beyond the analogy – Some speculations
 - 8.1) In Strong field? Example of theoretical frame dragging for a neutron star
 - 8.2) Analyse of the CMB power spectrum as a diffractogram X
 - 8.3) Geometrical torsion in CMB logical to take into account in Einstein-Cartan
 - 8.4) Self-repair/self-clogging of space after the passage of a rotating black hole, a sign of its great plasticity
- 9) Conclusion

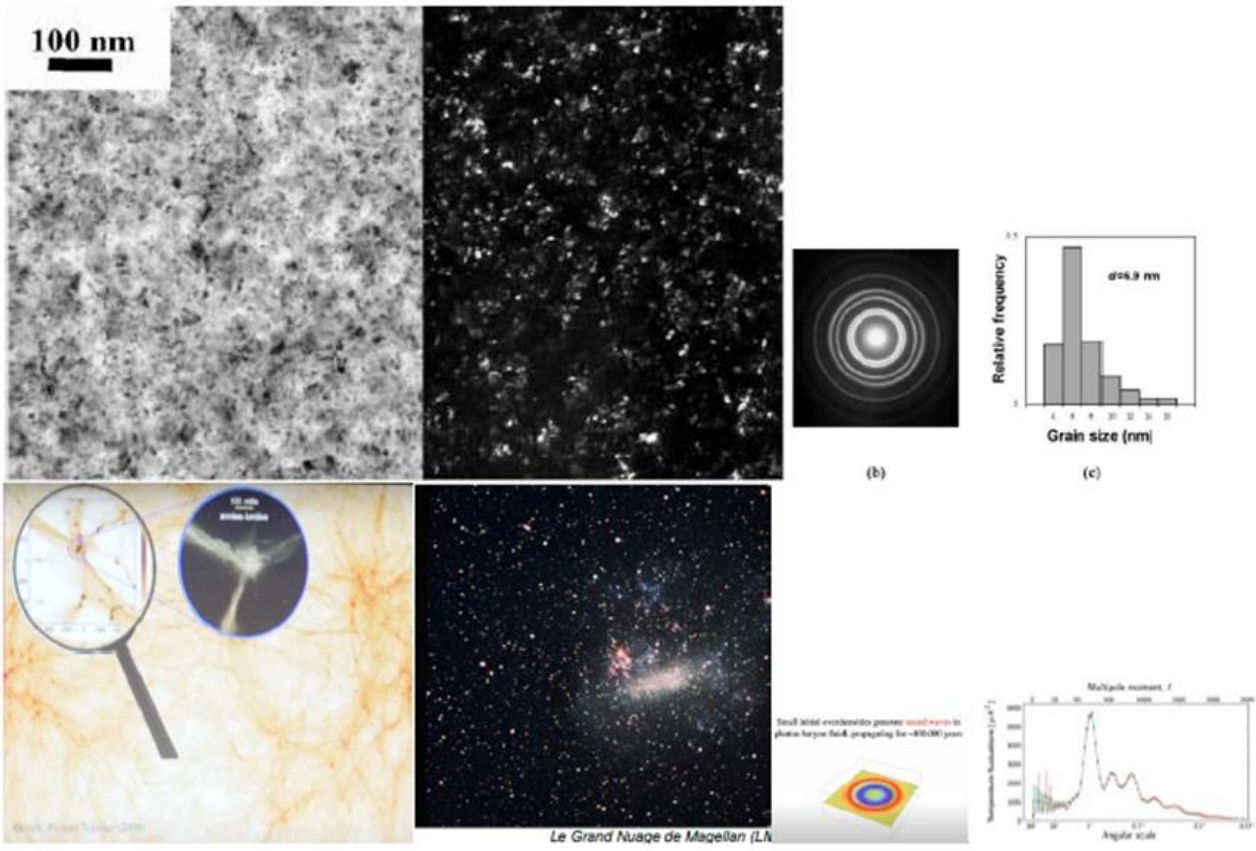


Figure 142<: Electron micrograph Bright and dark, diffraction X and grain size of a nanocrystal Ni-2.0 wt% P [318]

Structure of the cosmic web, sky with stars and galaxy andromeda, variation density and power spectrum of the Young universe at 380000 years-

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Y. Zhou et al.: Young's modulus in nanostructured metals

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Young's modulus in nanostructured metals

- Aim of the analogy and study,
- 1) Confirm links between elastic analogy and general relativity in weak field by several fundamental principles
 - 2) Propose several type of Timoshenko's mechanical models compatible with the strains measured data of the general relativity
 - 3) Look for via these adequate mechanical models the Young's modulus necessary (adjustment variable) to find the different strains observed by the different general relativity test experiments (calibration of the models)
 - 4) Analyse these Young's modulus and see if there is a link between them – junction approaches in the plane and perpendicular at the plane
 - 5) Come back from analogy to physic to extract potential didactic information's about general relativity and predictive informations for physic

1) Principles of equivalence between the elastic analogy of the space medium and general relativity in weak field

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$$
$$\textit{Curvature} = \frac{\textit{Angle}}{\textit{Surface}} = \frac{8\pi G}{c^4} \times \frac{\textit{Energy}}{\textit{Volume}}$$

6 principles of equivalences between classical relativity in weak field and general relativity (1/5)

Principle 1:

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The Mechanics of Spacetime – A Solid Mechanics Perspective on the Theory of General Relativity

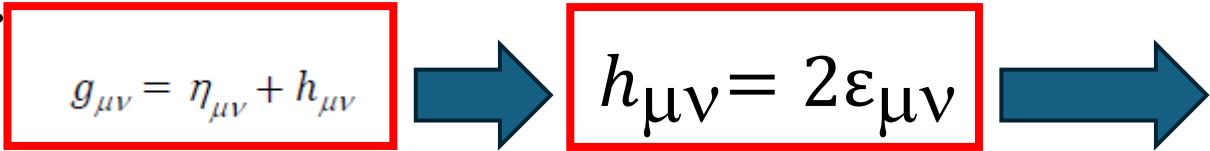
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$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$$

$$\text{Curvature} = \frac{\text{Angle}}{\text{Surface}} = \frac{8\pi G}{c^4} \times \frac{\text{Energy}}{\text{Volume}}$$

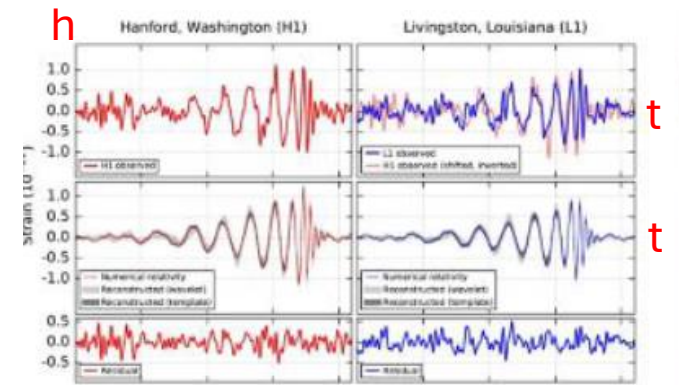
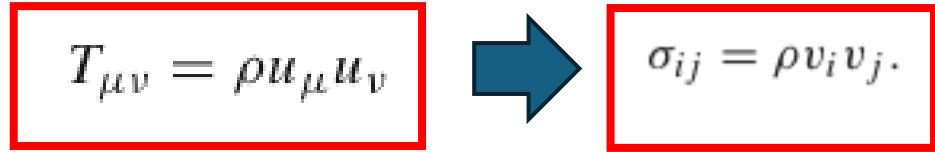
The perturbation of the metric tensor in weak field is equivalent at a strain tensor:



The components of the polarisation of the gravitational waves can be seen as components of strain tensor

Principle 2:

The stress Energy tensor is equivalent at stress tensor (4d =>3d):



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Mechanical conversion of the gravitational Einstein's constant κ

6 principles of equivalences between classical relativity in weak field and general relativity (2/5)

Principle 3: $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$ $F = k \delta \Rightarrow \delta \frac{1}{k} = F$ $\Rightarrow \sigma = E \varepsilon \Rightarrow \varepsilon \frac{1}{E} = \sigma$

The Einstein constant κ can be seen as the flexibility characteristic of the space time in 4 D “Timoshenko theory of the space time”

Solicitations	Curvature/energy formula	Dimensional Equations
Bending moment of $M_{(x)}$	$\frac{1}{R^2} = \frac{2}{EI} \times \frac{U}{L} = \frac{2}{YI} \times \frac{U}{L}$	$\frac{1}{m^2} = \frac{s^2}{kgm^3} \times \frac{U}{m}$
Twisting torque $T_{(x)}$	$\frac{1}{R_t^2} = \frac{2}{GI_t} \times \frac{U}{L} = \frac{2}{\mu I_t} \times \frac{U}{L}$	$\frac{1}{m^2} = \frac{s^2}{kgm^3} \times \frac{U}{m}$
Normal effort $N_{(x)}$	$\left(\frac{du}{dx}\right)^2 = \frac{2}{ES} \times \frac{U}{L} = \frac{2}{YS} \times \frac{U}{L}$	$1 = \frac{s^2}{kgm} \times \frac{U}{m}$
Shear effort $V_{(x)}$	$\left(\frac{dy}{dx}\right)^2 = \frac{2}{GS_r} \times \frac{U}{L} = \frac{2}{\mu S_r} \times \frac{U}{L}$	$1 = \frac{s^2}{kgm} \times \frac{U}{m}$
General relativity	$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$	$\frac{1}{m^2} = \frac{s^2}{kgm} \times \frac{U}{m^3}$

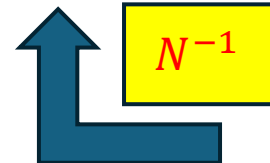
Curvature	= K	Energy density
$\left(\frac{du}{dx}\right)^2$	$= \frac{2}{ES}$	$\frac{U}{L}$
$\left(\frac{dy}{dx}\right)^2$	$= \frac{2}{GS_r}$	$\frac{U}{L}$
$\frac{1}{R^2}$	$= \frac{2}{EI}$	$\frac{U}{L}$
$\frac{1}{z^2} \left[(\varepsilon_{xx})^2 + (\varepsilon_{yy})^2 + 2(1-\nu) \frac{1}{4} \{\varepsilon_{xy}\}^2 + 2\nu \{\varepsilon_{xx}\varepsilon_{yy}\} \right]$	$= \frac{24(1-\nu^2)}{Eh^2}$	$\frac{dU}{dx dy h}$
$G_{\mu\nu} + \Lambda g_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu}$	$= 2.0766 \cdot 10^{-43}$	$T_{\mu\nu}$

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Mechanical conversion of the gravitational Einstein's constant κ



$$\kappa = \frac{8\pi G}{c^4}$$

Equivalent flexibility of space-time



6 principles of equivalences between classical relativity in weak field and general relativity (3/5)

Principle 4:

In consequences of the principles 1 to 3, the Einstein's field equation can be seen as a Hooke's law in 4 dimensions:

$$\frac{1}{m^2} = \frac{s^2}{kgm} \times \frac{U}{m^3}$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4}T_{\mu\nu}$$

Weak field case



$$\partial^\lambda \partial_\lambda \bar{h}_{\mu\nu} = \square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu}$$

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\bar{h}$$

Principe 3

Principe 2

$$h_{\mu\nu} = 2\varepsilon_{\mu\nu}$$

Principe 1

$$\varepsilon = \frac{1}{E} \sigma$$

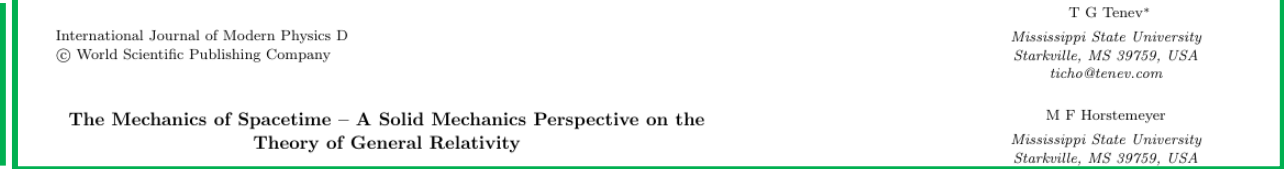
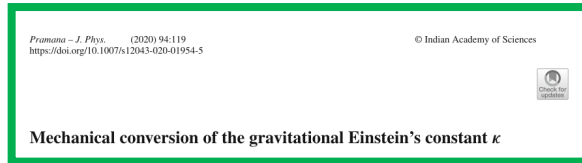
A FOUR-DIMENSIONAL HOOKE'S LAW CAN ENCOMPASS LINEAR ELASTICITY AND INERTIA

S. ANTOCI AND L. MIHICH

ABSTRACT. The question is examined, whether the formally straightforward extension of Hooke's time-honoured stress-strain relation to the four dimensions of special and of general relativity can make physical sense. The four-dimensional Hooke's law is found able to account for the inertia of matter; in the flat space, slow motion approximation the field equations for the "displacement" four-vector field ξ^i can encompass both linear elasticity and inertia. In this limit one just recovers the equations of motion of the classical theory of elasticity.

6 principles of equivalences between classical relativity in weak field and general relativity (4/5)

Principle 5 :



In the case of wave (mechanical or gravitational) there is correspondence between the energy density and the young's modulus of the medium

$$\left\{ \begin{aligned} \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 u(x,t)}{\partial t^2} &= 0. \\ \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\rho}{E} \times \frac{\partial^2 u(x,t)}{\partial t^2} &= 0. \end{aligned} \right.$$

$$Y_{longitudial} = \rho c^2$$

$$Y_{(shear)} = 2(1 + \nu)\rho c^2$$

$$\begin{aligned} \frac{1}{c^2} &= \frac{\rho}{E} \\ c &= \sqrt{\frac{E}{\rho}} \end{aligned}$$

$$c_{shear} = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{2(1 + \nu)\rho}}$$

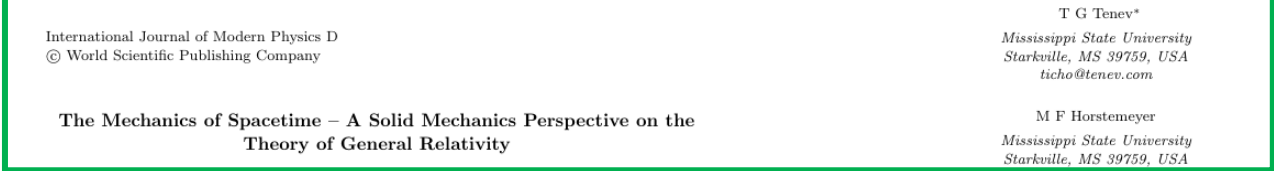
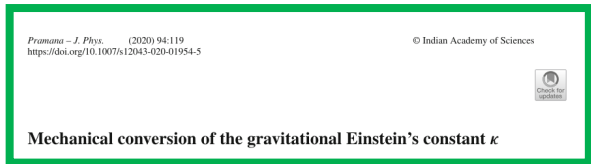
$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + \mu) \vec{\nabla} (\vec{\nabla} \cdot \vec{u}) + \mu \vec{\nabla}^2 \vec{u} + f_{external}$$

$$\partial^\lambda \partial_\lambda \bar{h}_{\mu\nu} = \square \bar{h}_{\mu\nu} = 0.$$

$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$h_{\mu\nu} = A_\times \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

6 principles of equivalences between classical relativity in weak field and general relativity (5/5)



Principe 6 :

From principle 1: In the case of gravitational waves, the component of the perturbation tensor $h_{\mu\nu}$ (polarizations) can be read as the component of an associate strain tensor

Measured by Ligo

$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \varepsilon_{xy}(A_+) = \frac{1}{2} A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

General relativity in weak field

Mechanical analogy

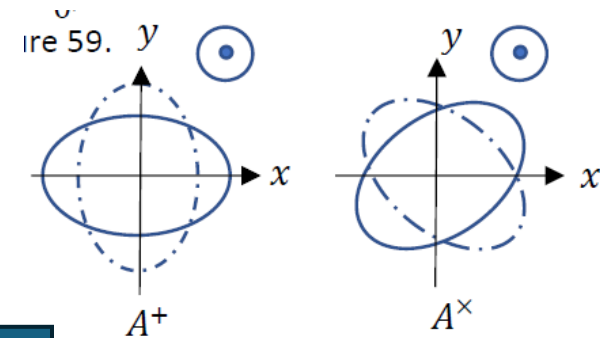
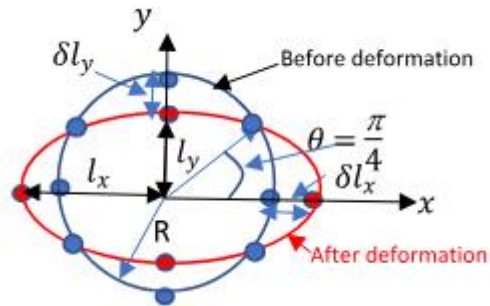
$$h_{\mu\nu} = A_\times \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \varepsilon_{xy}(A_\times) = \frac{1}{2} A_\times \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Not Measured by Ligo

2) Consequence of these 6-equivalence principles on the characteristics of the equivalent elastic medium associated at the space time

- 2.1) Based on classical general relativity of gravitational waves - Basic polarizations and strains in transverse planes –

isotropic transverse medium in sheets not connected out of plane to gather



Classical polarisation of the gravitational wave of the general relativity not modified

An isotropic transverse material worked as a succession of unconnected sheets

Under gravitational wave

« Sheet of particles » created by dynamic screen effect by GW

$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \varepsilon_{xy}(A_+) = \frac{1}{2} A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

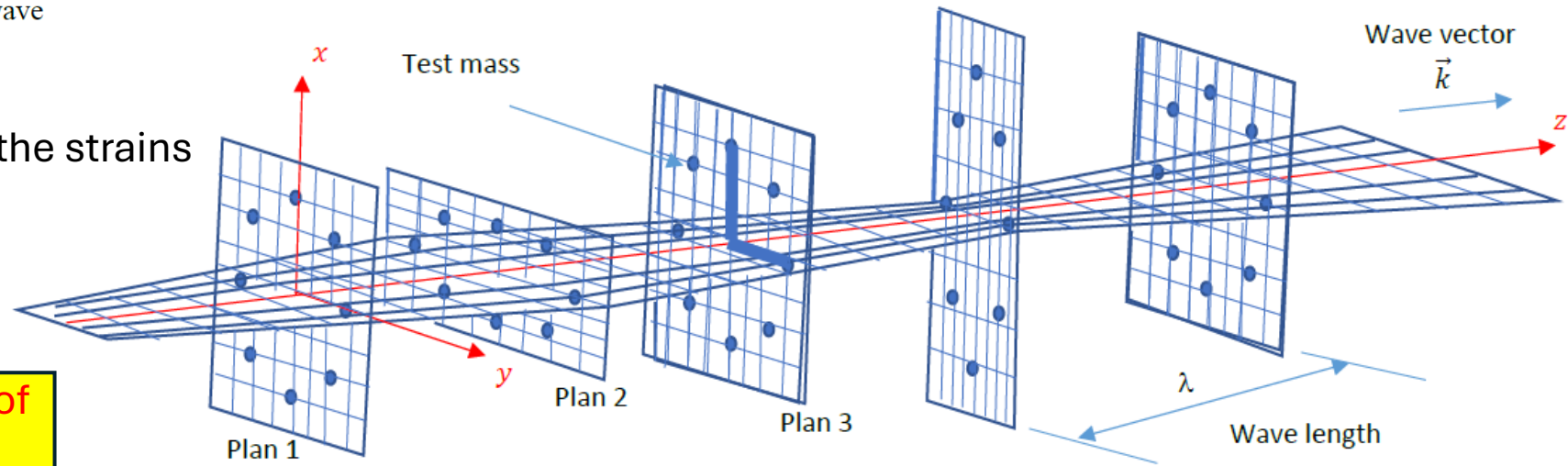
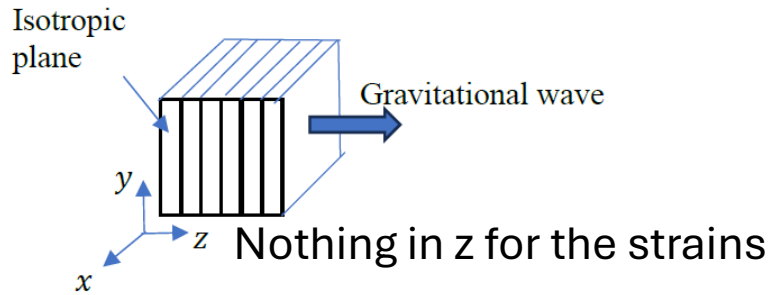
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Mechanical conversion of the gravitational Einstein's constant κ

Nothing in z

$$h_{\mu\nu} = A_\times \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \varepsilon_{xy}(A_\times) = \frac{1}{2} A_\times \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



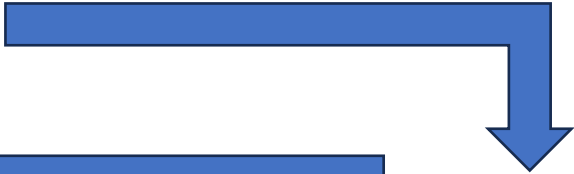
An elastic medium made of independent sheets => isotropic transverse

Classical General relativity

Law of passage in elasticity between classical general relativity and the associated deformation tensors in 2D - via continuum mechanic

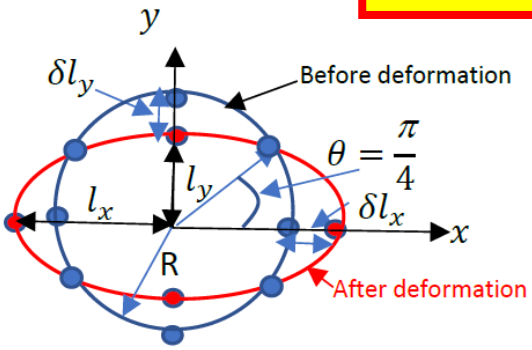
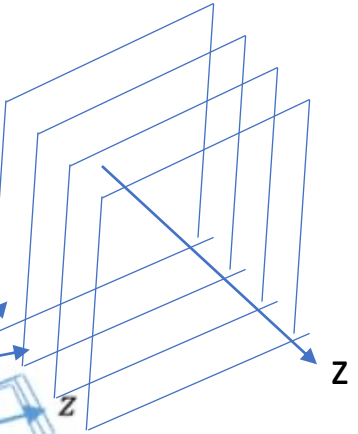
$2\varepsilon^{\mu\nu}$ (in mechanics) \rightarrow $h^{\mu\nu}$ (in GR in weak field)

Torsion pure: 2 expressions of the strain tensor



Pure mechanical Torsion strain tensors

No connection between the sheets



Continuum Mechanical in elastic

For a polarised wave A_+ :

$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For a polarised wave A_x :

$$h_{\mu\nu} = A_x \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

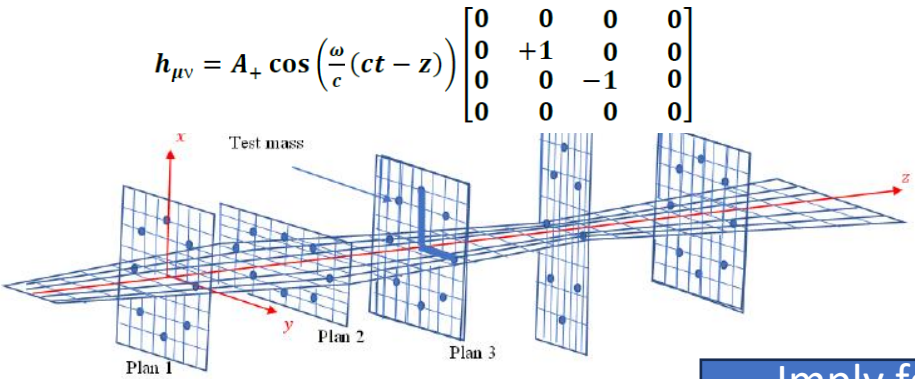
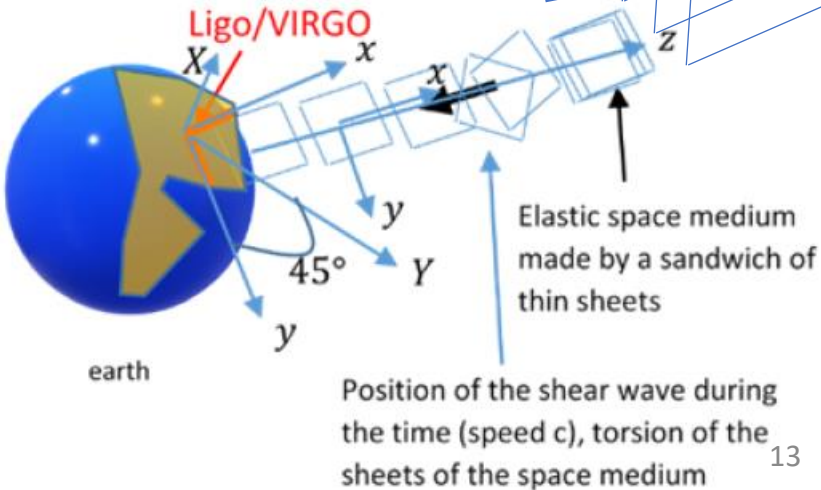
For a polarised wave A_y :

$$h_{\mu\nu} = A_y \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For a polarised wave A_z :

$$h_{\mu\nu} = A_z \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Imply for a mechanical model of medium made of stacking of thin sheets : space isotope transverse (anisotropic of space)



$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$h_{\mu\nu} = A_x \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

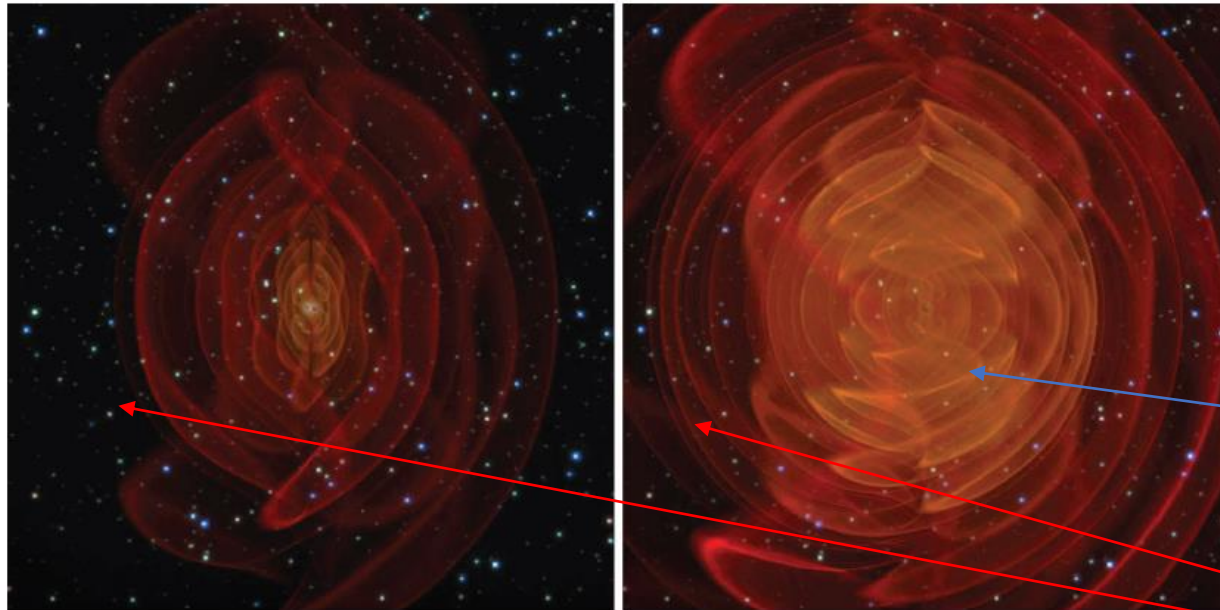
Numerical gravitation in 3D confirm the field of strain deformation/force (thin shell/thin sheet + screen effect)

SciDAC 2007

Journal of Physics: Conference Series 78 (2007) 012010

IOP Publishing

doi:10.1088/1742-6596/78/1/012010



Journal of Physics: Conference Series

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Binary black holes, gravitational waves, and numerical relativity

To cite this article: Joan M Centrella *et al* 2007 *J. Phys.: Conf. Ser.* 78 012010

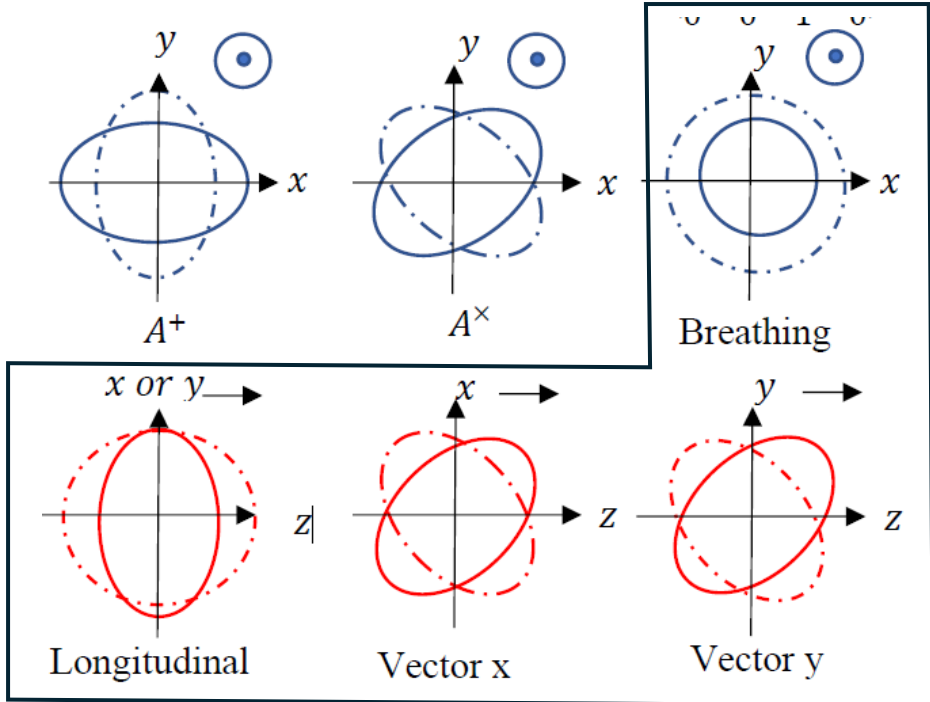
Torsion of space

Equivalent Sheets of space of identical gravity/strains

Figure 5. Contours of gravitational radiation for the merger of equal mass binary black holes. The radiation amplitude is denoted by the colors, increasing from red through orange and into yellow. (left) just before the black holes merge (right) shortly after the merger.

- 2.2) Based on modified general relativity of gravitational waves - complementary polarizations and strains in the propagation direction – **isotropic transverse medium in sheets connected out of plane to gather**

Complementary polarisations



This time there are some possible strain in z

Approach 1: Einstein-Cartan Theory and elasticity Theory (1/2) - 3 intersecting publications

Connection between the **Einstein-Cartan theory** (general relativity modified with geometric torsion added in the Riemann tensor) and defect theory plastic **crystallography**



Einstein-Cartan theory as a theory of defects in space-time

M. L. Ruggiero* and A. Tartaglia†
Dip. Fisica, Politecnico and INFN, Torino, Italy, I-10129

The Einstein-Cartan theory of gravitation and the classical theory of defects in an elastic medium are presented and compared. The former is an extension of general relativity and refers to four-dimensional space-time, while we introduce the latter as a description of the equilibrium state of a three-dimensional continuum. Despite these important differences, an analogy is built on their common geometrical foundations, and it is shown that a space-time with curvature and torsion can be considered as a state of a four-dimensional continuum containing defects. This formal analogy is useful for illustrating the geometrical concept of torsion by applying it to concrete physical problems. Moreover, the presentation of these theories using a common geometrical basis allows a deeper understanding of their foundations.

Polarisation following the **propagation direction of the gravitational wave** in the case of Einstein-Cartan theory



Gravitational Waves in ECSK theory: Robustness of mergers as standard sirens and nonvanishing torsion

Emilio Elizalde¹, Fernando Izaurieta^{2†}, Cristian Riveros^{3‡}, Gonzalo Salgado^{2§}
and Omar Valdivia^{1,4,5¶}

Nonlinear Passage law between the **perturbation of the metric** in Einstein Cartan-theory and equivalent strains of the medium



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<https://doi.org/10.1140/epjc/s10052-021-08862-x>

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PHYSICAL JOURNAL C



Regular Article - Theoretical Physics

Non-linear plane gravitational waves as space-time defects

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Example of complementary polarisations with Einstein-Cartan theory in link with defect theory

(2/2)

Einstein Cartan theory

$$R_{ab} - \frac{1}{2} R g_{ab} = \kappa P_{ab}$$

$$T_{ab}{}^c + g_a{}^c T_{bd}{}^d - g_b{}^c T_{ad}{}^d = \kappa \sigma_{ab}{}^c$$

Z components appears

Non linear passage law (plasticity)

$$\varepsilon^{ab} = e^{a\mu} e^{b\nu} \varepsilon_{\mu\nu} = \frac{1}{2} \begin{pmatrix} H/A & -\sqrt{2}a_1/A & -\sqrt{2}a_2/A & H/A \\ -\sqrt{2}a_1/A & 0 & 0 & -\sqrt{2}a_1/A \\ -\sqrt{2}a_2/A & 0 & 0 & -\sqrt{2}a_2/A \\ H/A & -\sqrt{2}a_1/A & -\sqrt{2}a_2/A & H/A \end{pmatrix} \quad \varepsilon^{(i)(j)} = \frac{1}{2} \begin{pmatrix} 0 & 0 & \frac{\sqrt{2}J}{\rho A} \sin \phi \\ 0 & 0 & -\frac{\sqrt{2}J}{\rho A} \cos \phi \\ \frac{\sqrt{2}J}{\rho A} \sin \phi & -\frac{\sqrt{2}J}{\rho A} \cos \phi & H/A \end{pmatrix}$$

Corresponding polarisations (GR pol+4 pol)

With as basis of polarizations:

$$P_{ab} = p_{(+)} P_{ab}^{(+)} + p_{(\times)} P_{ab}^{(\times)} + p_{(b)} P_{ab}^{(b)} + p_{(l)} P_{ab}^{(l)} + p_{(xz)} P_{ab}^{(xz)} + p_{(yz)} P_{ab}^{(yz)}$$

$$P_{ab}^{(+)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad P_{ab}^{(\times)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P_{ab}^{(b)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad P_{ab}^{(l)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P_{ab}^{(xz)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad P_{ab}^{(yz)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Strains component in the 3 directions

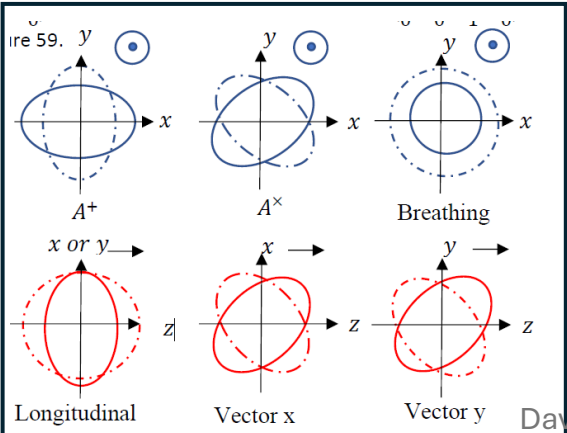
Which corresponds to pure torsion in the mechanical sense of the term studied in this thesis:

$$\varepsilon_{ab}^{(+)} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \varepsilon_{xx} & 0 & 0 \\ 0 & 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \varepsilon_{ab}^{(\times)} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_{xy} & 0 \\ 0 & \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

But taking into account the torsion in the sense of Einstein Cartan potentially generates other types of deformations and constraints concerning for the last 3 in red the direction of propagation of the wave (z here).

$$\varepsilon_{ab}^{(b)} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \varepsilon_{xx} & 0 & 0 \\ 0 & 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \varepsilon_{ab}^{(l)} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon_{zz} \end{pmatrix}$$

$$\varepsilon_{ab}^{(xz)} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon_{xz} \\ 0 & 0 & 0 & 0 \\ 0 & \varepsilon_{zx} & 0 & 0 \end{pmatrix} \quad \varepsilon_{ab}^{(yz)} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon_{yz} \\ 0 & 0 & \varepsilon_{zy} & 0 \end{pmatrix}$$



Abstract

We consider non-linear plane gravitational waves as propagating space-time defects, and construct the Burgers vector of the waves. In the context of classical continuum systems, the Burgers vector is a measure of the deformation of the medium, and at a microscopic (atomic) scale, it is a naturally quantized object. One purpose of the present article is ultimately to probe an alternative way on how to quantize plane gravitational waves.

Law of passage in plasticity between Einstein Cartan polarization and the associated deformation tensors in 3D - via the theory of defects

$$\varepsilon_{\mu\nu} = \frac{1}{2} \begin{pmatrix} H & -\sqrt{2}a_1 & -\sqrt{2}a_2 & H \\ -\sqrt{2}a_1 & 0 & 0 & -\sqrt{2}a_1 \\ -\sqrt{2}a_2 & 0 & 0 & -\sqrt{2}a_2 \\ H & -\sqrt{2}a_1 & -\sqrt{2}a_2 & H \end{pmatrix} \quad \varepsilon^{(i)(j)} = \frac{1}{2} \begin{pmatrix} 0 & 0 & \frac{\sqrt{2}J}{\rho A} \sin\Phi \\ 0 & 0 & -\frac{\sqrt{2}J}{\rho A} \cos\Phi \\ \frac{\sqrt{2}J}{\rho A} \sin\Phi & -\frac{\sqrt{2}J}{\rho A} \cos\Phi & H/A \end{pmatrix}$$

Additional deformation direction xz and yz

Einstein-Cartan Polarisation

The first two terms are those of general relativity giving the two classical polarizations as measured by Ligo/Virgo.

$$P_{ab}^{(+)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Which corresponds to pure torsion in the mechanical sense of the term studied in this thesis:

$$\varepsilon_{ab}^{(+)} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \varepsilon_{xx} & 0 & 0 \\ 0 & 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

But taking into account the torsion in the sense of Einstein Cartan potentially generates other types of deformations and stresses concerning for the last 3 in red the direction of propagation of the wave (z here).

$$\varepsilon_{ab}^{(b)} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \varepsilon_{xx} & 0 & 0 \\ 0 & 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\varepsilon_{ab}^{(xz)} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon_{xz} \\ 0 & 0 & 0 & 0 \\ 0 & \varepsilon_{zx} & 0 & 0 \end{pmatrix}$$

$$P_{ab}^{(\times)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\varepsilon_{ab}^{(\times)} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_{xy} & 0 \\ 0 & \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\varepsilon_{ab}^{(l)} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon_{zz} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\varepsilon_{ab}^{(yz)} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon_{yz} \\ 0 & 0 & \varepsilon_{zy} & 0 \end{pmatrix}$$

Continuum Mechanical in plastic

Non-linear plane gravitational waves and space-time defects share many features. Both field configurations (i) are established over a flat space-time background, (ii) induce a local deformation in the background geometry, (iii) may have an axial symmetry (along the z axis, for instance), (iv) may have a singularity along an axis (the z axis, for instance). Therefore, it is possible to define and evaluate the Burgers vector for non-linear plane gravitational waves. The Burgers vector in a crystalline lattice or inside a metal determines the nature of the defect.

269] Carneiro F L,Ulhoa S C, Maluf J W, da Rocha-Neto J F (2021) «Non-linear plane gravitational waves as space-time defects» Europran Physical Journal C 81 67

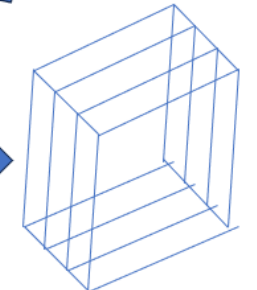
Reconstitution of an equivalent 3 D medium isotrope transverse

Plasticity

Defect theory // Einstein-Cartan theory

Defect theory // Crystallography

Equivalent elastic Medium



Connection between the sheets

Approach 2: Other general relativity modified theories

Not only Einstein-Cartan theory allows to have complementary polarisations

Theories	Polarization modes
Metric $f_{(R)}$ gravity	$A+, A \times, \textit{breathing}, \textit{longitudinal}$
Palatini $f_{(R)}$ gravity	$A+, A \times,$
Scalar tensor theory (massive)	$A+, A \times, \textit{breathing}, \textit{longitudinal}$
Brans-Dicke theory (massive)	$A+, A \times, \textit{breathing}, \textit{longitudinal}$
Brans-Dicke theory (mass less)	$A+, A \times, \textit{breathing}$

Theory	+	×	x	y	B	L
General relativity	Yes	Yes	No	No	No	No
GR in noncompactified 4/6D Minkovski	Yes	Yes	Yes	Yes	Yes	Yes
Einstein/Aether	Yes	Yes	Yes	Yes	Yes	Yes
5D Kaluza-Klein	Yes	Yes	Yes	Yes	Yes	No
Randall-Sundrum braneworlds	Yes	Yes	No	No	No	No
Dvali-Gabadadze-Porrati braneworld	Yes	Yes	Dep	Dep	Dep	Dep
Brans-Dicke	Yes	Yes	No	No	Yes	Yes
F(R) gravity	Yes	Yes	No	No	Yes	Yes
Bimetric Theory	Yes	Yes	Yes	Yes	Yes	Yes

On the Polarization of Gravitational Waves

Dissertation
zur
Erlangung der naturwissenschaftlichen Doktorwürde
(Dr. sc. nat.)
vorgelegt der
Mathematisch-naturwissenschaftlichen Fakultät
der
Universität Zürich
von

Lionel Antoine Philippoz
von
Leytron VS

Gravitational Wave Polarizations: A test of General Relativity using Binary Black hole mergers

Thesis by
Sudhi Mathur

In Partial Fulfillment of the Requirements for the
Degree of
Bachelor of Science, Physics

Approach 3: General relativity in second order

Elasticity solid medium 2nd order

Second order general relativity also forecast strains in z direction

Classical general relativity in 2nd order gravito-electromagnetism

Gravitational Wave polarisations A^+ and A^\times and combination of these polarisations

Transition via Fermi coordinates

Associated elastic medium

$$\frac{d^2X}{dt^2} = -E$$

$$E = E^{(0)} + E^{(1)}$$

$$E^{(0)} = -\nabla\phi^{(0)}$$

$$E^{(1)} = -\nabla\phi^{(1)} - \frac{2}{c} \frac{\partial A}{\partial T}$$

$$\phi = -\frac{c^2}{2} R_{0i0j}(T) X^i X^j - \frac{c^2}{6} R_{0i0j,m}(T) X^i X^j X^m$$

$$\phi = \phi^{(0)} + \phi^{(1)}$$

$$\phi^{(0)} = \frac{c^2}{2} R_{0i0j}(T) X^i X^j$$

$$\phi^{(1)} = -\frac{1}{12} \frac{\omega^3}{c} [A^+ \cos(\omega T) (Y^2 - Z^2) - 2A^\times \sin(\omega T) YZ]$$

Components of the gravitational force in the field of a gravitational wave

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† Department of Physics and Astronomy, Cardiff University, Cardiff CF24 3YB, United Kingdom
‡ Sternberg Astronomical Institute, Moscow University, Moscow 119899, Russia

Abstract.

Gravitational waves bring about the relative motion of free test masses. The detailed knowledge of this motion is important conceptually and practically, because the mirrors of laser interferometric detectors of gravitational waves are essentially free test masses. There exists an analogy between the motion of free masses in the field of a gravitational wave and the motion of free charges in the field of an electromagnetic wave. In particular, a gravitational wave drives the masses in the plane of the wave-front and also, to a smaller extent, back and forth in the direction of the wave's propagation. To describe this motion, we introduce the notion of "electric" and "magnetic" components of the gravitational force. This analogy is not perfect, but it reflects some important features of the phenomenon. Using different methods, we demonstrate the presence and importance of what we call the "magnetic" component of motion of free masses. It contributes to the variation of distance between a pair of particles. We explicitly derive the full response function of a 2-arm laser interferometer to a gravitational wave of arbitrary polarization. We give a convenient description of the response function in terms of the spin-weighted spherical harmonics. We show that the previously ignored "magnetic" component may provide a correction of up to 10%, or so, to the usual "electric" component of the response function. The "magnetic" contribution must be taken into account in the data analysis, if the parameters

$$X_{(T)} = X_0 + \frac{1}{4} \frac{\omega}{c} A^+ [1 - \cos(\omega T)] (Y_0^2 - Z_0^2) + \frac{1}{2} \frac{\omega}{c} A^\times \sin(\omega T) Z_0 Y_0$$

$$Y_{(T)} = \left[1 - \frac{A^+}{2} \sin(\omega T) \right] Y_0 + \frac{A^\times}{2} [1 - \cos(\omega T)] Z_0 + \frac{1}{2} \frac{\omega}{c} A^+ [\cos(\omega T) - 1] X_0 Y_0 - \frac{1}{2} \frac{\omega}{c} A^\times \sin(\omega T) X_0 Z_0$$

$$Z_{(T)} = \left[1 + \frac{A^+}{2} \sin(\omega T) \right] Z_0 + \frac{A^\times}{2} [1 - \cos(\omega T)] Y_0 + \frac{1}{2} \frac{\omega}{c} A^+ [1 - \cos(\omega T)] X_0 Z_0 - \frac{1}{2} \frac{\omega}{c} A^\times \sin(\omega T) X_0 Y_0$$

$$\frac{|E^{(1)}|}{|E^{(0)}|} \approx \frac{\omega L}{c} \approx \frac{L}{\lambda} \approx 8 \times 10^{-2} \left(\frac{L}{4km} \right) \left(\frac{f}{1000Hz} \right)$$

Gravitomagnetic induction in the field of a gravitational wave

Matteo Luca Ruggiero*

Dipartimento di Matematica "G. Peano", Università degli studi di Torino,

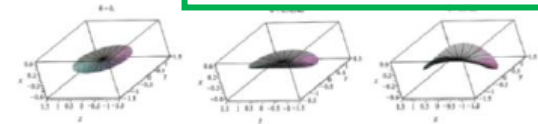
Via Carlo Alberto 10, 10123 Torino, Italy and

INFN - LNL, Viale dell'Università 2, 35020 Legnaro (PD), Italy

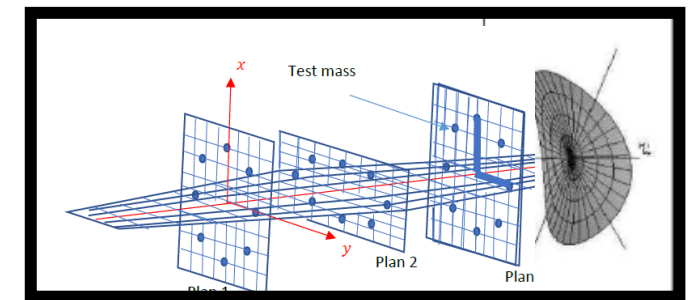
(Dated: August 30, 2022)

Abstract

The interaction of a plane gravitational wave with test masses can be described in the proper detector frame, using Fermi coordinates, in terms of a gravitoelectric and a gravitomagnetic field. We use this approach to calculate the displacements produced by gravitational waves up to second order in the distance parameter and, in doing so, we emphasize the relevance of the gravitomagnetic contribution related to gravitational induction. In addition, we show how this approach can be generalized to calculate displacements up to arbitrary order.



Motion of the test mass out of the transverse plane



Gravito electromagnetism

Approach 4 : General relativity by hydrodynamic approach (1/2)

c the speed of sound relative to the fluid,
 v the speed of the moving fluid,
 \vec{n} the unit vector,

We can therefore write with respect to the laboratory that the sound ray propagates with respect to the fluid at a speed:

$$\frac{d\vec{x}}{dt} = c\vec{n} + \vec{v}$$

$$dx - vdt = cndt$$

Either by defining: $n^2 = 1$ normalization condition that defines a sound cone:

$$-c^2 dt^2 + (dx - vdt)^2 = 0$$

Or by expanding the expression below:

$$\begin{aligned} -c^2 dt^2 + dx^2 - 2vdxdt + v^2 dt^2 &= 0 \\ [v^2 - c^2]dt^2 + 1 dx^2 - 2vdxdt &= 0 \end{aligned}$$

So, we have a metric of the following form:

$$g = \Omega^2 \begin{bmatrix} [v^2 - c^2] & -v^T \\ -v & I \end{bmatrix}$$

Ω a function

I the Identity matrix 3×3

In the case of the fluid dynamic the metric becomes:

$$g_{\mu\nu}(t,x) = \frac{\rho}{c} \begin{bmatrix} -[c^2 - v^2] & -v^T \\ -v & I \end{bmatrix}$$

Wit $I = [I]$ the identity matrix.

Acoustic wave on a fluid in motion can recreate the behaviour of the gravitational wave with also z direction strains

Parameter	Classical theory of the gravitational wave (see appendix A of this thesis for the proof)	Analogy acoustic binary in a fluid in movement (Aeroacoustic quadrupole)
Expression of the strain h generated by the binary on the medium (transverse strain)	$h_{ij}^{TT} = \bar{h}_{ij}^{TT} = \frac{2G}{Rc^4} \frac{d^2}{dt^2} I_{ij}^{TT} \left(t - \frac{R}{c} \right)$ <p>Transverse component</p>	$h_{jk}^{TT} = \bar{h}_{jk}^{TT} = \frac{kG}{4\pi r c_0^4} \frac{d^2}{dt^2} I_{jk}^{TT} \left(t - \frac{r}{c_0} \right)$ <p>Magnitude of the metric perturbation see Formula 3.27 [250] with $k=8\pi$</p>
Strain in the direction Longitudinal (propagation direction of the wave)	<p>Does not exist we suppose in general that space is a incompressible medium</p> <p>Longitudinal component</p>	<p>If compressible medium of density ρ_0 in a linear theory with specific Newtonian gauge at higher order of the compactness of the source that imply that an additional compression wave is possible.</p> $ h_{00}^{lg} \frac{c_0}{\Omega_r} = \frac{2}{5} \left(\frac{\Omega D}{2c_0} \right)^2 \bar{h}_{jk}^{TT} $ <p>Magnitude of the metric perturbation see Formula 3.28 [250]</p>

$$\square_A h_{\mu\nu}(z, t) = 0$$

$$h_{\mu\nu}(z, t) = \begin{bmatrix} -c_0^2 \frac{(\gamma+1)}{2} \times \frac{\rho'_{(1)}(z, t)}{\rho_0} & 0 & 0 & -v'_{(1)}(z, t) \\ 0 & \frac{(3-\gamma)}{2} \times \frac{\rho'_{(1)}(z, t)}{\rho_0} & 0 & 0 \\ 0 & 0 & \frac{(3-\gamma)}{2} \times \frac{\rho'_{(1)}(z, t)}{\rho_0} & 0 \\ -v'_{(1)}(z, t) & 0 & 0 & \frac{(3-\gamma)}{2} \times \frac{\rho'_{(1)}(z, t)}{\rho_0} \end{bmatrix} \quad 21$$

Approach 4 : General relativity by hydrodynamic approach (2/2)

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 Fakultät V - Verkehrs- und Maschinensysteme
 Institut für Strömungsmechanik und Technische Akustik
 Fachgebiet Technische Akustik
 Sekr. TA 7 - Einsteinufer 25 - 10587 Berlin

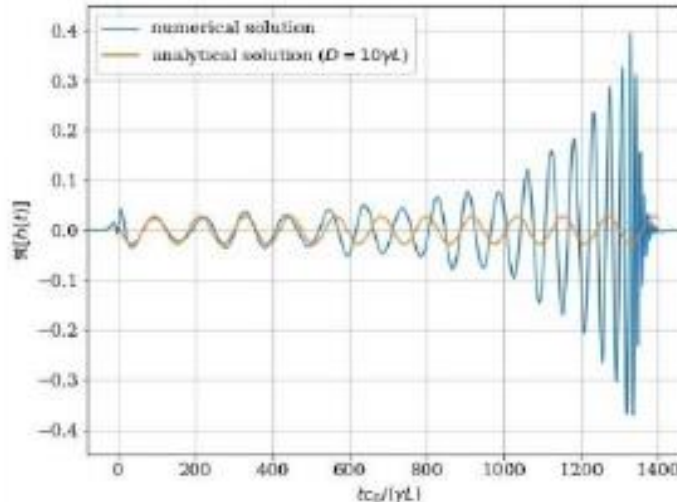
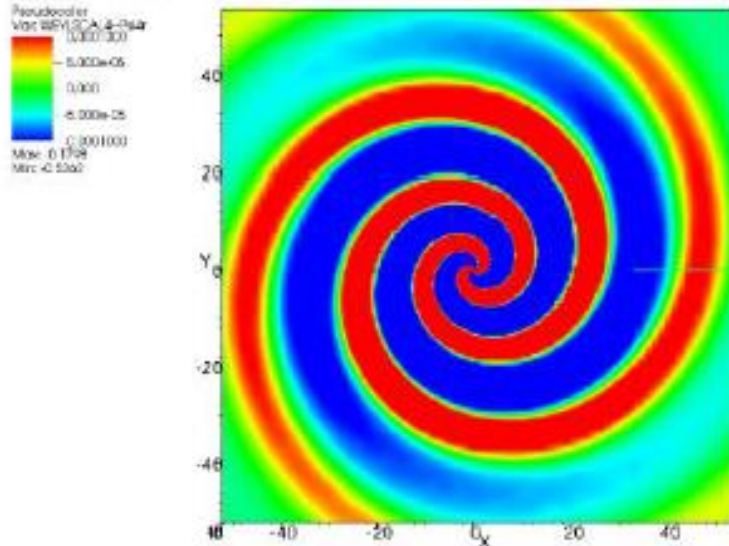
Acoustic analogies with general relativity,
 quantum fields, and thermodynamics

Drasko Masovic, TU Berlin, 2018 (last update: August 15, 2022)

$$h' = \begin{bmatrix} -c_0^2 \frac{(\gamma+1) \rho'_{(1)}}{2 \rho_0} & v_{(1)}^3 \sin \theta & 0 & -v_{(1)}^3 \cos \theta \\ v_{(1)}^3 \sin \theta & \frac{(3-\gamma) \rho'_{(1)}}{2 \rho_0} & 0 & 0 \\ 0 & 0 & \frac{(3-\gamma) \rho'_{(1)}}{2 \rho_0} & 0 \\ -v_{(1)}^3 \cos \theta & 0 & 0 & \frac{(3-\gamma) \rho'_{(1)}}{2 \rho_0} \end{bmatrix}$$

Example GW150914
 reproduced by acoustic
 binary in rotation

DB: Psl4r.0.xy.h5
 Cycle: 560128 Time: 1350.27



Space time is model by a dynamic fluid
 and the acoustic wave model
 gravitational wave

parameter	value
binary separation	10L
black hole mass	32.5M _⊙
black hole spin	-0.385L ² c ₀ ³ /(4G)
radial linear momentum	-0.0008454Lc ₀ ² /G
azimuthal linear momentum	-0.0953Lc ₀ ³ /G

Synthesis of the Modified General relativity

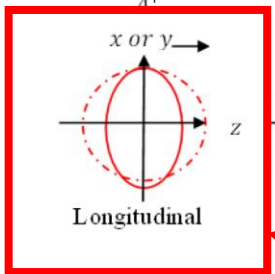
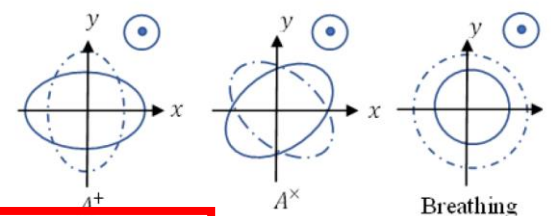
Einstein-Cartan theory

Longitudinal polarisation

Second order general relativity

Hydrodynamic and acoustic

Other general relativity modified



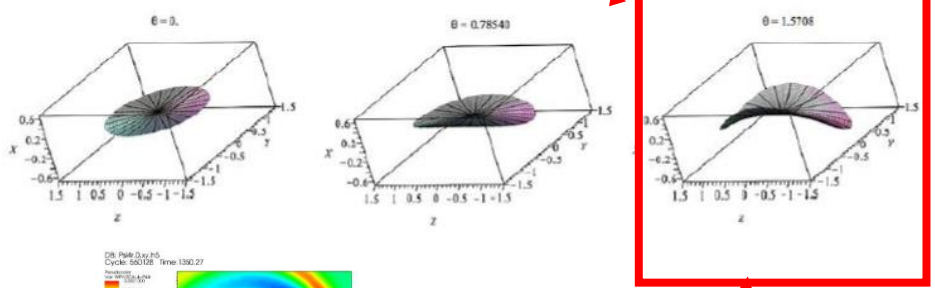
$$P_{ab}^{(+)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad P_{ab}^{(\times)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (55)$$

$$P_{ab}^{(b)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad P_{ab}^{(l)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (56)$$

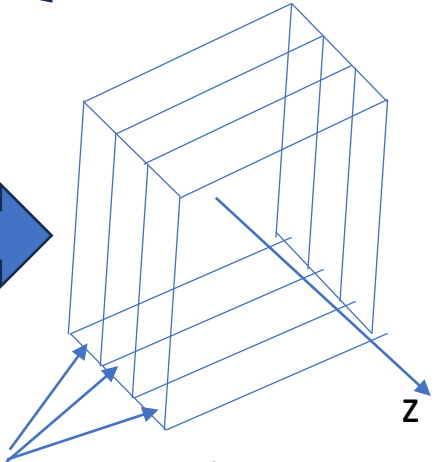
$$P_{ab}^{(xz)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad P_{ab}^{(yz)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

If we read the additional component of polarisation as xx xz and yz additional strains we obtain a coherent 3D material

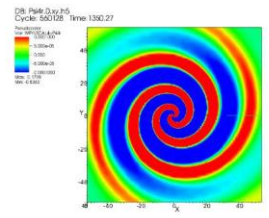
Equivalent elastic Medium



Strain in the longitudinal direction



Connection between the sheets



$$h_{ij}^{TT} = \bar{h}_{ij}^{TT} = \frac{2G}{Rc^4} \frac{d^2}{dt^2} I_{ij}^{TT} \left(t - \frac{R}{c} \right)$$

$$h_{jk}^{TT} = \bar{h}_{jk}^{TT} = \frac{kG}{4\pi r c_0^4} \frac{d^2}{dt^2} I_{jk}^{TT} \left(t - \frac{r}{c_0} \right)$$

Theory	+	×	x	y	b	l
General relativity	Yes	Yes	No	No	No	No
GR in noncompactified 4/D Minkovski	Yes	Yes	Yes	Yes	Yes	Yes
Einstein/Aether	Yes	Yes	Yes	Yes	Yes	Yes
5D Kaluza-Klein	Yes	Yes	Yes	Yes	Yes	No
Randall-Sundrum braneworlds	Yes	Yes	No	No	No	No
Dvali-Gabadadze-Porrati braneworld	Yes	Yes	dep	dep	dep	dep
Brans-Dicke	Yes	Yes	No	No	Yes	Yes
F(R) gravity	Yes	Yes	No	No	Yes	Yes
Bimetric Theory	Yes	Yes	Yes	Yes	Yes	Yes

$$|h_{00}^{lg}| \frac{c_0}{\Omega_r} = \frac{2}{5} \left(\frac{\Omega D}{2c_0} \right)^2 |\bar{h}_{jk}^{TT}|$$

Compression strain in the longitudinal direction

What are the message of the different tentative of modified general relativity?

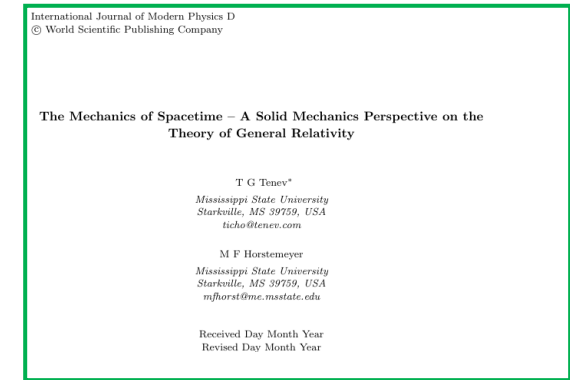
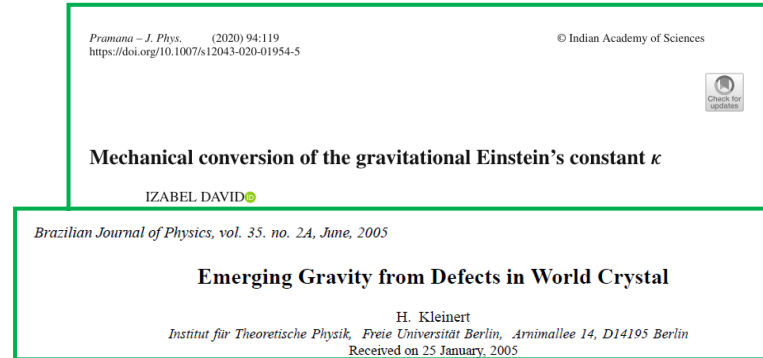
- The different approaches to modify the general relativity converge all in direction of **additional polarisations at $A+ A^x$**
- Based on the principle 1 between the polarisation and the strain, these approaches allow to «rebuild « **a 3 D medium with sheets of space connected together**
- But this third dimension is associated:
 - For Einstein Cartan-theory at defect theory so at plasticity in crystallography
 - For Einstein's general relativity at second order deformation out of the plane
 - For Einstein's general relativity at an hydroacoustic fluid theoryFor all several theories at complementary polarisation that are not measured until to day

3) Consequences about the potential models that can be used to reproduce the forecast and measured strains of the space-time

3.1) Practical characteristic of the elastic medium isotropic transverse

- Grain size thickness?
 - Tenev and Horstemeyer: 10^{-35} m
 - Quantum gravity : 10^{-35} m
 - String theory : 10^{-35} m

=> we keep this hypothesis



- Structure in sheet by screen dynamic effect under gravitational wave?
 - Tenev and Horstemeyer : hyper surface

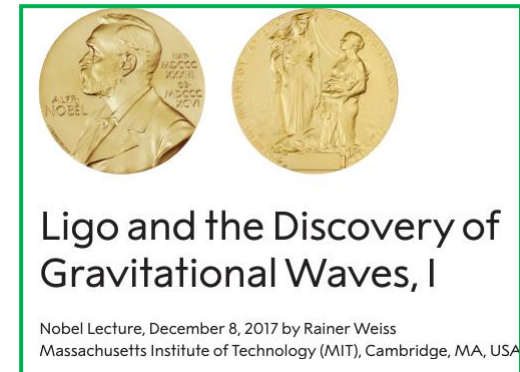
Elastic constants:

$$Y = 6c^7/2\pi\hbar G^2, \quad \nu = 1$$

- Young's modulus?
 - Following quantum field Theory 10^{113} Pa
 - Following gravitational Wave (R Weiss Nobel Prize lecture) 10^{31} Pa

=> we will extract Y of our models basing GR strains

Rainer Weiss Lecture



- Poisson's ratio?
 - Tenev and Horstemeyer $\nu=1$ compatible with strains measured in the interferometers

The power per area in the wave is proportional to the square of the rate of change of the strain times a gigantic factor which tells that a small amount of strain in space is accompanied by a huge amount of energy. In other words, it takes enormous amounts of energy to distort space. One way to say it is, the stiffness (Young's modulus) of space at a distortion frequency of 100 Hz is 10^{20} larger than steel.

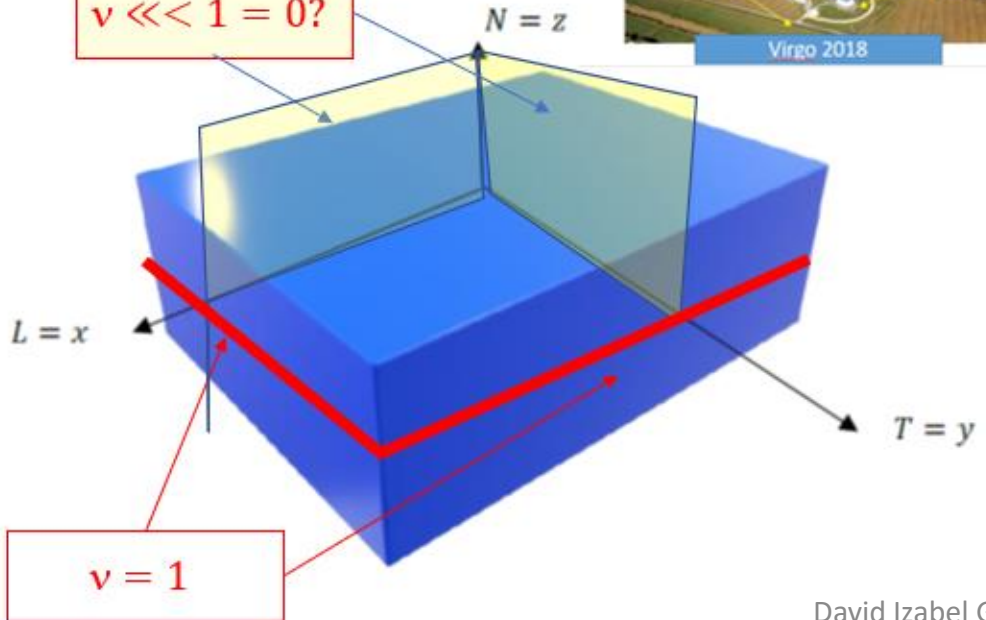
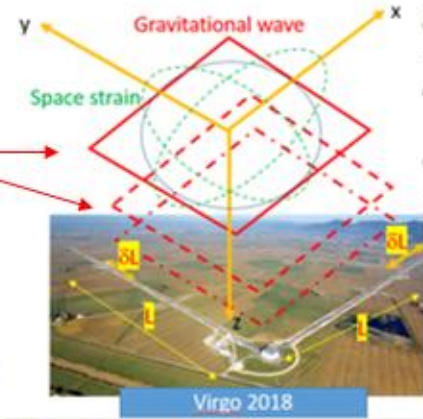
Consequences => pass isotropy => non isotropy of the medium? Limit validity analogy?

Direction of the gravitational waves



Planes deforming independently of each other
($v_{xz} = v_{yz} = 0$)

$\nu \ll 1 = 0?$



a) In the LT plan (x, y)
- For Young's modulus $Y=E$:

$$Y_L = E_L = Y_T = E_T$$

- For the Poisson's ratio:

$$\nu_{TL} = \nu_{LT}$$

$$G_{LT} = \frac{E_L}{2(1 + \nu_{LT})}$$

b) In LN (xz) and TN plans (yz)

- For the Poisson's ratio:

$$\nu_{NT} = \nu_{NL}$$

$$\nu_{LN} = \nu_{TN}$$

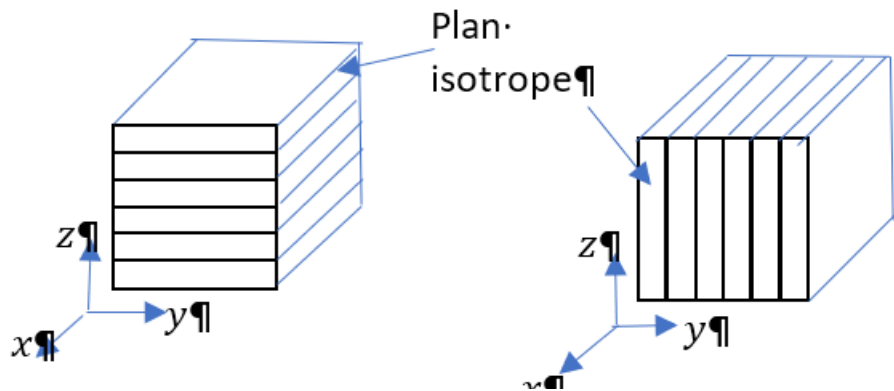
- For shear modulus:

$$G_{TN} = G_{LN}$$

- For the following key relationships between Poisson's ratio and Young's modulus:

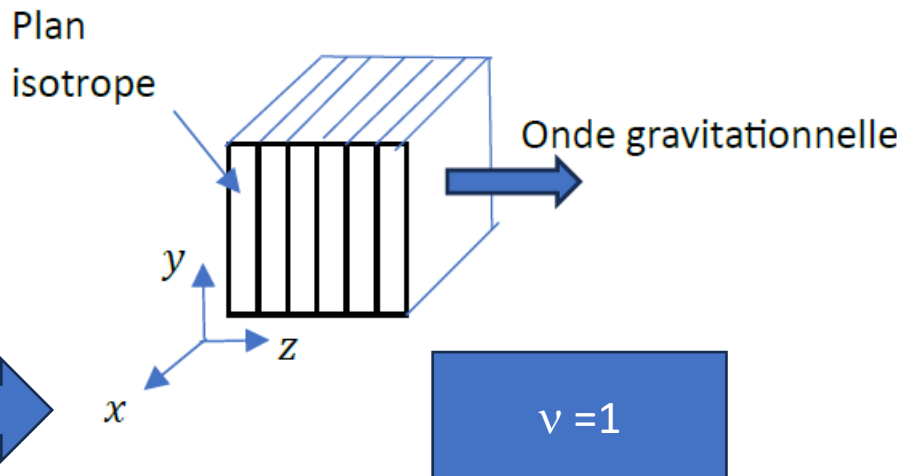
$$\frac{\nu_{NT}}{E_N} = \frac{\nu_{LN}}{E_L}$$

Clay HDL



Analogy of the space medium behaviour as clay in sheets

Space under gravitational wave



Same different nu ratio as space time following different direction

$$\begin{cases} \varepsilon_h = \varepsilon_{xx} \\ \varepsilon_h = \varepsilon_{yy} \\ \varepsilon_v = \varepsilon_{zz} \\ \varepsilon_{hv} = \varepsilon_{yz} \\ \varepsilon_{hv} = \varepsilon_{xz} \\ \varepsilon_{hh} = \varepsilon_{xy} \end{cases} = \begin{bmatrix} \frac{1}{E_h} & -\frac{\nu_{hh}}{E_h} & -\frac{\nu_{vh}}{E_v} & 0 & 0 & 0 \\ -\frac{\nu_{hh}}{E_h} & \frac{1}{E_h} & -\frac{\nu_{vh}}{E_v} & 0 & 0 & 0 \\ -\frac{\nu_{vh}}{E_h} & -\frac{\nu_{vh}}{E_h} & \frac{1}{E_v} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G_{hv}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G_{hv}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1 + \nu_{hh}}{E_h} \end{bmatrix} \begin{cases} \sigma_h = \sigma_{xx} \\ \sigma_h = \sigma_{yy} \\ \sigma_v = \sigma_{zz} \\ \sigma_{hv} = \sigma_{yz} \\ \sigma_{hv} = \sigma_{xz} \\ \sigma_{hh} = \sigma_{xy} \end{cases}$$

Example of the clay

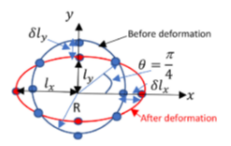
Références	E_v (MPa)	E_h (MPa)	ν_{vh}	ν_{hh}	ν_{hv}	G_{hv} (MPa)
Résultats expérimentaux	400	500	0.4	0.8	0.6- 1.0	-
Proposition pour le modèle anisotrope	280	500	0.3	0.3	0.54	130
François et al (2012)	200	400	0.125	0.125	-	178
Bernier et al (2007)	300	300	0.125	0.125	-	-
Yu et al (2013)	700	1400	0.125	0.125	-	-

Necessity to have an anisotropic model of space to be in accordance with the Poisson's ratio $\nu=1$

Gravitational wave = transverse wave Horstemeyer Poisson's ratio $\nu=1$

We demonstrate based on 3 different approaches:

First Approach: Analysis of the Movements of Particles Positioned in Space on a Circle Undergoing the Passage of a Gravitational Wave $\varepsilon_{xx} = -\nu\varepsilon_{yy}$. Strains measured in the arms of Virgo



Checked on the Ligo/Virgo measurements

Second Approach: In the z Direction, the Gravitational Wave is a Transverse Wave and is Not a Compression Wave

$$c_{pressure} = \sqrt{\frac{\lambda + 2\mu}{\rho}} = 0 \quad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

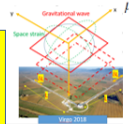
Third Approach: Based on Available Datas $\rho = \frac{\mu}{c^2} \rightarrow \nu = \frac{E}{2c^2\rho} - 1$

Consequence 1 $\mu = \frac{E}{2(1+\nu)}$

Transverse shear wave implies $\nu = 1$ (outside the range -1, 0,5) so we have to consider an anisotropic medium

Consequence 2

Imply also a behaviour of space as a some of plans deformed independently during the passage of the gravitational wave



Continuous medium?

3.2) Study of several mechanical S. Timoshenko models of the space time that can reproduce the order of magnitude of the strains forecast and measured in general relativity in weak field

Weak field general relativity is continuous and deterministic. It can be modeled by an elastic analogy, a Hooke's law via the theory of elasticity in a continuous medium and in weak fields with resistance models of type strength of materials.

Timochenko if:

- we place ourselves far from the point of application of the efforts, that is to say (far from the point of coalescence of black holes, far in space and time from the big bang).
- We reduce the dynamics of space-time to a sum of equivalent static cases “screen shot” which follow one another following the arrow of time.

Link between the metric tensor perturbation $h_{\mu\nu}$ and gravitation experiments

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$$

$$\text{Curvature} = \frac{\text{Angle}}{\text{Surface}} = \frac{8\pi G}{c^4} \times \frac{\text{Energy}}{\text{Volume}}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

h_{00}

Newton Gravitation

$$\Delta\phi = 4\pi G\rho$$

Einstein in weak Field

$$\Delta h_{00} = \frac{2}{c^2}\Delta\phi = \frac{8\pi G}{c^4}\rho c^2$$

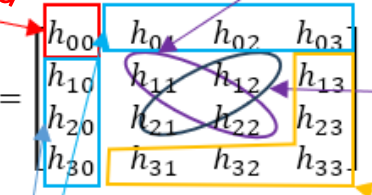
$h_{0i}; h_{j0}$

Lense Thirring frame dragging and geodetic effect

$$\frac{\partial^2 x}{\partial t^2} = \frac{GM}{r^2} \frac{\omega l^2}{r} \left[\frac{4x^2 + y^2 - 2z^2}{5r^2} \frac{\partial y}{\partial t} + \frac{12yz}{5r^2} \frac{\partial z}{\partial t} \right] - \frac{GM}{r^2} \frac{x}{r}$$

$$\frac{\partial^2 y}{\partial t^2} = -\frac{GM}{r^2} \frac{\omega l^2}{r} \left[\frac{4x^2 + y^2 - 2z^2}{5r^2} \frac{\partial x}{\partial t} + \frac{12xz}{5r^2} \frac{\partial z}{\partial t} \right] - \frac{GM}{r^2} \frac{y}{r}$$

$$\frac{\partial^2 z}{\partial t^2} = \frac{GM}{r^2} \frac{\omega l^2}{r} \left[\frac{12zx}{5r} \frac{\partial y}{\partial t} - y \frac{\partial x}{\partial t} \right]$$

$$T_{\mu\nu} = \rho_0 \left(\frac{dt}{ds} \right)^2 \begin{bmatrix} 1 & ir' \cos\theta \sin\phi & -ir' \cos\theta c \\ ir' \cos\theta \sin\phi & 0 & 0 \\ -ir' \cos\theta \sin\phi & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$


Lengthening and shortening

Angle

Angle

Gravitational wave polarisation A^+

$$\partial^\lambda \partial_\lambda \bar{h}_{\mu\nu} = \square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} = 0$$

$h_{xx}; h_{yy}$

Gravitational wave polarisation A^\times

$h_{xy}; h_{yx}$

Gravitational wave with possible new polarisations (Einstein-Cartan theory etc)

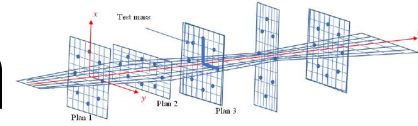
Hypothetic new polarisations with Einstein-Cartan theory (geometrical torsion)

$h_{zz}; h_{yz}$
 $h_{zy}; h_{zx}$
; h_{xz}

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} + \begin{bmatrix} \frac{2kM}{r} & i\frac{4kM}{5r^2} \frac{ly}{c} & -\frac{4kM}{5r^2} \frac{lx}{c} & 0 \\ i\frac{4kM}{5r^2} \frac{ly}{c} & -\frac{2kM}{r} & 0 & 0 \\ -\frac{4kM}{5r^2} \frac{lx}{c} & 0 & -\frac{2kM}{r} & 0 \\ 0 & 0 & 0 & -\frac{2kM}{r} \end{bmatrix}$$

$$= \begin{bmatrix} -1 + \frac{2kM}{r} & i\frac{4kM}{5r^2} \frac{ly}{c} & -\frac{4kM}{5r^2} \frac{lx}{c} & 0 \\ \frac{4kM}{5r^2} \frac{ly}{c} & -1 - \frac{2kM}{r} & 0 & 0 \\ -\frac{4kM}{5r^2} \frac{lx}{c} & 0 & -1 - \frac{2kM}{r} & 0 \\ 0 & 0 & 0 & -1 - \frac{2kM}{r} \end{bmatrix}$$

Consequence about the models that can reproduce the strain of the space-time



Source NASA

Pramana - J. Phys. (2020) 94:119
 https://doi.org/10.1007/s12043-020-01954-5
 © Indian Academy of Sciences
 Check for updates
 Mechanical conversion of the gravitational Einstein's constant κ

Consequence 1: In the plane approach (GW)

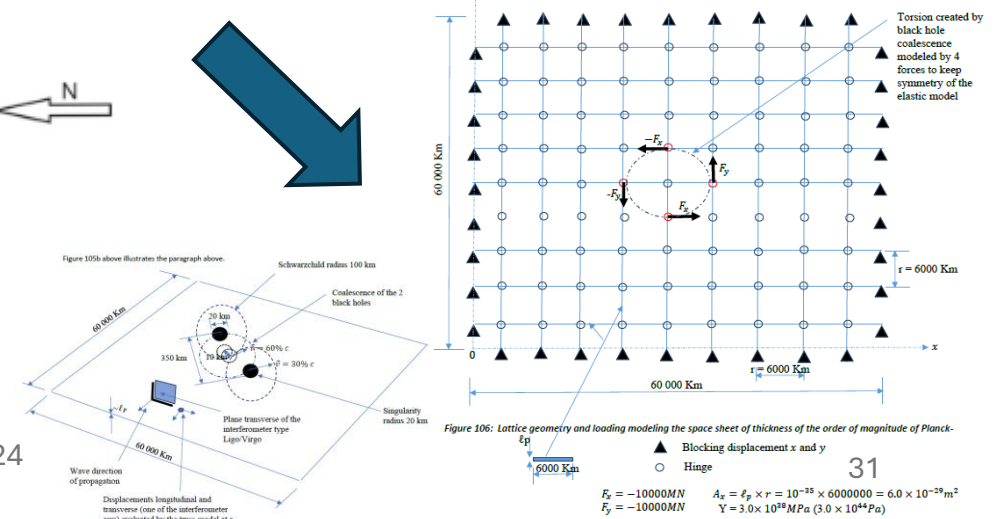
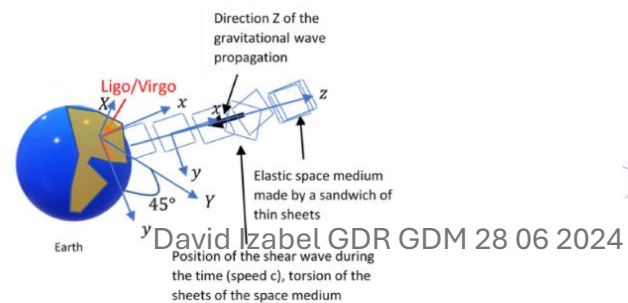
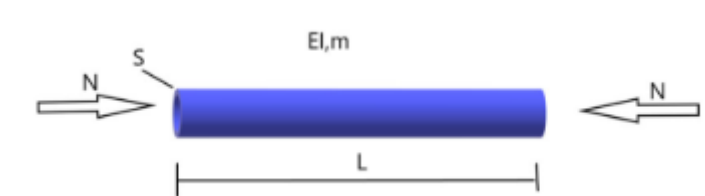
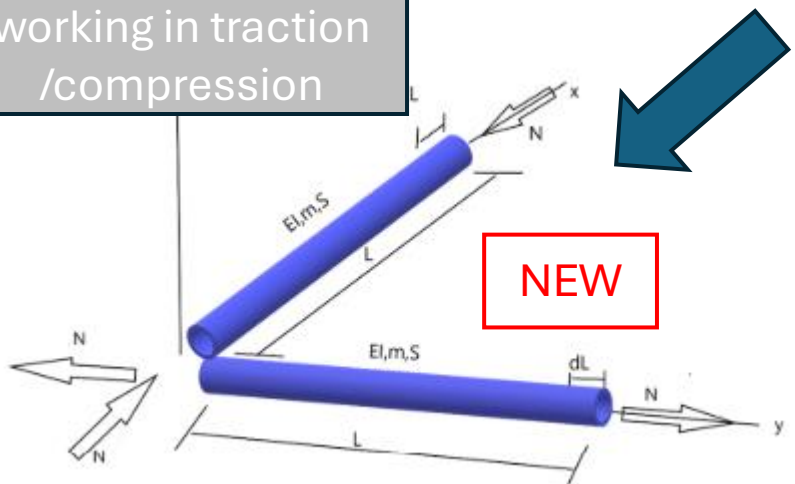
NEW

The Einstein's field equation in weak field is equivalent at structure in one or two dimensions in compression/traction:

Model of interferometer Ligo/Virgo as gauge of space by beam working in traction /compression

Normal effort $N(x)$	$\left(\frac{du}{dx}\right)^2 = \frac{2}{ES} \times \frac{U}{L} = \frac{2}{YS} \times \frac{U}{L}$	$1 = \frac{s^2}{kgm} \times \frac{U}{m}$
General relativity	$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$	$\frac{1}{m^2} = \frac{s^2}{kgm} \times \frac{U}{m^3}$

Model of space by a truss loaded in torsion by 2 black holes in rotation and coalescence : GW150914 and GW170817 strains



David Izabel GDR GDM 28 06 2024

Consequence about the models that can reproduce the strain of the space-time

Consequence 1: **In the plane** approach space put in torsion
(Lense-Thirring – frame dragging effect)

The Einstein's field equation in weak field is equivalent at structure in one or two dimensions in compression/traction:

NEW Timoshenko's model of truss of beam

Model of sheet of space by a truss put in torsion by the Earth rotation: Gravity probB experiment frame dragging effect

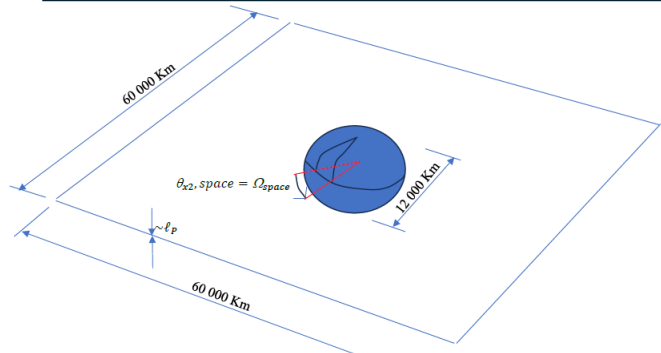
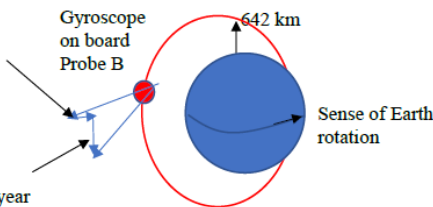


Figure 121: Elastic space sheet of Planck thickness subjected to torsion by the rotation of the Earth -

Space time frame dragging effect Ω
 -37.2 +/- 7.2 milli arcsecond /year (studied here)-



Geodetic effect (curvature)
 -6601.8 +/- 18.3 milli arcsecond /year

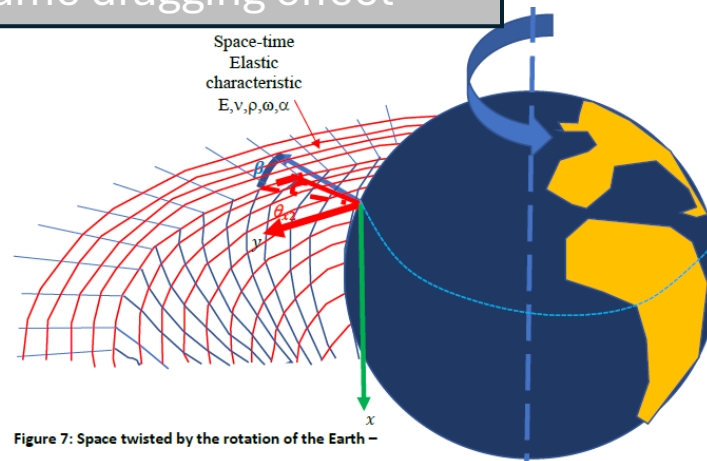
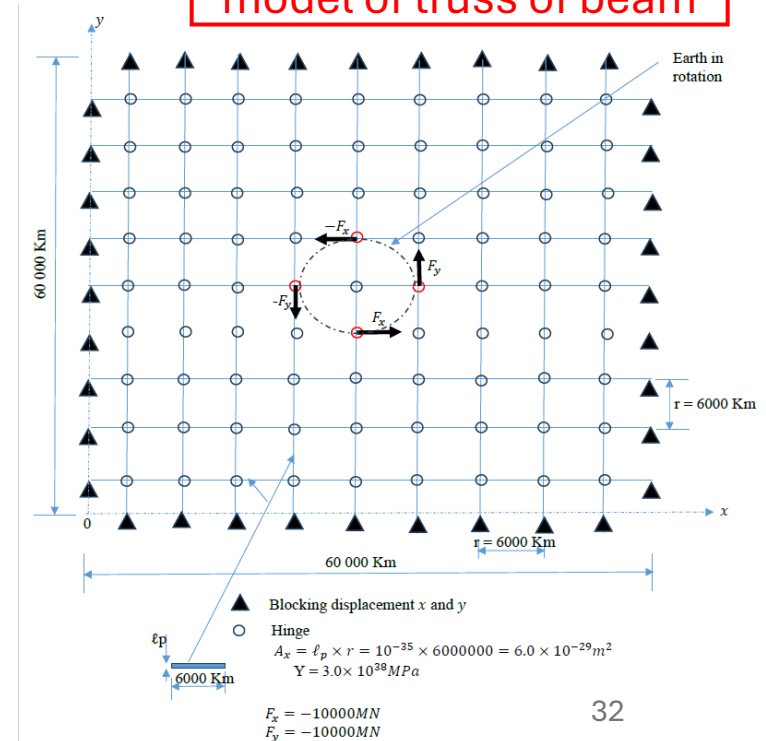


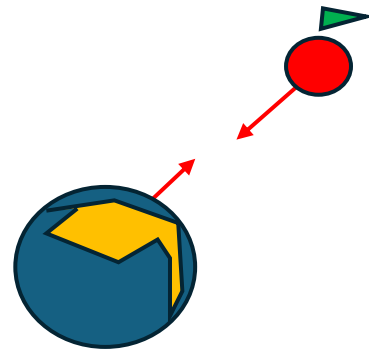
Figure 7: Space twisted by the rotation of the Earth -

Frame dragging: the Earth by its rotation drags space horizontally (**imposed strain**)

Dayid Izabel GDR GDM 28 06 2024



Consequence about the models that can reproduce the strain of the space-time



Consequence 2: **Perpendicular at the plan** approach (classical gravitation)

The Einstein's field equation in weak field is equivalent **at the Poisson equation that is equivalent in 2 dimension at membrane:**

Einstein's equation in weak field

$$\Delta h_{00} = \frac{2}{c^2} \Delta \phi = \frac{8\pi G}{c^4} \rho c^2$$

Poisson's equation/Newton in weak field

$$\frac{c^2}{2} \Delta h_{00} = \Delta \phi = 4\pi G \rho$$

Membrane equation in weak field (Timoshenko)

$$\frac{1}{L} \Delta w = \frac{1}{L} \left[\frac{\partial^2 w(x, y)}{\partial x^2} + \frac{\partial^2 w(x, y)}{\partial y^2} \right] = \frac{1}{ES} \times \frac{(gM) \times L}{\Delta L \times L^2}$$

Curvature $(1/R)^2 = \text{flexibility } (1/ES=1/N) \times \text{energy density } N.m/m^3$

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IOP Conf. Series: Journal of Physics: Conf. Series 845 (2017) 012003 doi:10.1088/1742-6596/845/1/012003

Introducing surface tension to spacetime

H A Perko¹
¹Koppa Research, Office 11, 140 E. 4th Street, Loveland, CO, USA 80537

IARD 2018 IOP Publishing
IOP Conf. Series: Journal of Physics: Conf. Series 1239 (2019) 012010 doi:10.1088/1742-6596/1239/1/012010

Gravitation in the surface tension model of spacetime

H A Perko¹
¹Office 14, 140 E. 4th Street, Loveland, CO, USA 80537

IARD 2020 IOP Publishing
Journal of Physics: Conference Series 1956 (2021) 012004 doi:10.1088/1742-6596/1956/1/012004

Dark matter and dark energy: cosmology of spacetime with surface tension

David Izabel GDR GDM 28 06 2024
H A Perko
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Consequence about the models that can reproduce the strain of the space-time

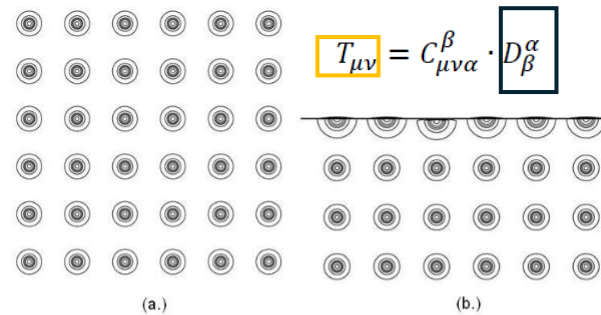
$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$$

$$\text{Curvature} = \frac{\text{Angle}}{\text{Surface}} = \frac{8\pi G}{c^4} \times \frac{\text{Energy}}{\text{Volume}}$$

Consequence 2: **Perpendicular at the plan** approach

$$D_{\beta}^{\alpha} = \begin{bmatrix} \frac{\partial u^0}{\partial x^0} & \frac{1}{2} \left(\frac{\partial u^0}{\partial x^1} + \frac{\partial u^1}{\partial x^0} \right) & \frac{1}{2} \left(\frac{\partial u^0}{\partial x^2} + \frac{\partial u^2}{\partial x^0} \right) & \frac{1}{2} \left(\frac{\partial u^0}{\partial x^3} + \frac{\partial u^3}{\partial x^0} \right) \\ \frac{1}{2} \left(\frac{\partial u^1}{\partial x^0} + \frac{\partial u^0}{\partial x^1} \right) & \frac{\partial u^1}{\partial x^1} - \frac{1}{2} \left| \frac{\partial u^0}{\partial x^1} \right|^2 d\tau' & \frac{1}{2} \left(\frac{\partial u^1}{\partial x^2} + \frac{\partial u^2}{\partial x^1} \right) - \frac{1}{2} \left| \frac{\partial u^0}{\partial x^1} \right| \left(\frac{\partial u^0}{\partial x^2} \right) d\tau' & \frac{1}{2} \left(\frac{\partial u^1}{\partial x^3} + \frac{\partial u^3}{\partial x^1} \right) - \frac{1}{2} \left| \frac{\partial u^0}{\partial x^1} \right| \left(\frac{\partial u^0}{\partial x^3} \right) d\tau' \\ \frac{1}{2} \left(\frac{\partial u^2}{\partial x^0} + \frac{\partial u^0}{\partial x^2} \right) & \frac{1}{2} \left(\frac{\partial u^2}{\partial x^1} + \frac{\partial u^1}{\partial x^2} \right) - \frac{1}{2} \left| \frac{\partial u^0}{\partial x^2} \right| \left(\frac{\partial u^0}{\partial x^1} \right) d\tau' & \frac{\partial u^2}{\partial x^2} - \frac{1}{2} \left| \frac{\partial u^0}{\partial x^2} \right|^2 d\tau' & \frac{1}{2} \left(\frac{\partial u^2}{\partial x^3} + \frac{\partial u^3}{\partial x^2} \right) - \frac{1}{2} \left| \frac{\partial u^0}{\partial x^2} \right| \left(\frac{\partial u^0}{\partial x^3} \right) d\tau' \\ \frac{1}{2} \left(\frac{\partial u^3}{\partial x^0} + \frac{\partial u^0}{\partial x^3} \right) & \frac{1}{2} \left(\frac{\partial u^3}{\partial x^1} + \frac{\partial u^1}{\partial x^3} \right) - \frac{1}{2} \left| \frac{\partial u^0}{\partial x^3} \right| \left(\frac{\partial u^0}{\partial x^1} \right) d\tau' & \frac{1}{2} \left(\frac{\partial u^3}{\partial x^2} + \frac{\partial u^2}{\partial x^3} \right) - \frac{1}{2} \left| \frac{\partial u^0}{\partial x^3} \right| \left(\frac{\partial u^0}{\partial x^2} \right) d\tau' & \frac{\partial u^3}{\partial x^3} - \frac{1}{2} \left| \frac{\partial u^0}{\partial x^3} \right|^2 d\tau' \end{bmatrix}$$

3D Space	4D Spacetime
Surface Energy = $\Phi = \frac{\text{Energy}}{\text{Area}}$	Surface Energy = $\Phi = \frac{\text{Energy}}{\text{Volume}}$
Surface Tension = $\frac{\text{Force}}{\text{Width}} = \frac{\text{Energy} / L}{\text{Area} / L} = \Phi$	Surface Tension = $\frac{\text{Force}}{\text{Area}} = \frac{\text{Energy} / L}{\text{Volume} / L} = \Phi$



$$T_{\mu\nu} = C_{\mu\nu\alpha}^{\beta} \cdot D_{\beta}^{\alpha}$$

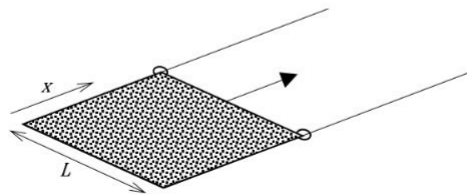
Metric

$$g_{\beta}^{\alpha} = I - 2D_{\beta}^{\alpha} d\tau'$$

Link with membrane

$$D_{\beta}^{\alpha} = \frac{1}{2} \left(\frac{\partial u_{\alpha}}{\partial x_{\beta}} + \frac{\partial u_{\beta}}{\partial x_{\alpha}} \right) - \frac{1}{2} \left| \left(\frac{\partial u_0}{\partial x_{\alpha}} \right) \left(\frac{\partial u_0}{\partial x_{\beta}} \right) \right| d\tau'$$

Figure 2. Probability Density of Interacting Particles (a. in continuum, b. with boundary)



$$D_{\beta}^{\alpha} g_{\mu\nu} = \frac{1}{2} g_{\mu\nu} g^{\mu\nu} \mathcal{L}_v g - \frac{1}{2} R g_{\mu\nu}$$

PERKO approach

dimensions [14]. Since α and β are dummy indices, one can change indices such that $D_{\beta}^{\alpha} g_{\mu\nu} = D_{\mu}^{\alpha} g_{\alpha\nu} \neq D_{\mu\nu}$. Hence,

$$D_{\mu}^{\alpha} g_{\alpha\nu} \neq D_{\mu\nu}$$

$$D_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

Link with GR

The rate of deformation tensor with two covariant components is the Einstein Tensor given in [14].

Returning attention to the constitutive relationship (5), it can be seen that,

$$T_{\mu\nu} = C_{\mu\nu\alpha}^{\beta} \cdot D_{\beta}^{\alpha} = C_{\mu\nu\alpha}^{\beta} \cdot \frac{D_{\mu\nu}}{g_{\mu\nu}}$$

Contracting the elasticity tensor, moving it to the stress energy side, and inserting (13) yields an equation closely analogous to general relativity,

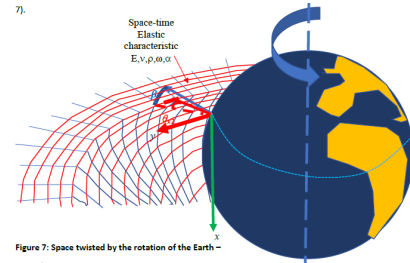
$$T_{\mu\nu} \cdot C_{\beta}^{\alpha} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \quad (14)$$

except the Einstein constant from [15] is replaced by a symmetric nondegenerate anisotropic elasticity tensor,

$$C_{\beta}^{\alpha} = g_{\mu\nu} C_{\beta}^{\mu\nu\alpha} = \frac{2}{ch} \begin{bmatrix} 4\pi l_p^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

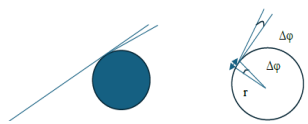
Figure 3. Hypersurface Stretched Across a Wire Frame Window with One Moveable Side

Consequence about the models that can reproduce the strain of the space-time



Consequence 2: **Perpendicular at the plan approach (Geodetic effect)**

The Einstein's field equation in weak field is equivalent **at the Poisson equation that is equivalent in 2 dimension at membrane:**



Model of membrane: gravitation in weak field around Earth and Sun, deviation of beam light

Model of membrane around the earth (Gravity prob B experiment **geodetic effect**)

$$\Delta w = \frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2} = \frac{g\mu}{T} = \frac{1}{R}$$

$w(x,y)$ The Vertical displacement of the membrane in m.

$T = N/L$ The force by meter along the membrane in Newton/m

μ The mass by square meter of the membrane in kg/m^2

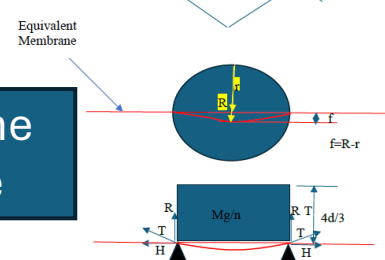
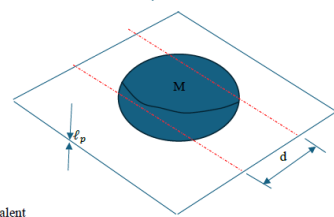
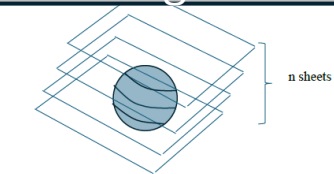
$g = \frac{GM}{r^2}$ The acceleration g : in m/s^2

$$\Delta w = \frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2} = \frac{g}{ES} \frac{M}{L^2} = \frac{1}{R}$$

$$\frac{1}{L} \Delta w = \frac{1}{L} \left[\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2} \right] = \frac{1}{ES} \times \frac{(gM) \times L}{\Delta L \times L^2}$$

Curvature $(1/R)^2 = \text{flexibility } (1/ES=1/N) \times \text{energy density } N.m/m^3$

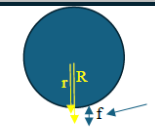
NEW



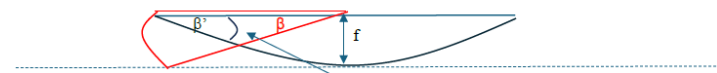
Tension of the membrane

Curvature of the membrane

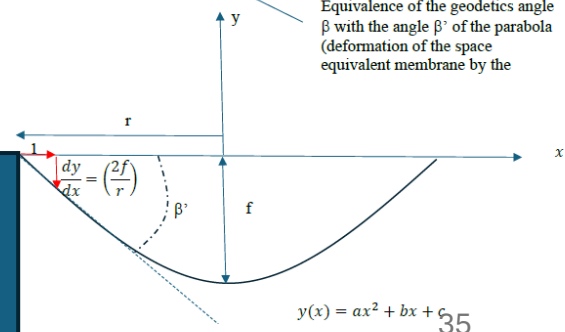
NEW



Curvature effect on space time by the Earth imply increase of Earth radius of $R-r=f=1.47mm$



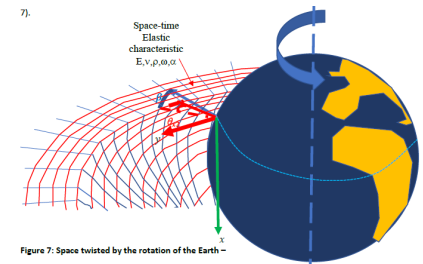
Equivalence of the geodetics angle β with the angle β' of the parabola (deformation of the space equivalent membrane by the



Consequence about the models that can reproduce the strain of the space-time

Consequence 3: In plane + spatial (Frame dragging)

The Einstein's field equation in weak field is equivalent **in 3D at equivalent at cylinder in torsion:**



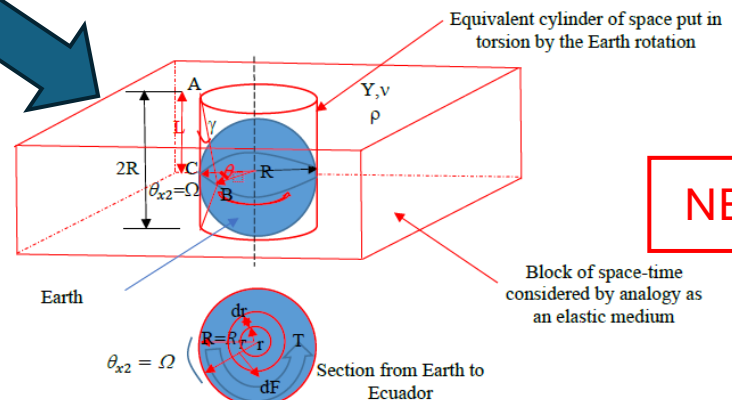
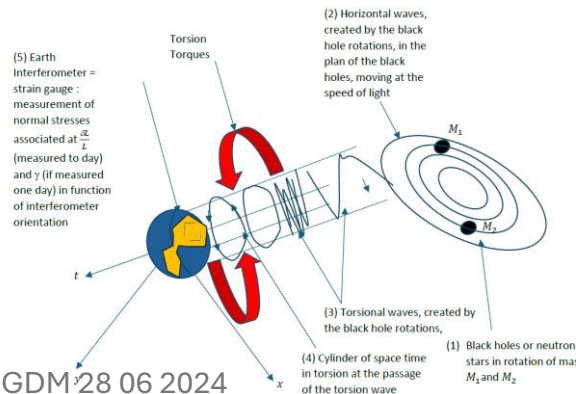
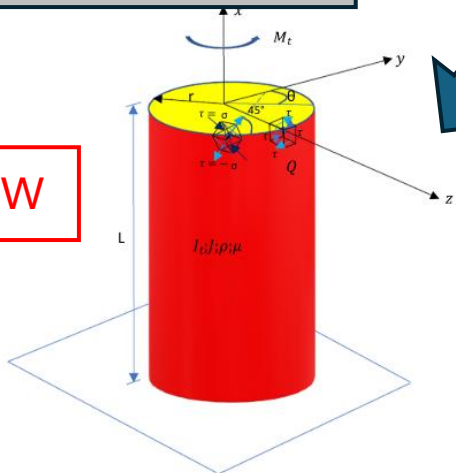
Gravitational waves - torsion space by black holes rotation

Twisting torque $T_{(x)}$	$\frac{1}{R_t^2} = \frac{2}{GI_t} \times \frac{U}{L} = \frac{2}{\mu I_t} \times \frac{U}{L}$	$\frac{1}{m^2} = \frac{s^2}{kgm^3} \times \frac{U}{m}$
General relativity	$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4}T_{\mu\nu}$	$\frac{1}{m^2} = \frac{s^2}{kgm} \times \frac{U}{m^3}$

Gravity prob B Experiment
Frame dragging

Pramana - J. Phys. (2020) 94:119
<https://doi.org/10.1007/s12043-020-01954-5>
© Indian Academy of Sciences
Check for updates
Mechanical conversion of the gravitational Einstein's constant κ

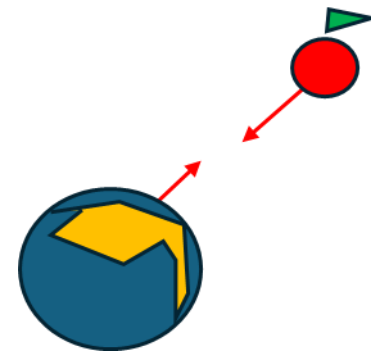
NEW



NEW

Rotation imposed by the Earth at the cylinder of space

Consequence about the models that can reproduce the strain of the space-time



Consequence 4: **spatial shell** (classical gravitation)

The Einstein's field equation in weak field is equivalent **in 3D at equivalent at sphere with internal pression :**

$$\oint_{\sigma} \varrho d\sigma = \oint_V (\nabla \cdot \varrho) dV \quad (5)$$

which means by direct comparison of (4) and (5),

$$\nabla \cdot \varrho = -dP \quad (6)$$

Differential temporal pressure (mass energy) is the spatial divergence of surface tension.

This line of logic is somewhat analogous to the treatment of corpuscular, capillary, and meniscus geometry in physical chemistry of surfaces. An example of corpuscular geometry is shown in Figure 5. For two-dimensional curved surfaces, surface tension acts against differential surface pressure, dP .

$$\left. \begin{aligned} \varrho 2\pi R &= dP \pi R^2 \\ \frac{2\varrho}{R} &= dP \end{aligned} \right\} \text{2-D Surface in 3-Space}$$

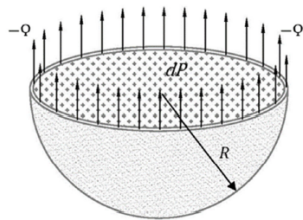


Figure 5. Corpuscular Analog of the Divergence Theorem in Physical Chemistry of Surfaces

From this analogy, one can intuitively derive a similar relationship for spatial three-surfaces intrinsic in time,

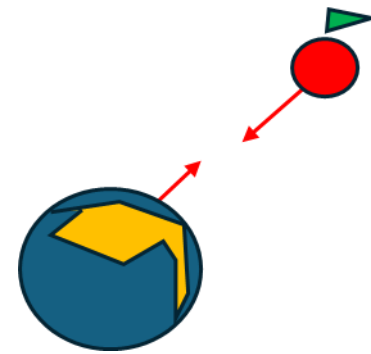
$$\left. \begin{aligned} \varrho 4\pi R^2 &= dP \frac{4}{3}\pi R^3 \\ \frac{3\varrho}{R} &= dP \end{aligned} \right\} \text{3-D Surface in 4-Space} \quad (7)$$

Dark Matter and Dark Energy: Cosmology of Spacetime with Surface Tension

H A Perko¹

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Consequence about the models that can reproduce the strain of the space-time



Consequence 4: spatial (classical gravitation)

The Einstein's field equation in weak field is equivalent **in 3D at equivalent at sphere with internal pression:**

Deviation of the sun beam light
Eddington Experiment

The data about the sphere with an internal pression is given at the figure 129.

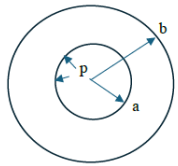


Figure 129: Notation for a sphere loading by an internal pression

In elasticity, we have the differential equation [308]:

$$\frac{d^2 u_r}{dr^2} + \frac{2}{r} \frac{du_r}{dr} = \frac{2}{r^2} u_r$$

So, the beginning of the equation follow the form of:

$$\Delta \phi = \frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \times \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \times \frac{\partial^2 \phi}{\partial \theta^2} + \frac{1}{r^2 \tan \theta} \times \frac{\partial \phi}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \times \frac{\partial^2 \phi}{\partial \varphi^2}$$

$$\Delta u_r = \frac{2}{r^2} u_r$$

So, it's a development of the Poisson's equation that is this modified by a distribution f that is not constant. The solution is on the form:

$$u_r = C_1 r + \frac{C_2}{r^2}$$

With for the 2 constants:

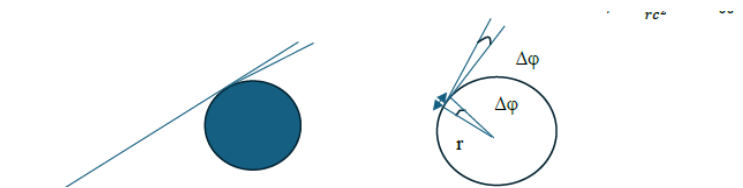
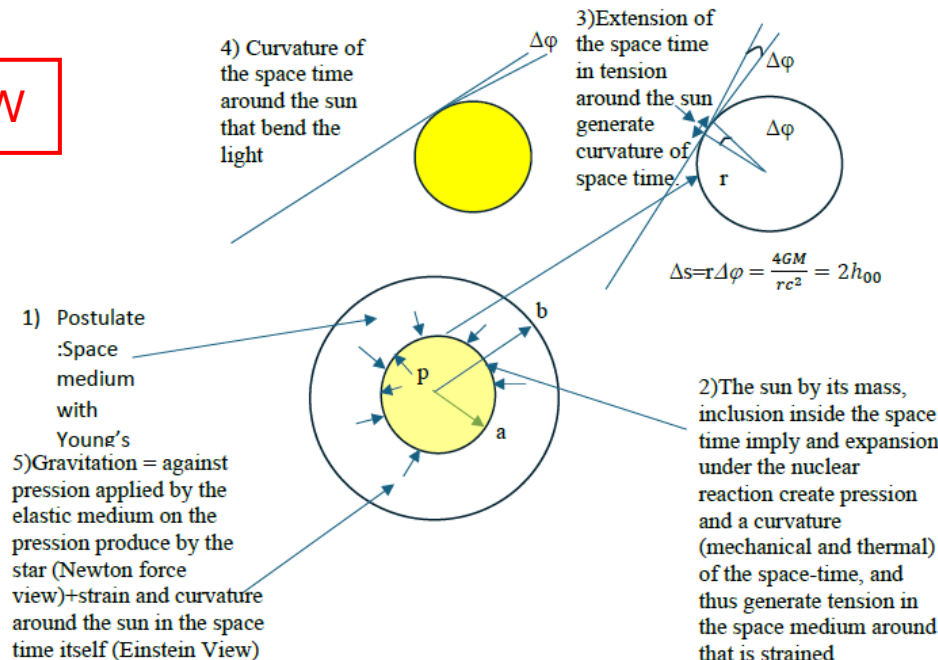
$$C_2 = \frac{1 + \nu}{2(1 + 2\nu)} b^3 C_1$$

$$C_1 = \frac{(1 - 2\nu)}{E} \frac{a^3}{b^3 - a^3} p$$

We know the displacement u_r , so we can extract the Young's modulus $E = Y$:

$$E = Y = \frac{a^3 p}{u_r (b^3 - a^3)} \left[(1 - 2\nu)r + (1 + \nu) \frac{b^3}{2r^2} \right]$$

NEW



The Sun tries to explode because of the nuclear reaction but it can't because space-time is in tension around it, trying to compress it uniformly. As space is in tension it extends like a membrane locally or a sphere globally

Consequence about the models that can reproduce the strain of the space-time

Consequence 5: **spatial in 4D** (classical gravitation)

The Einstein's field equation in weak field is equivalent **in 4D** at **Hypersurface membrane**:

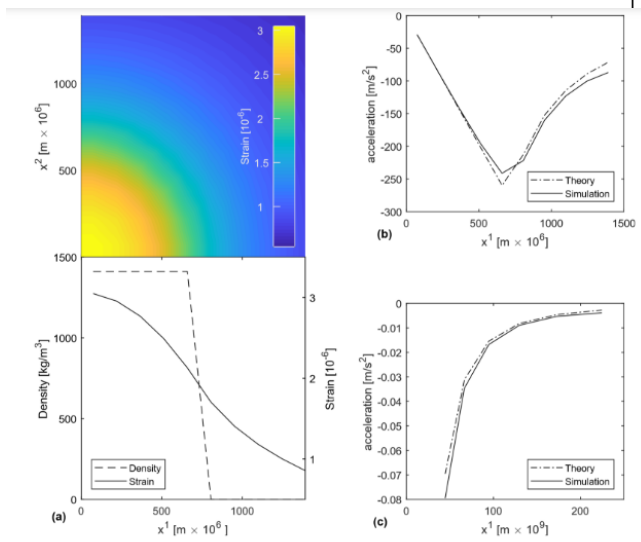
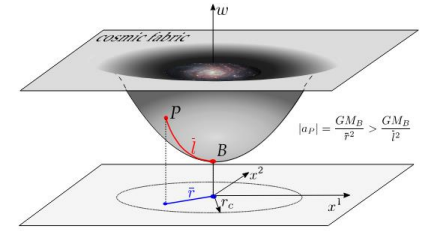


Figure 7.3
Fabric strain and acceleration due to the Sun

Panel (a) shows the variation of density ρ and the fabric's volumetric strain in the vicinity of the Sun. Panels (b) and (c) compare the theoretical value for the gravitational acceleration due to the Sun with the simulated value that was output from simulating the Sun's gravity. The comparison is shown within two radial ranges: in the vicinity of the Sun (b), and in the vicinity of the Earth (c). The mesh used in this experiment had $58 \times 58 \times 58 = 195,112$ nodes.

Bending moment of $M(x)$	$\frac{1}{R^2} = \frac{2}{EI} \times \frac{U}{L} = \frac{2}{YI} \times \frac{U}{L}$	$\frac{1}{m^2} = \frac{s^2}{kgm^3} \times \frac{U}{m}$
General relativity	$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$	$\frac{1}{m^2} = \frac{s^2}{kgm} \times \frac{U}{m^3}$

Hyper surface of Planck Thickness

Mississippi State University
Scholars Junction
 Theses and Dissertations
 12-14-2018
An Elastic Constitutive Model of Spacetime and its Applications
 Tichomir G. Tenev

Model of sun gravity

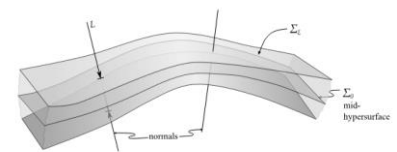


Figure 3.3
Hypersurfaces

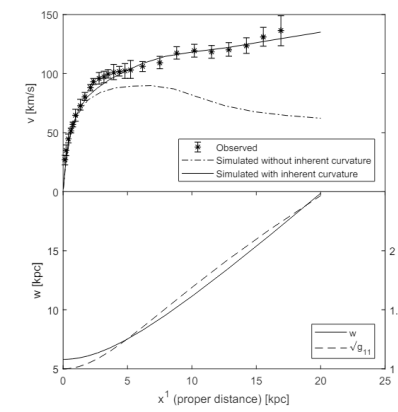


Figure 7.5
Rotation curve of the M33 galaxy

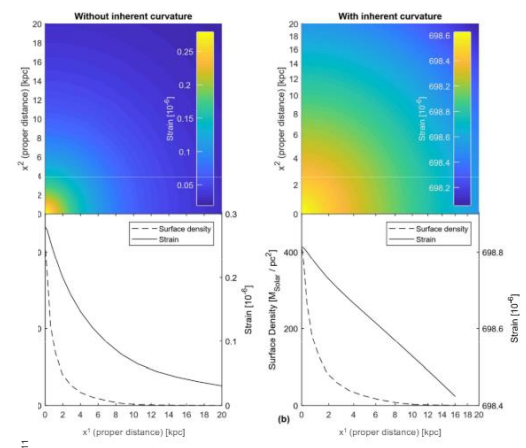


Figure 7.4
Fabric strain due to the M33 galaxy.

Figure 7.4: Fabric strain due to the M33 galaxy without (a) and with (b) inherent curvature. The top panels visualize a quadrant of the strain field, while the bottom panels show the density and strain profiles. The density profile used here is from figure 10 of Corbelli et al. [37].

Model of galaxy M33

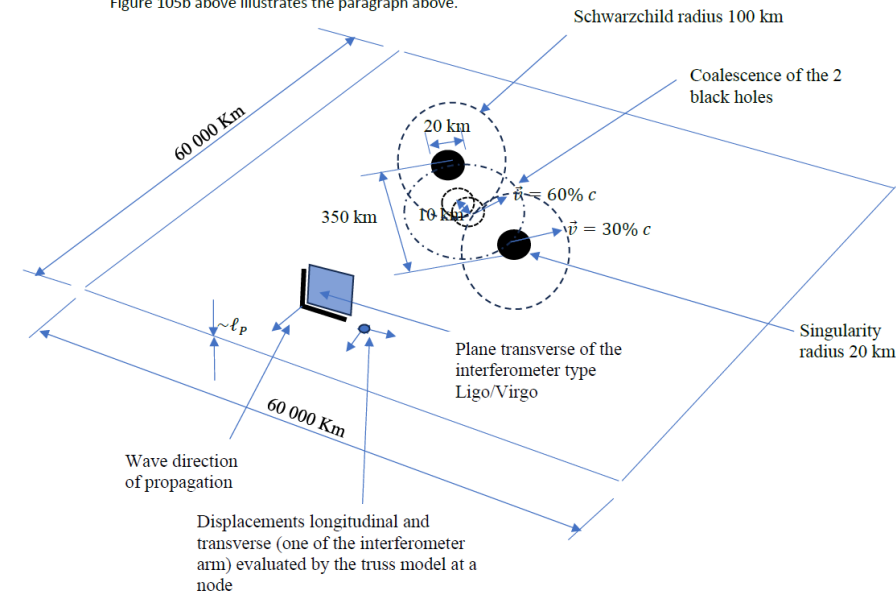
4) Numerical applications of the different models

4.1 **Models in plane** with spatial component of strains (h special associated at h_{ij} for Gravitational wave GW or space part of gravity prob B experiment h_{0i}, h_{j0})

Case1 : GW150914 - Coalescence of 2 black holes

$$h_{ij}$$

Figure 105b above illustrates the paragraph above.



Principle: we impose a Torsion torque equivalent at the black holes coalescence in the middle of the sheet modelled by a truss and look for the Young modulus intensity that allow to refind the strains of the space time fabric measured by the interferometer Ligo/Virgo

Data

Data	Value	Unit
Mass black hole1	36	Solar mass
Mass black hole 2	29	Solar mass
speed before coalescence	0,3c	
Speed at coalescence	0,6c	
Duration of coalescence	0,2s	
Thikness sheet (Planck)	1,00E-35	m
Young modulus Y	1,00E+44	Pa
Diameter Black hole	20	km
Distance between force	10	km
Mesch of the truss	6000	km
Area of the bar	6,00E-29	m ²
Solar Mass	1,99E+30	kg
Speed c	299792458	m/s

Case 1 : GW150914 - Finite element model of a truss (bars working in compression/traction)

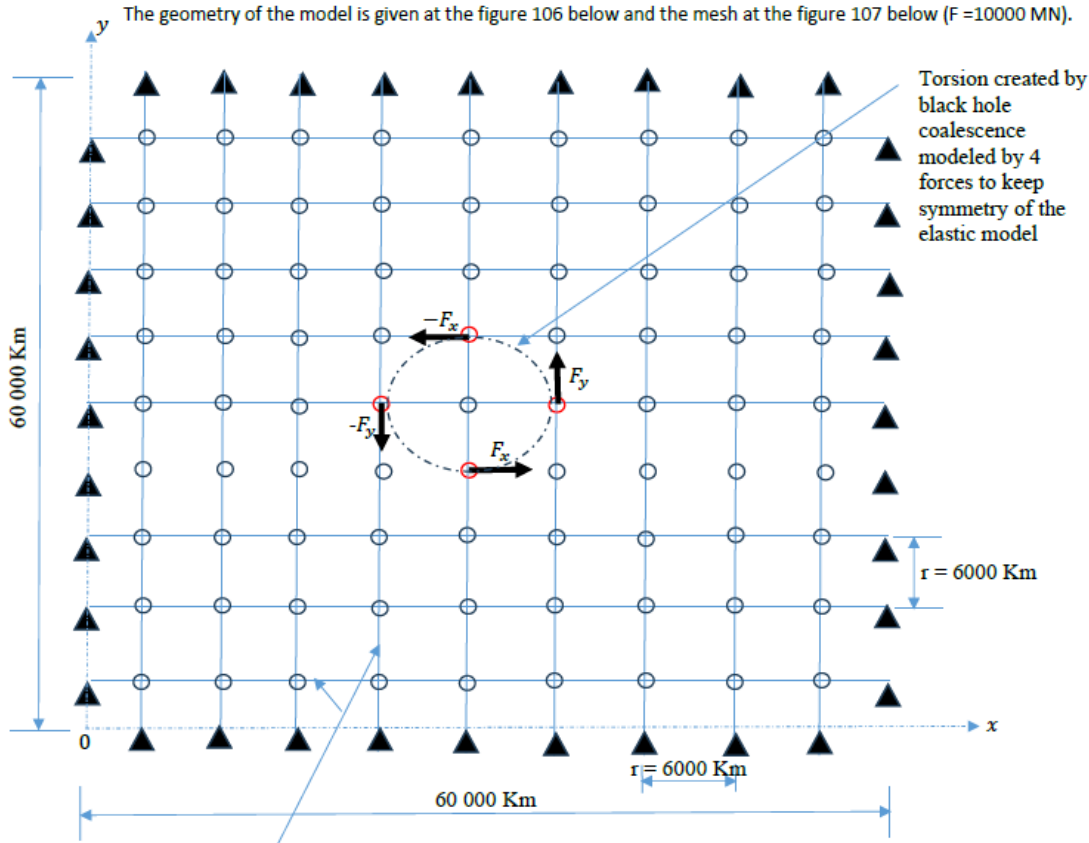


Figure 106: Lattice geometry and loading modeling the space sheet of thickness of the order of magnitude of Planck-

ℓ_p

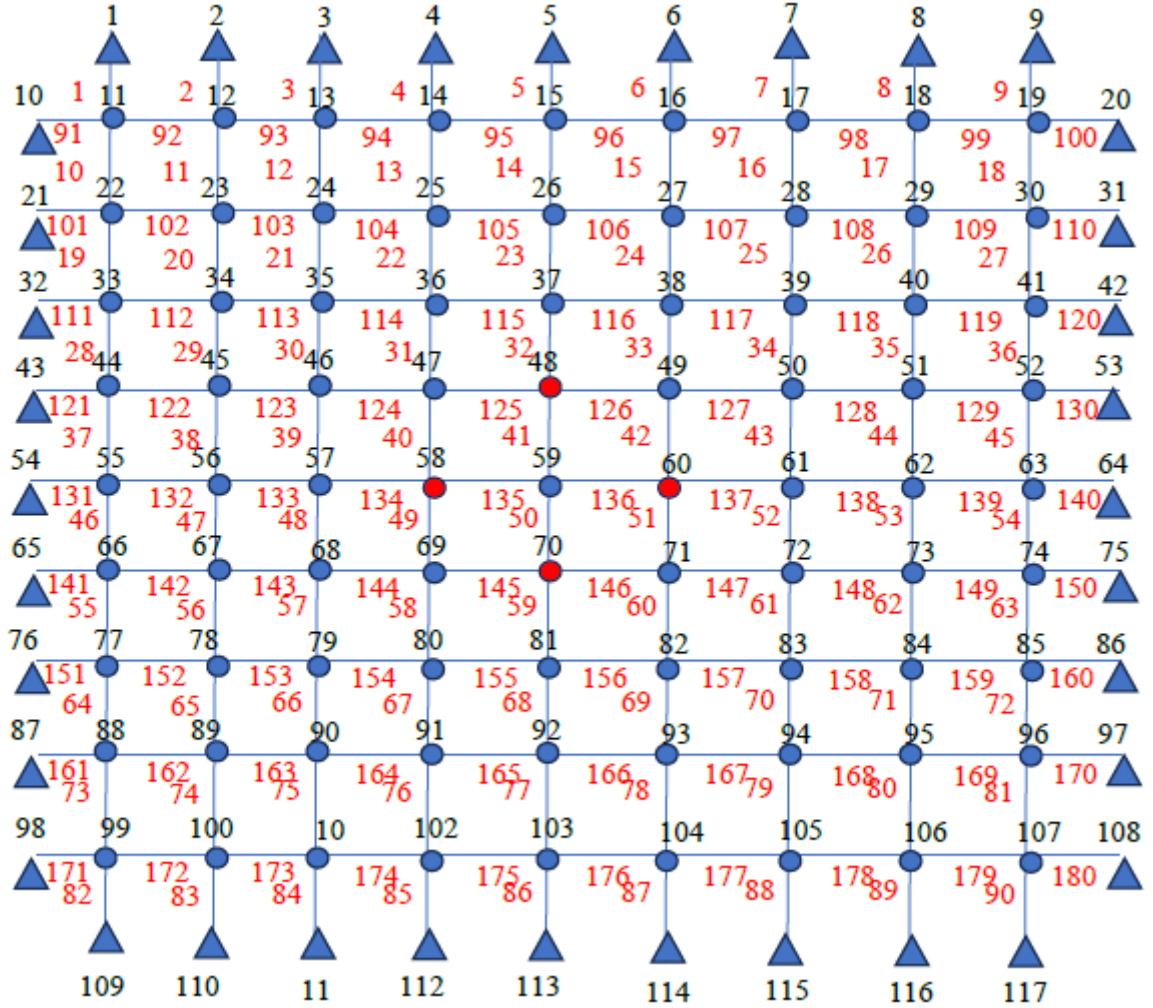
6000 Km

▲ Blocking displacement x and y

○ Hinge

$F_x = -10000 \text{ MN}$ $A_x = \ell_p \times r = 10^{-35} \times 6000000 = 6.0 \times 10^{-29} \text{ m}^2$

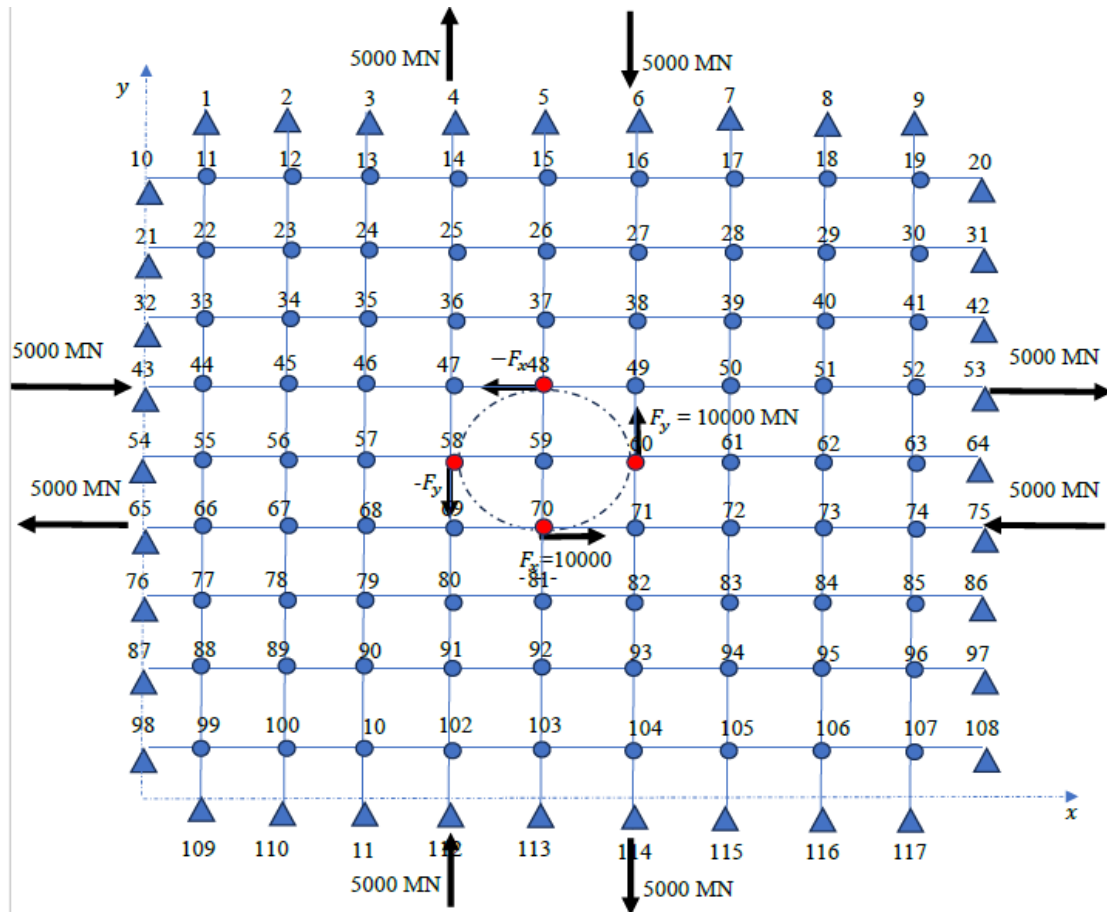
$F_y = -10000 \text{ MN}$ $Y = 3.0 \times 10^{38} \text{ MPa} (3.0 \times 10^{44} \text{ Pa})$



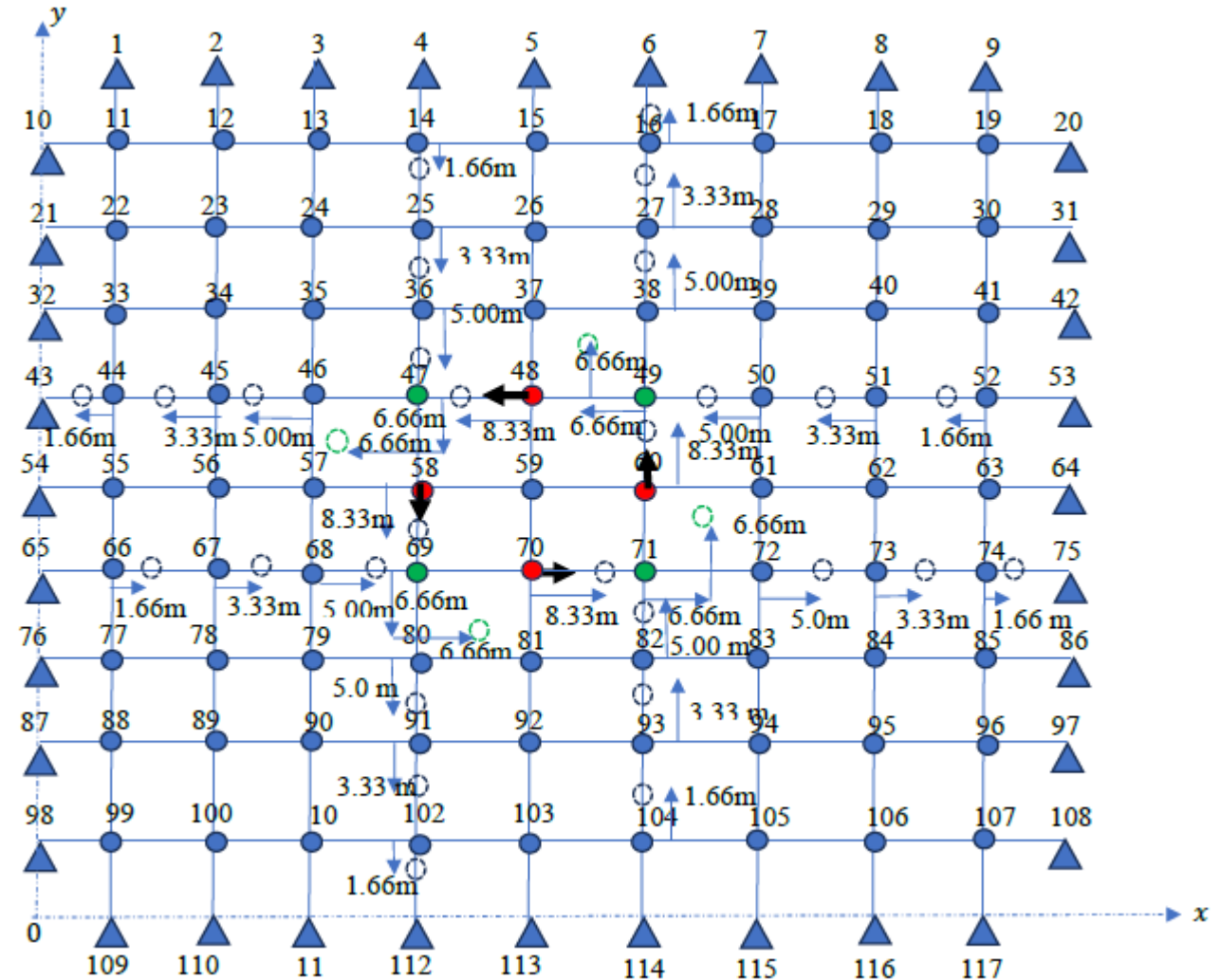
Mesh and loading

Bar number and nodes

Case 1: GW150914 - Results obtained with arbitrary loading of 4 loads of 10 000 MN



Reaction on the supports



In displacements

Case 1: GW150914 - Numerical application and comparison with the strain measurements

1) Calculus of the acceleration of the 2 masses	
$a = \frac{v_{coalescence} - v_{350km}}{\Delta t} =$	449688687 m/s ²
2) Calculus black hole mass	
$m = (M_1 + M_2)/2 \times \text{solar mass}$	= 6,46E+31 kg
3) Calculus of the Force created by each black hole	
$F_{black\ hole} = m \times a$	2,91E+40 N
4) Calculus of the torsion Torque applied	
$T = 2F_{Black\ hole} \times \frac{L_1}{2}$	2,91E+44 N.m
5) Calculus of the number of planck sheets concerned	
$n = \frac{d_{black-hole}}{l_p}$	2,00E+39 sheets

6) Calculus of the Torsion torque by sheet	
$T_{Planck-sheet} = \frac{T}{n}$	1,45E+05 N.m/sheet
7) Calculus of the corresponding forces applied at the model	
$F_{Planck-sheet} = \frac{T_{Planck-sheet}}{4r}$	6,05E-03 N/sheet
8) Calculus of the longitudinal strain with the fictive force applied	
$h_{F=10000MN} = \frac{\Delta L}{r}$	2,77E-07
9) Calculus of the real longitudinal strain with the coalescence loading	
$h_{F_{Planck\ sheet}} = h_{F(for\ F=10000MN)} \times \frac{F_{Planck-sheet}}{F}$	1,67E-19
10) Calculus of the real transversal strain with the coalescence Loading	
$h_{F_{Planck\ sheet\ node\ 47}} = h_{F(for\ F=10000MN)} \times \frac{F_{Planck-sheet}}{F}$	6,72E-19

Case 1: GW150914 - Comparison measured value and numerical model for

GW150914

Data	Value	Unit
Mass black hole 1	36	Solar mass
Mass black hole 2	29	Solar mass
speed before coalescence	0,3	c
Speed at coalescence	0,6	c
Duration of coalescence	0,2	s
Thikness sheet (Planck)	1,00E-35	m
Young modulus Y	1,00E+44	Pa
Diameter Black hole	20	km
L1 Distance between the 2 forces	10	km
Mesch of the truss	6000	km
Area of the bar	6,00E-29	m ²
Solar Mass	1,99E+30	kg
Fictive force applied on the model	1,00E+04	MN
Displacement longitudinal of the model	1,66E+00	m
Displacement transversal of the model	6,66E+00	m
Speed c	299792458	m/s

9) Calculus of the real longitudinal strain with the coalescence loading

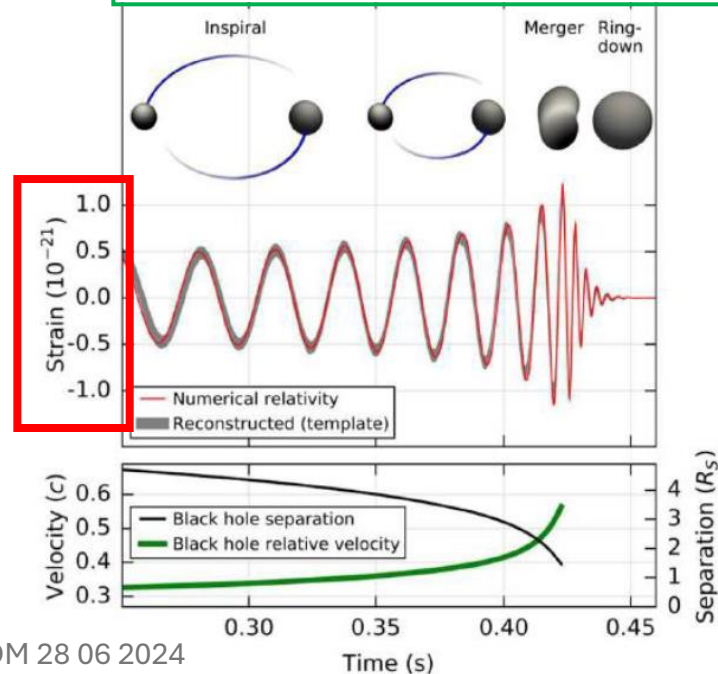
$$h_{F_{Planck\ sheet}} = h_{F(for\ F=10000MN)} \times \frac{F_{Planck-sheet}}{F} \quad 1,67E-19$$

10) Calculus of the real transversal strain with the coalescence Loading

$$h_{F_{Planck\ sheet\ node\ 47}} = h_{F(for\ F=10000MN)} \times \frac{F_{Planck-sheet}}{F} \quad 6,72E-19$$

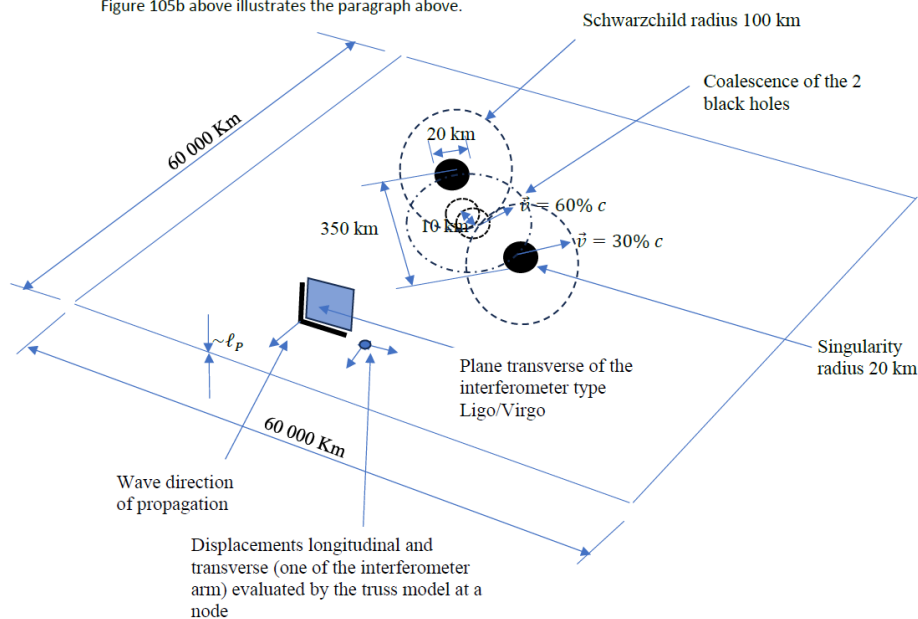
GW150914: Implications for the Stochastic Gravitational-Wave Background from Binary Black Holes

B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration)
Phys. Rev. Lett. **116**, 131102 – Published 31 March 2016



Order of magnitude ok with experiment if $Y = 1 \times 10^{44}$ Pa and Planck thickness

Figure 105b above illustrates the paragraph above.



Case 1: GW 150914 - Comparison measured value and numerical model for GW150914

Data	Value	Unit
Mass black hole 1	36	Solar mass
Mass black hole 2	29	Solar mass
speed before coalescence	0,3	c
Speed at coalescence	0,6	c
Duration of coalescence	0,2	s
Thickness sheet (Planck)	1,00E-35	m
Young modulus Y	1,00E+44	Pa
Diameter Black hole	100	km
L1 Distance between the 2 forces	10	km
Mesch of the truss	6000	km
Area of the bar	6,00E-29	m ²
Solar Mass	1,99E+30	kg
Fictive force applied on the model	1,00E+04	MN
Displacement longitudinal of the model	1,66E+00	m
Displacement transversal of the model	6,66E+00	m
Speed c	299792458	m/s
Strain Measured value	1,00E-21	

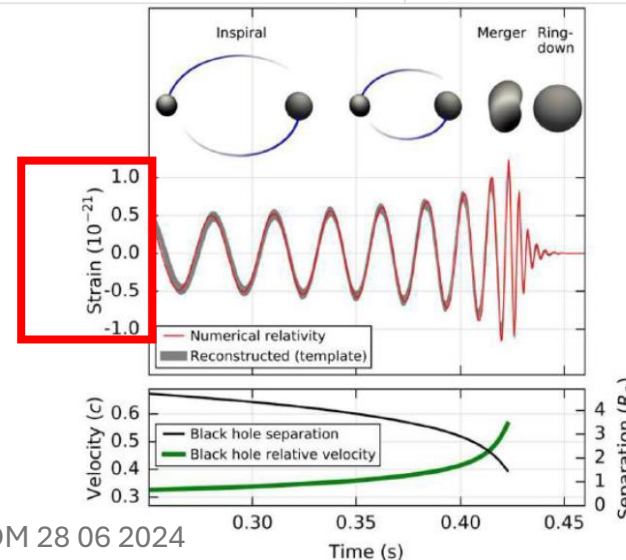
With diameter of black hole = horizon

$$r_{black\ hole} = \frac{2Gm}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 6.4 \times 10^{31}}{299792458^2} = 94993.6m \rightarrow 100km$$

$$h_{F_{Planck\ sheet}} = h_{F(for\ F=10000MN)} \times \frac{F_{Planck-sheet}}{F} = 3,35E-20$$

10) Calculus of the real transversal strain with the coalescence Loading

$$h_{F_{Planck\ sheet\ node\ 47}} = h_{F(for\ F=10000MN)} \times \frac{F_{Planck-sheet}}{F} = 1,34E-19$$



1,0 x 10⁻²¹

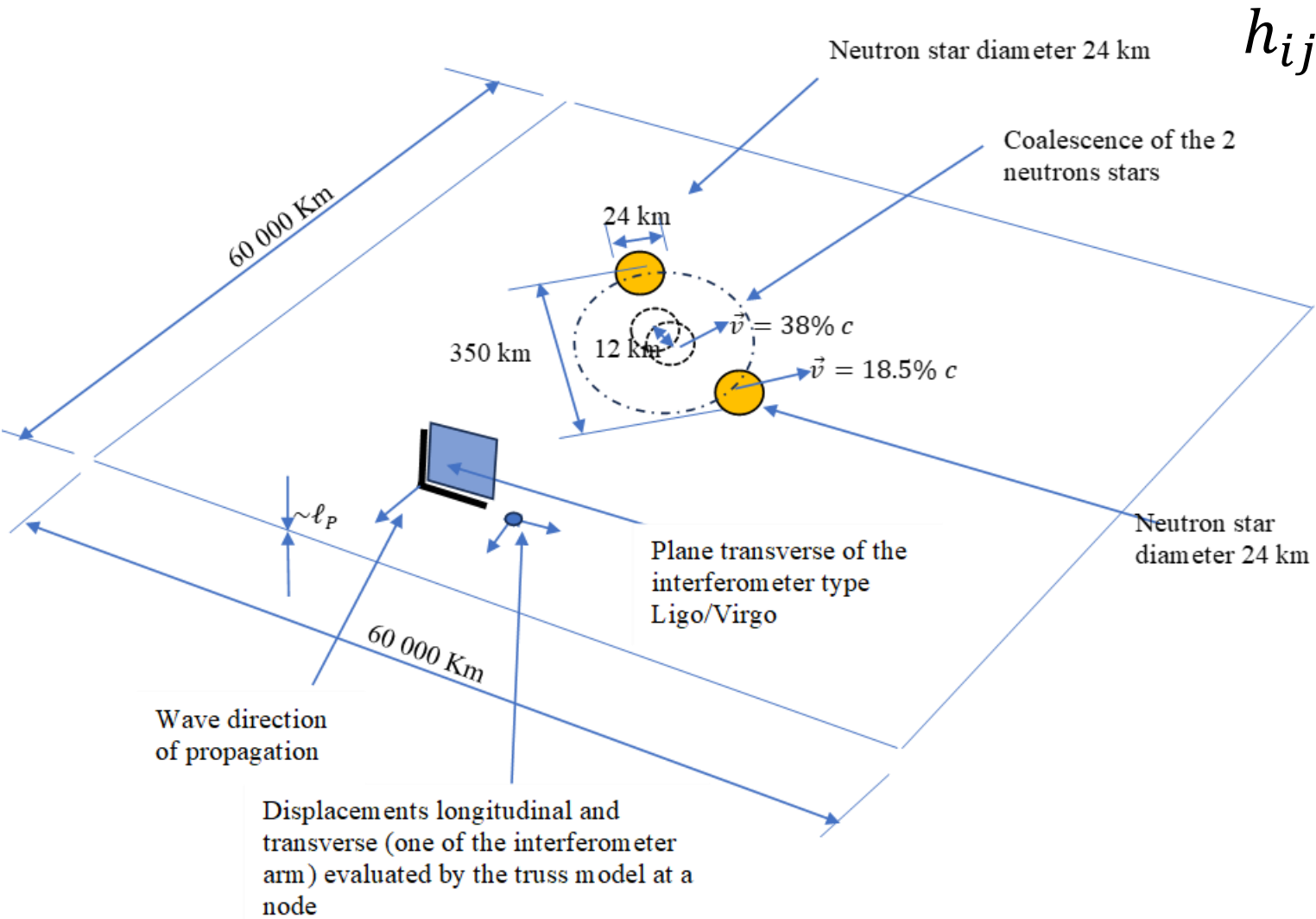
Order of magnitude ok with experiment if Y = 1x10⁴⁴ Pa and Planck thickness

Conclusion Y = 10⁴⁴ Pa is an acceptable value for h_{ij} strains

GW150914: Implications for the Stochastic Gravitational-Wave Background from Binary Black Holes

B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration) Phys. Rev. Lett. **116**, 131102 – Published 31 March 2016

Case2 : GW170817 - Coalescence of 2 neutrons stars



h_{ij}

Data

Data	Value	Unit
Mass neutron star 1	1,17	Solar mass
Mass neutrojn star 2	1,17	Solar mass
speed before coalescence	0,38	c
Speed at coalescence	0,185	c
Duration of coalescence	0,0833	s
Thikness sheet (Planck)	1,00E-35	m
Young modulus Y	1,00E+44	Pa
Diameter neutron star	24	km
L1 Distance between the 2 forces	12	km
Mesch of the truss	6000	km
Area of the bar	6,00E-29	m ²
Solar Mass	1,99E+30	kg
Fictive force applied on the model	1,00E+04	MN
Displacement longitudinal of the model	1,66E+00	m
Displacement transversal of the model	6,66E+00	m
Speed c	299792458	m/s
Strain Measured value	1,00E-21	

Case 2: GW180817 Comparison measured value and numerical model for GW170817

Data	Value	Unit
Mass neutron star 1	1,17	Solar mass
Mass neutrojn star 2	1,17	Solar mass
speed before coalescence	0,38	c
Speed at coalescence	0,185	c
Duration of coalescence	0,0833	s
Thikness sheet (Planck)	1,00E-35	m
Young modulus Y	1,00E+44	Pa
Diameter neutron star	24	km
L1 Distance between the 2 forces	12	km
Mesch of the truss	6000	km
Area of the bar	6,00E-29	m ²
Solar Mass	1,99E+30	kg
Fictive force applied on the model	1,00E+04	MN
Displacement longitudinal of the model	1,66E+00	m
Displacement transversal of the model	6,66E+00	m
Speed c	299792458	m/s
Strain Measured value	1,00E-21	

With diamètre of black hole = horizon

$$h_{F_{Planck\ sheet}} = h_{F(For\ F=10000MN)} \times \frac{F_{Planck-sheet}}{F} = 2.766 \times 10^{-7} \times \frac{3.418 \times 10^{-10}}{10000} = \frac{\Delta L}{L} = 9.456 \times 10^{-21}$$

Transversally nodes 47 49 69 71:

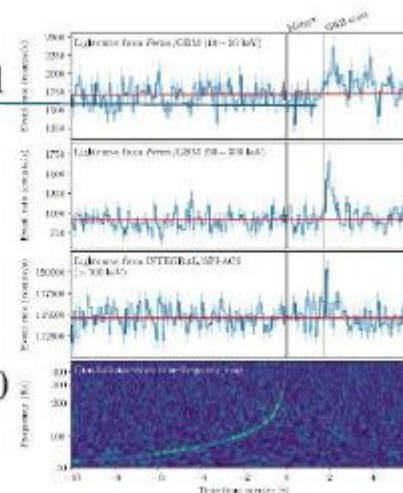
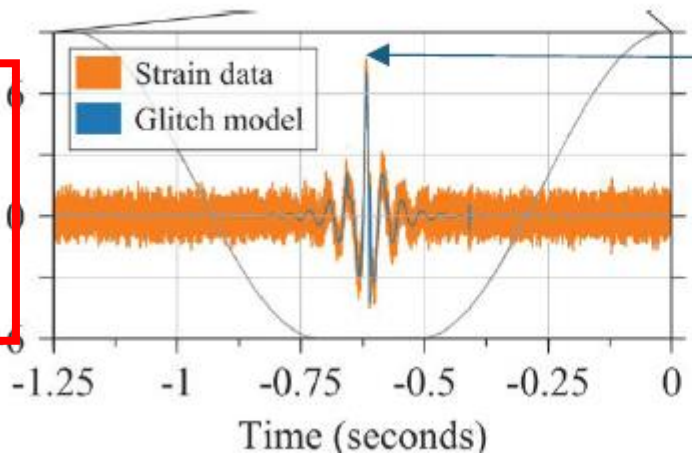
$$h_{F_{Planck\ sheet\ node\ 47}} = h_{F(For\ F=10000MN)} \times \frac{F_{Planck-sheet}}{F} = \frac{6.66}{6000000} \times \frac{3.418 \times 10^{-10}}{10000} = \frac{\Delta L}{L} = 3.793 \times 10^{-20}$$

Order of magnitude
OK



Conclusion Y = 10⁴⁴ Pa is an acceptable value for h_{ij} strains

Strain (×10⁻²⁰)



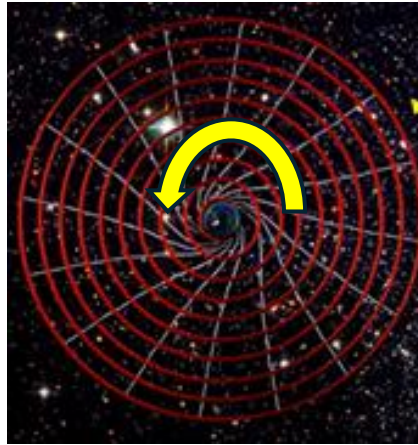
8 × 10⁻²⁰

Order of magnitude ok with experiment if Y = 1 × 10⁴⁴ Pa and Planck thickness

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration)
Phys. Rev. Lett. **119**, 161101 – Published 16 October 2017

Case 3 : Approach 1 - Frame Dragging effect around the earth measured by Gravity prob B



$$h_{0i}, h_{j0}$$

Principle: we impose the frame dragging rotation angle θ_{x2} measured during the gravity prob B experiment at a cylinder of space in rotation inside an elastic medium (Torsion) and extract the corresponding Young's modulus Y of the space time fabric to be correlated with gravity prob B measurement

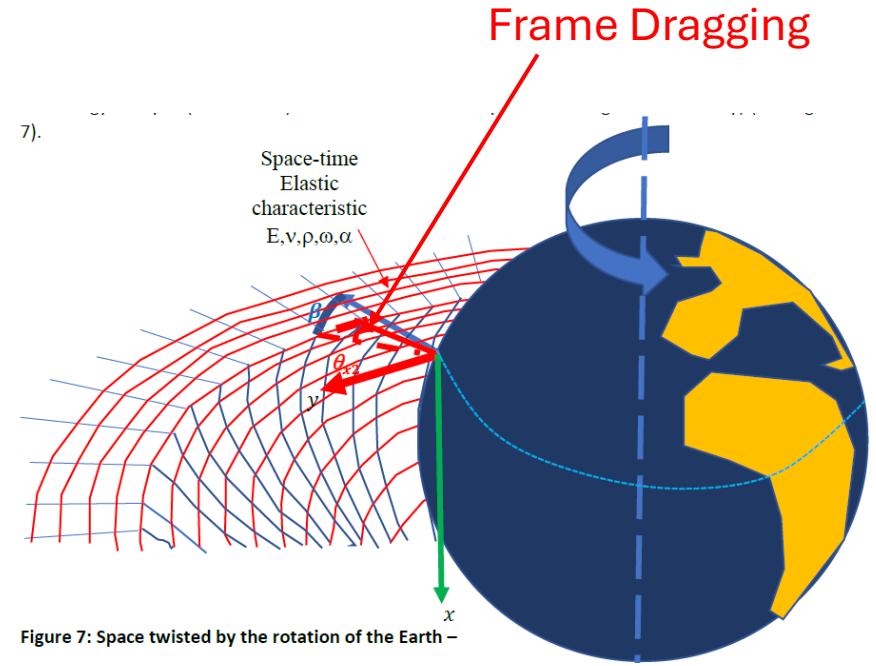
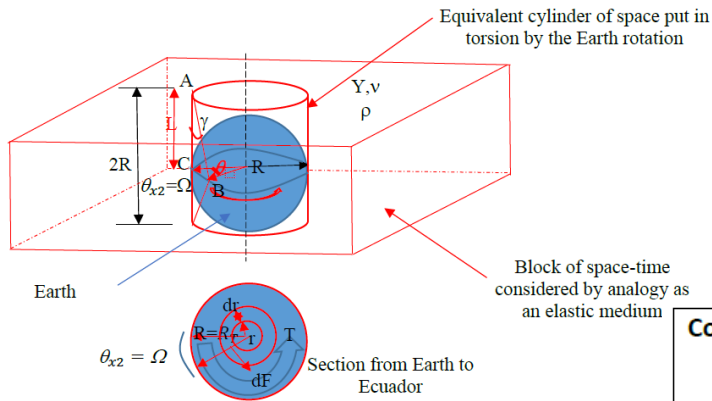


Figure 7: Space twisted by the rotation of the Earth -



Gravity Prob B experiment

Considered effect	Prediction of general relativity in milliarc second by year (*)	Measurements made by Gravity probe B in milliarc second by year (*)	Error %
Geodetic drift rate	-6606.1	-6601.8+/-18.3	0.28
Frame dragging	-39.2	-37.2+/-7.2	19

1 milliarc seconde = 4.848×10^{-9} rad

Lense-thirring effect: Part of $h_{\mu\nu} = \gamma_{\mu\nu}$ concerned

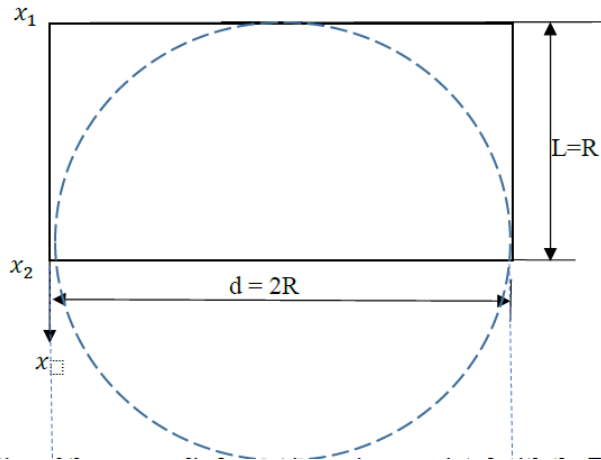
$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} + \begin{bmatrix} \frac{2kM}{r} & i\frac{4kM ly}{5r r^2} \omega l & -i\frac{4kM lx}{5r r^2} \omega l & 0 \\ i\frac{4kM ly}{5r r^2} \omega l & -\frac{2kM}{r} & 0 & 0 \\ -i\frac{4kM lx}{5r r^2} \omega l & 0 & -\frac{2kM}{r} & 0 \\ 0 & 0 & 0 & -\frac{2kM}{r} \end{bmatrix}$$

$$= \begin{bmatrix} -1 + \frac{2kM}{r} & i\frac{4kM ly}{5r r^2} \omega l & -i\frac{4kM lx}{5r r^2} \omega l & 0 \\ i\frac{4kM ly}{5r r^2} \omega l & -1 - \frac{2kM}{r} & 0 & 0 \\ -i\frac{4kM lx}{5r r^2} \omega l & 0 & -1 - \frac{2kM}{r} & 0 \\ 0 & 0 & 0 & -1 - \frac{2kM}{r} \end{bmatrix} \quad T_{\mu\nu} = \rho_0 \left(\frac{dt}{dS} \right)^2 \begin{bmatrix} 1 & ir' \omega \sin \theta \sin \phi & -ir' \omega \sin \theta \cos \phi & 0 \\ ir' \omega \sin \theta \sin \phi & 0 & 0 & 0 \\ -ir' \omega \sin \theta \cos \phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

156 Lense u. Thirring, Einfluß der Eigenrotation der Zentralkörper. Physik. Zeitschr. XIX, 1918.

Über den Einfluß der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie.

Von J. Lense und H. Thirring.



We have so the following expression for the $\theta_{(x)} \cdot \theta_{(x)} = A + Bx$

With the limit conditions described above and the frame defined in figure 120, we obtain

$$\theta_{(x1)} = \theta_{(0)} = A + Bx_1 = A$$

$$\theta_{(x2)} = \theta_{(L)} = \theta_{(x1)} + BL$$

That imply the two constants are:

$$B = \frac{\theta_{(x2)} - \theta_{(x1)}}{L}$$

$$A = \theta_{x1}$$

By asking: $\theta_{x1} = 0$ we have:

$$\theta_{(x)} = \theta_{x1} + \frac{\theta_{(x2)} - \theta_{(x1)}}{L} x$$

And finally:

$$\theta_{(x)} = \frac{\theta_{(x2,space)}}{L} x$$

$$\theta_{(x)} = \frac{\theta_{(x2,space)}}{L} x$$

By reporting the expression above of the torsion torque function of the θ variable, we obtain the expression function of x of this one:

$$T_{(x)} = \frac{\pi G d^4 \theta_{x2,space}}{32 L^2} x$$

We can therefore calculate the strain energy U of the equivalent torsional bar:

$$U = \frac{1}{2} \int_0^L \frac{T_{(x)}^2}{G I_t} dx = \frac{1}{2 G I_t} \int_0^L \left(\frac{\pi G d^4 \theta_{x2,space}}{32 L^2} x \right)^2 dx$$

So finally:

$$U = \frac{\pi^2 G d^8 \theta_{x2,space}^2}{6144 I_t L}$$

In the case of a solid tube, with the equation of the torsional inertia I_t , we obtain:

$$U = \frac{\pi G d^4 \theta_{x2,space}^2}{192 L}$$

With the definition of the shear modulus of the elastic medium associated:

$$G = \frac{Y}{2(1 + \nu)}$$

We obtain finally:

$$U = \frac{\pi Y d^4 \theta_{x2,space}^2}{384(1 + \nu)L}$$

Strain elastic energy

Case 3 : Approach 1 - Frame dragging Gravity prob B

The kinetic energy of rotation of the Earth is equal to the energy of deformation by torsion of the associated space-time cylinder driven by the Earth.

$$E_{Kinetic} = \frac{1}{2} J \omega^2$$

With angular velocity ω in rad/s

In the case of a rotating ball the moment of kinetic inertia is:

$$J = \frac{2}{5} M_T R_T^2$$

We can define the angular velocity in rad/s by the expression above function of the time taken by the Earth to do a complete tour in 24h:

$$\omega_T = \frac{1 \text{ tour}}{24 \times 60 \times 60} \times \frac{2\pi \text{ rad}}{1 \text{ tour}} = \frac{\pi}{43200} \text{ rad/s}$$

We obtain the following expression of the Kinetic energy:

$$E_{Kinetic,T} = \frac{1}{2} \times \frac{2}{5} M_T R_T^2 \times \left(\frac{\pi}{43200} \right)^2$$

So, the final expression of the Kinetic energy by torsion of the Earth:

$$E_{Kinetic,T} = \frac{\pi^2}{9331200000} M_T R_T^2$$

Kinetic energy

$$E_{cinetic,T} = \frac{\pi^2}{9331200000} M_T R_T^2 = U = 2 \frac{\pi Y (2R_T)^4 \theta_{x2,space}^2}{384(1 + \nu) R_T}$$

We extract an expression of Young's modulus Y of spacetime:

$$Y = \frac{\pi^2 \times 384(1 + \nu) R_T}{2\pi \times 9331200000 (2R_T)^4 \theta_{x2,space}^2} M_T R_T^2$$

Or after some mathematics:

$$Y = \frac{\pi \times 12(1 + \nu)}{9331200000 \times R_T \times \theta_{x2,space}^2} M_T$$

Cas 3 : Approach 1 - Numerical Applications and comparison and conclusion about the necessary young's modulus

- Estimation of the spatial part and time part of the space-time measurement angle of the gravity prob B frame dragging

If we place ourselves in the equatorial plane of the Earth, the interval becomes roughly speaking:

$$ds^2 = c^2 dt^2 - (d\ell^2)$$

Graviti prob B measured at $r = 6642 \text{ km}$ an angle variation of $d\theta = 6.04 \times 10^{-15} \text{ rad/s}$ of space-time, so the associated variation in length is .

$$ds = r d\theta = 6642000 \times 6.04 \times 10^{-15} = 4.01 \times 10^{-8} \text{ m}$$

In parallel, we have an estimate of the distance variation related to the entrainment effect of the time:

$$d_{t(\Delta t)} = c \times dt = 299792458 \times 1.0 \times 10^{-16} = 2.99792458 \times 10^{-8} \text{ m}$$

From these two values, we can therefore deduce the variation of length in strict spatial distance:

$$|d_\ell| = \sqrt{ds^2 - c^2 dt^2}$$

$$d_\ell = \sqrt{(4.01 \times 10^{-8})^2 - (2.99792458 \times 10^{-8})^2}$$

$$d_{\ell(\text{space})} = 2.66 \times 10^{-8} \text{ m}$$

This corresponds to a spatial angle to be found in our elastic model of:

$$\theta_{x2(\text{space})} = \Omega_{\text{space}} = \frac{d_\ell}{r} \sim \frac{2.66351 \times 10^{-8}}{6642000} = 4.0 \times 10^{-15} \text{ rad/s}$$

For memory the "time" angle is so:

$$\theta_{x2(\Delta t)} = \Omega_{(\Delta t)} = \frac{c \Delta t}{r} \sim \frac{2.99792458 \times 10^{-8}}{6642000} = 4.51 \times 10^{-15} \text{ rad/s}$$

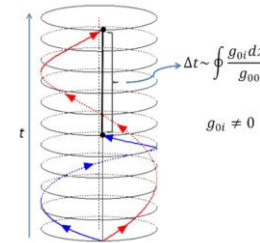


Figure 2. Two pulses of radiation counter-propagating in a circuit where $g_{0i} \neq 0$. $\oint \frac{g_{0i} dx^i}{g_{00}}$ for a gravitomagnetic field $g_{0i} \sim \frac{1}{c} \Omega$, and $\oint \frac{g_{0i} dx^i}{g_{00}} \sim \Omega r$ for a circuit rotating with angular velocity Ω .

So, we postulate 50% time 50% space as for the beam light around the sun (0,84" for Newton for space alone and 1,75" space time Einstein)

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Time travel, Clock Puzzles and Their Experimental Tests

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Abstract. Is time travel possible? What is Einstein's theory of relativity mathematically predicting in that regard? Is time travel related to the so-called clock 'paradoxes' of relativity and if so how? Is there any accurate experimental evidence of the phenomena regarding the different flow of time predicted by General Relativity and is there any possible application of the temporal phenomena predicted by relativity to our everyday life? Which temporal phenomena are predicted in the vicinities of a rotating body and of a mass-energy current, and do we have any experimental test of the occurrence of these phenomena near a rotating body? In this paper, we address and answer some of these questions.

6 Frame Dragging

In Fig. 3 is described a clock 'puzzle' owed to the spin of a central body. For this effect to occur, the clocks, or twins, would not need to move close to the speed of light (as in the case of the well-known 'twin-paradox' of special relativity). For example, if two such twins meet again, having flown arbitrarily slowly around the whole Earth in opposite directions on the equatorial plane and exactly at the same altitude, the difference in their ages owed to the Earth's spin would be approximately 10^{-16} (for an altitude of about 6,000 km), which would in principle be detectable if not for the other, much larger, relativistic clock effects. These clock effects are striking around a rotating black hole, however.

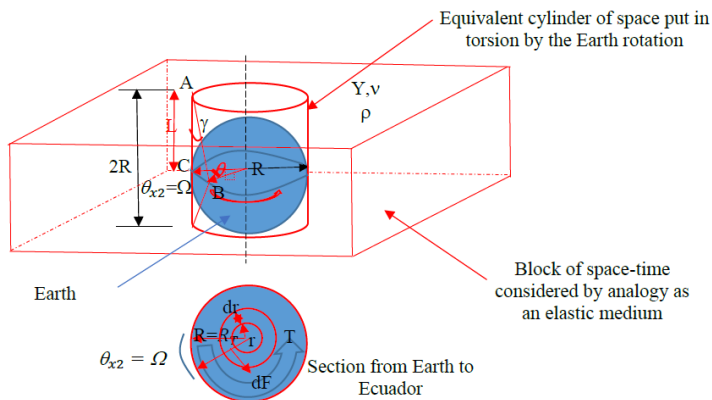
Case 3 : Approach 1 – Gravity prob B space angle via cylinder of space in torsion

Data	Value	Unit
Mass of the Earth M_T	5,97E+24	kg
Poisson's ratio	1	
Radius of the Earth R_T	6371	km
Gravity Prob B (space)	4,00E-15	rad

39milliarc second/year for Prob B

1milliarc second 4,85E-09rad

1,23668E-06milliarc second /s 6,00E-15rad/s

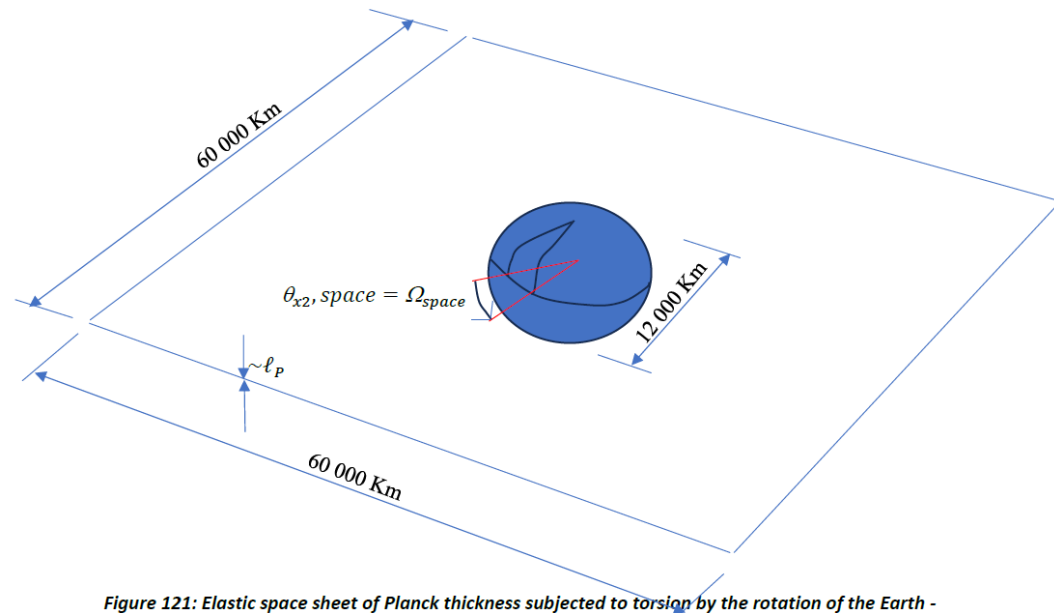


$$4,73E+38Pa$$

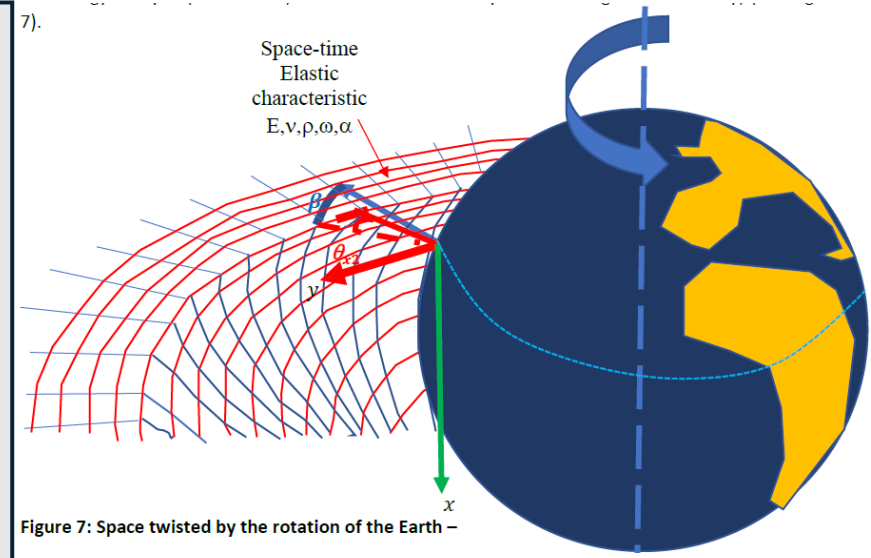
$$Y = \frac{\pi \times 12 \times (1 + \nu)}{9331200000 \times R_T \times \theta_{x2,space}^2} M_T$$

Conclusion not to far of $Y = 10^{44}$ Pa obtained with the GW150914 and GW170817

Case 3: Approach 2 :Gravity prob B frame dragging via fine elements model



Principle: we transform the gravity prob B frame dragging angle θ_{x_2} imposed by the Earth rotation in a torsion torque equivalent that we put in the middle of the sheet modelled by a truss (same that use for GW) and look for the Young modulus intensity that allow to refind the strains and rotation of the space time fabric



Case 3: Approach 2 - Gravity prob B frame dragging via finite element (truss)

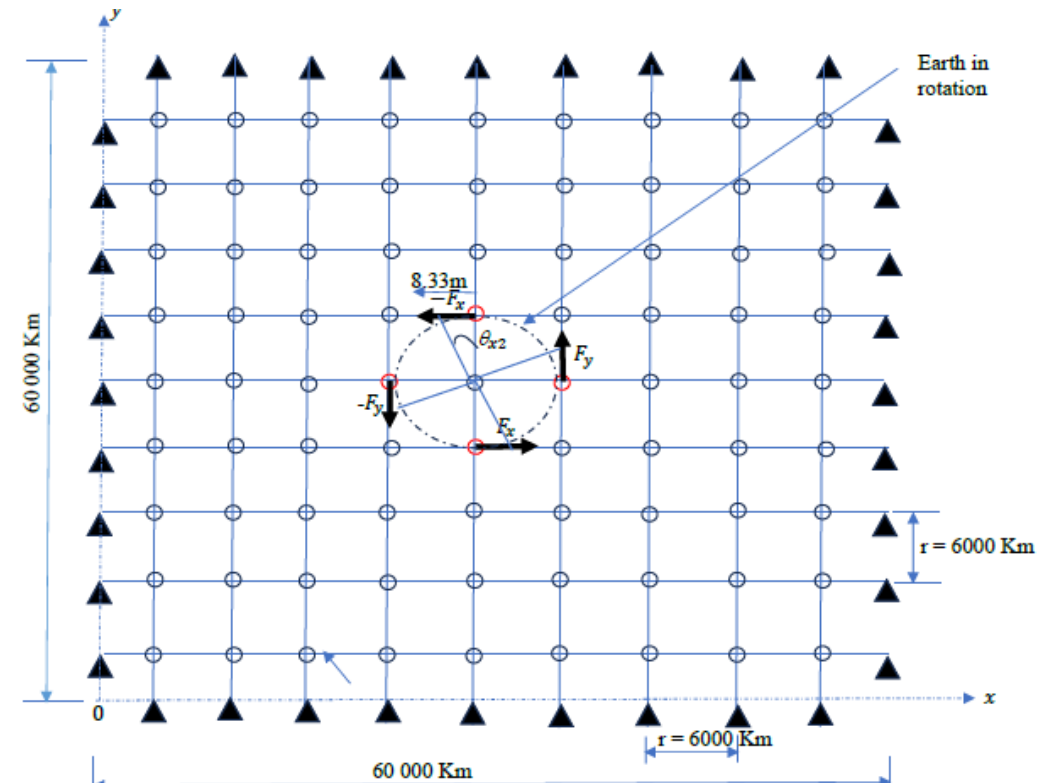
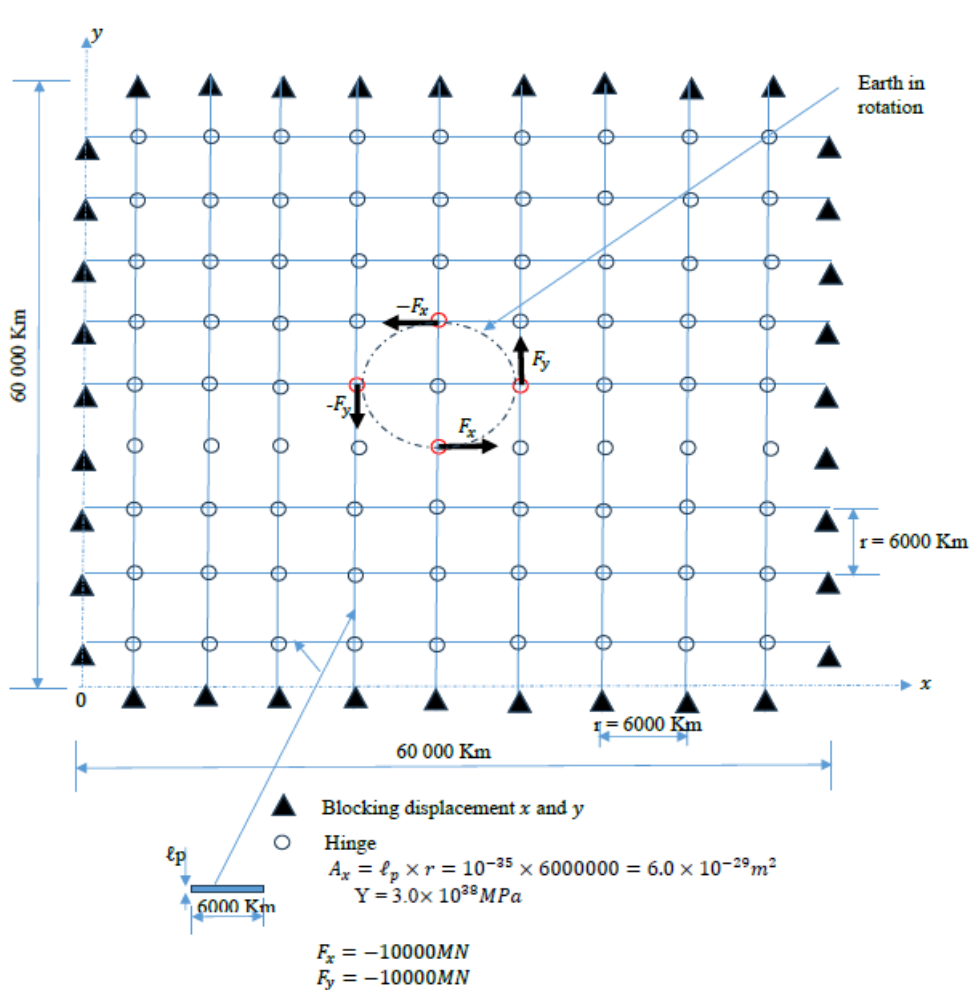


Figure 126: Determination of the rotation θ_{x2} of the elastic lattice model modeling a space sheet of

Planck thickness -

Displacement and rotation with
a fictive load of 10000 MN

Case 3: Approach 2 - Gravity prob B frame dragging approach 2 via finite element (truss)

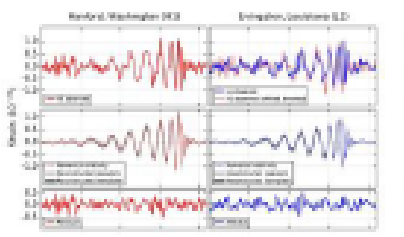
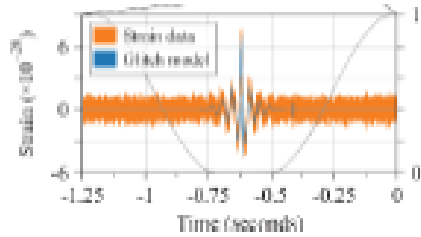
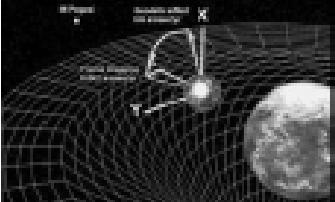
Data

Data	Value	Unit
Thickness sheet (Planck)	1,62E-35	m
Young modulus Y	3,00E+44	Pa
Poisson's ratio	1	
Mesch of the truss r	6000	km
Fictive force applied on the model	10000	MN
Area of the bar	6,00E-29	m ²
Fictive force applied on the model	1,00E+04	MN
Distance L (Radius of Earth)	6,37E+03	km
Diameter d of the Earth	1,27E+04	km
Area of the bar	6,00E-29	m ²
Displacement for F = 10000	8,33E+00	m
Gravity Prob B (space) θ_{x2}	26,02	milliarcsec/Y
Gravity Prob B (space) θ_{x2}	4,00E-15	rad/s

Conclusion Y = 10⁴⁴ Pa is an acceptable value for space strains

1) Calculus of the rotation angle of the model for F=10 000 MN		
$\theta_{x2,space} = \frac{\text{displacement model}}{\text{width of the mesh}}$	8,33E+00 6000000	1,39E-06 rad
2) Calculus of the Shear modulus of the space		
$G = \frac{E}{2(1+\nu)}$		7,50E+43 Pa
3) Calculus of the equivalent torque at the imposed angle		
$T = 2 \frac{\pi G d^4 \theta_{x2,space}}{32L}$		2,44E+50 N.m/s 2,44E+44 MN.m/s
4) Calculus of the equivalent load on the finite elements models		
$F_{x,real} = \frac{T}{4r}$		1,02E+43 N 1,02E+37 MN
5) Number of Planck sheets concerned		
$n = \frac{d}{\ell_p}$		7,87E+41 sheets

6) Calculus of the real load calibrated with gravity prob B		
$F_{x,real \text{ by } \ell_p} = \frac{F_{x,real}}{n}$		1,29E+01 N 1,29E-05 MN
9) Calculus of the angle associated with the finite element model and gravity prob B		
$\theta_{x2,space,real} = \Omega_{space,real} = \theta_{x2,space} \times \frac{F_{x,real \text{ by } \ell_p}}{F_x}$		1,79E-15 rad/s
10) Conversion of the rad /sin milliarc second /year		
	1 milliarc second	4,85E-09 rad
	3,70E-07 milliarc second /s	1,79E-15 rad/s
	11,66 milliarc second /year	
	26,02 measured by gravity prob B (space part)	

Case of general relativity study GW/Lense thirring effect	Type of parameter measured or calculated following general relativity	Theoretical results of the general relativity	Measured results GW => Ligo/Virgo Lense Thirring => G prob B	Mechanical model of the Planck sheet associated with an elastic truss	Young's modulus used for the calculation (Pa) (space aspect)																					
GW150914 (coalescence of 2 black holes) (Weak Gravitational field)	Elongation and shortening transverse strain h measured on Earth 	$\pm 10^{-21}$	$\pm 1 \times 10^{-21}$	Transverse h $\pm 1.33 \times 10^{-19}$ (1) Longitudinal h $\pm 3.32 \times 10^{-20}$ (1) Transverse h $\pm 2.0 \times 10^{-6}$ (1) Longitudinal h $\pm 5.0 \times 10^{-7}$ (1)	3×10^{44} 2×10^{31}																					
GW180717 (coalescence of 2 neutron stars) (Weak Gravitational field)	Elongation and shortening transverse strain h measured on +Earth 	$\pm 10^{-20}$	$\pm 8 \times 10^{-20}$	Transverse h $\pm 3.799 \times 10^{-20}$ (1) Longitudinal h $\pm 9.47 \times 10^{-21}$ (1) $\pm 1.508 \times 10^{-23}$ (2) Transverse h $\pm 3.7 \times 10^{-8}$ (1) Longitudinal h $\pm 9.28 \times 10^{-9}$ (1)	3×10^{44} 2×10^{31}																					
Frame dragging created by the Earth on the space-time (Weak Gravitational field)	Horizontal angle of distortion Ω measured on Earth at r =6700 km <table border="1" data-bbox="529 1073 853 1202"> <thead> <tr> <th>Source</th> <th>Dec. (J2000.0)</th> <th>RA. (J2000.0)</th> </tr> </thead> <tbody> <tr> <td>Gravitope 1</td> <td>-0.384 06 ± 0.11 7</td> <td>-41.7 ± 0.0</td> </tr> <tr> <td>Gravitope 2</td> <td>-0.397 03 ± 0.11 1</td> <td>-39.1 ± 0.0</td> </tr> <tr> <td>Gravitope 3</td> <td>-0.405 03 ± 0.11 2</td> <td>-29.9 ± 0.0</td> </tr> <tr> <td>Gravitope 4</td> <td>-0.384 07 ± 0.11 2</td> <td>-40.3 ± 0.0</td> </tr> <tr> <td>Planet (our local)</td> <td>-0.401 04 ± 0.11 3</td> <td>-27.3 ± 0.0</td> </tr> <tr> <td>Earth position</td> <td>-0.400 01</td> <td>-30.7</td> </tr> </tbody> </table> 	Source	Dec. (J2000.0)	RA. (J2000.0)	Gravitope 1	-0.384 06 ± 0.11 7	-41.7 ± 0.0	Gravitope 2	-0.397 03 ± 0.11 1	-39.1 ± 0.0	Gravitope 3	-0.405 03 ± 0.11 2	-29.9 ± 0.0	Gravitope 4	-0.384 07 ± 0.11 2	-40.3 ± 0.0	Planet (our local)	-0.401 04 ± 0.11 3	-27.3 ± 0.0	Earth position	-0.400 01	-30.7	39.2 milliarc second/year	37.2 milliarc second/year (space time) 25.8 milliarc second/year (space estimation only)	- $\Omega = 11.55$ milliarc second/ year (1) Model (3)	3×10^{44} 4.73×10^{38}
Source	Dec. (J2000.0)	RA. (J2000.0)																								
Gravitope 1	-0.384 06 ± 0.11 7	-41.7 ± 0.0																								
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Earth position	-0.400 01	-30.7																								

4.2) Models perpendicular at the plane with temporal components of the strains (associated at h_{00})

Strains of the space calculated from component h_{00} (time component)

Transformation of h_{00} in radius of curvature variation

$$R_S = \frac{2GM}{c^2} \quad (16)$$

Calculations on space-time curvature within the Earth and Sun
 Wm. Robert Johnston
 last updated 3 November 2008



The true circumference of a non-rotating black hole with a given mass is $C_{bh} = 2\pi R_S$
 Specific observed and derived data for the Sun and Earth are as follows:

	Sun	Earth
GM	$1.32712438 \times 10^{20} \text{ m}^3/\text{s}^2$	$3.98600441 \times 10^{14} \text{ m}^3/\text{s}^2$
R	695,990 km	6,371.0 km
R_S	2.95325003 km	8.87005606 mm
ΔR	492 m	1.48 mm

The table lists GM rather than M , since GM for the Earth and Sun is known with greater accuracy than the mass. The measured radii of the Sun and Earth would correspond to R in our formulation, not s . For both bodies $R_S \ll R$, justifying use of the final expression of the relativistic correction to the radius as $\Delta R = M/3$ —assuming uniform density.

In weak Field

Einstein Gravitational field equation in weak field

$$\left\{ \begin{array}{l} \phi = \frac{GM}{r} \\ g_{00} = \eta_{00} + h_{00} = 1 + \frac{2\phi}{c^2} \end{array} \right.$$

$$\Delta h_{00} = \frac{2}{c^2} \Delta \phi = \frac{8\pi G}{c^4} \rho c^2$$

Numerical values of the strains h_{00}

General relativity results

$$h_{00} \approx 2 \frac{GM}{rc^2} = \frac{2\phi}{c^2}$$

Numerical application for the Earth, the value of the strain is:

$$h_{00} = 2 \times \frac{6.67430 \times 10^{-11} \times 5.972 \times 10^{24}}{6371000 \times 299792458 \times 299792458} = 1.39222 \times 10^{-9}$$

For the Sun: $h_{00} = 2 \times \frac{6.67430 \times 10^{-11} \times 1.9891 \times 10^{30}}{695990000 \times 299792458 \times 299792458} = 4.244 \times 10^{-6}$

Associated curvature of the space : eg Earth

$$\Delta R = \frac{1}{6} R_S = \frac{GM}{3c^2} = \frac{6.6726 \times 10^{-11} \times 5.972 \times 10^{24}}{3 \times 299792458^2} = 0.00147792m$$

$$\Delta R = \frac{1}{6} R_S = \frac{h_{00} R c^2}{6c^2} = \frac{h_{00} R}{6} = \frac{1.39222 \times 10^{-9} \times 6371000}{6} = 0.001477m \rightarrow 1.47792mm$$

Demonstration of the variation of radius of the Earth or the Sun following the general relativity in weak field

$$ds^2 = g_{00}c^2 dt^2$$

$$\Delta s^2 = (1 + h_{00})c^2 \Delta t^2$$

$$ds^2 = c^2 \left(1 - \frac{a}{r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{a}{r}\right)} - r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$a = R_S = \frac{2GM}{c^2}$$

$$ds = \frac{dr}{\sqrt{\left(1 - \frac{2GM}{rc^2}\right)}}$$

$$s = \int_0^R \frac{dr}{\sqrt{\left(1 - \frac{2Gm(r)}{rc^2}\right)}}$$

$$m(r) = \int_0^r 4\pi r^2 \rho dr$$

$$m(r) = \frac{4}{3} \pi r^3 = \frac{Mr^3}{R^3}$$

weak field

$$s = \int_0^R \frac{dr}{\sqrt{\left(1 - \frac{2GM}{rc^2}\right)}}$$

$$s = \int_0^R \frac{dr}{\sqrt{\left(1 - \frac{1}{3} \frac{8\pi G}{c^4} \rho c^2 r^2\right)}}$$

$$s = \int_0^R \frac{dr}{\sqrt{\left(1 - \frac{\kappa t_{00}}{3} r^2\right)}}$$

$$s = \int_0^R \frac{dr}{\sqrt{\left(1 - \frac{\Delta h_{00}}{6} r^2\right)}}$$

$$\left\{ \begin{array}{l} M = \frac{4}{3} \pi R^3 \rho \\ \kappa = \frac{8\pi G}{c^4} \\ t_{00} = \rho c^2 \end{array} \right.$$

$$\Delta h_{00} = -2\kappa \left(T_{00} - \frac{1}{2} \eta_{00} T \right) = 2\kappa \rho c^2 = 2\kappa t_{00}$$

$$\frac{\Delta h_{00}}{2} = \kappa t_{00}$$

$$\Delta R = \frac{1}{6} R_S = \frac{GM}{3c^2} = \frac{6.6726 \times 10^{-11} \times 5.972 \times 10^{24}}{3 \times 299792458^2} = 0.00147792m$$

$$\Delta R = \frac{1}{6} R_S = \frac{h_{00} R c^2}{6c^2} = \frac{h_{00} R}{6} = \frac{1.39222 \times 10^{-9} \times 6371000}{6} = 0.001477m \rightarrow 1.47792mm$$

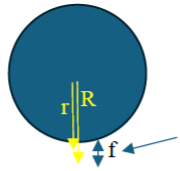
$$\frac{\Delta R}{R} = \varepsilon = \frac{h_{00}}{6}$$

Cas 4: Geodetic effect around the Earth in link with gravity prob B

Timoshenko

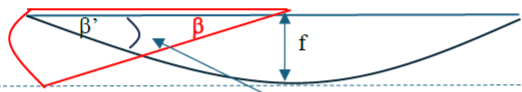
Einstein

$$R-r=\Delta R=f$$



$$\Delta R = \frac{1}{6} R_S = \frac{h_{00} R c^2}{6 c^2} = \frac{h_{00} R}{6}$$

Curvature effect on space time by the Earth imply increase of Earth radius of $R-r=f=1.47\text{mm}$



$$y(x) = \frac{f}{r^2} x^2 - f = f \left(\frac{x^2}{r^2} - 1 \right)$$

$$\frac{dy}{dx} = f \left(\frac{2x}{r^2} \right)$$

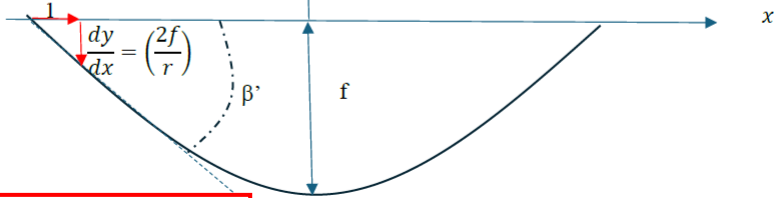
Equivalence of the geodetics angle β with the angle β' of the parabola (deformation of the space equivalent membrane by the

$$\frac{dy}{dx} = \left(\frac{2f}{r} \right)$$

For $x = r$

And the estimation of the pitch equivalent at the geodetic angle is:

$$\tan \beta = \frac{(2f)}{r} = \beta = \left(\frac{2f}{r} \right)$$



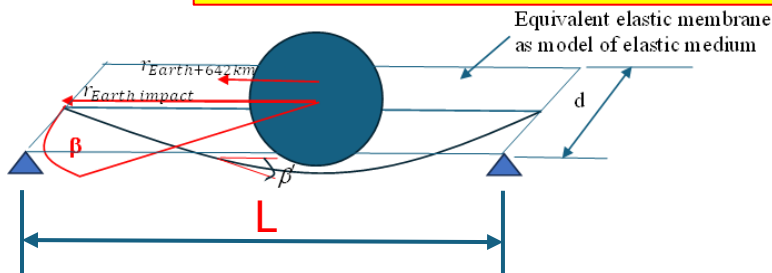
$$y(x) = ax^2 + bx + c$$

Corelation between β and the membrane deflection f

$$\frac{dy}{dx} = f \left(\frac{2x}{r^2} \right)$$

$$\frac{dy}{dx} = f \left(\frac{2(r_{\text{Earth}} + 642 \text{ km})}{r_{\text{Earth impact}}^2} \right) = \beta$$

g near 0



gravity prob B

Principle: The variation of Radius of the Earth ΔR due to gravity, associated at the space time curvature is transformed in variation of curvature (deflection f) of an equivalent membrane of span corresponding at the quasi nul gravity at each extremity. The angle β' is compared by reciprocity with the geodetic angle β determined by the gravity prob B experiment. From the deflexion f of the membrane it is possible to come back at the tension of the membrane and its Young's modulus of the space time fabric

David Izabel GDR.GDM 28 06 2024

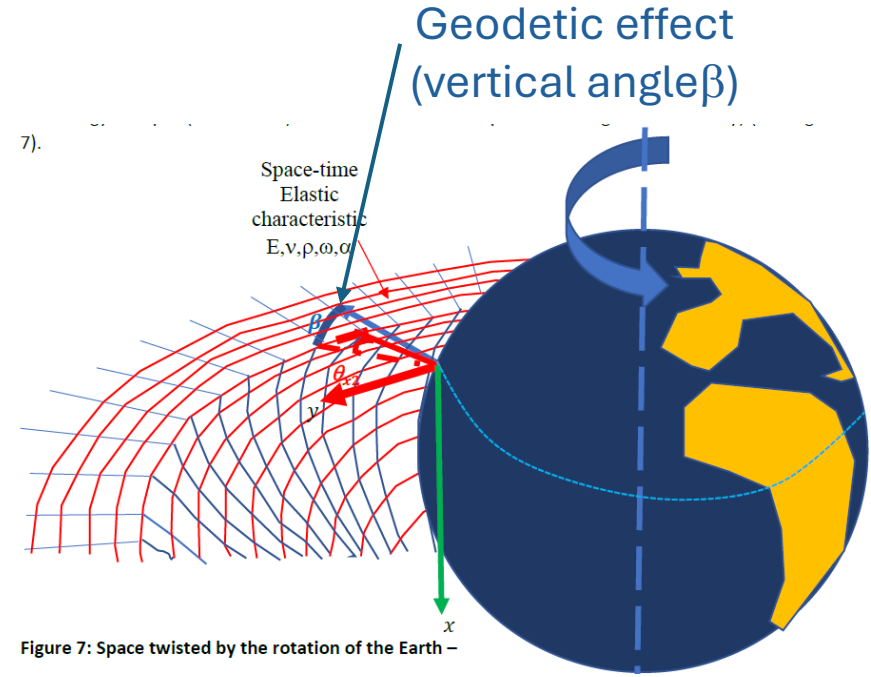


Figure 7: Space twisted by the rotation of the Earth -

Considered effect	Prediction of general relativity in milliarc second by year (*)	Measurements made by Gravity probe B in milliarc second by year (*)	Error %
Geodetic drift rate	-6606.1	-6601.8+/-18.3	0.28
Frame dragging	-39.2	-37.2+/-7.2	19

1 milliarc seconde = 4.848×10^{-9} rad

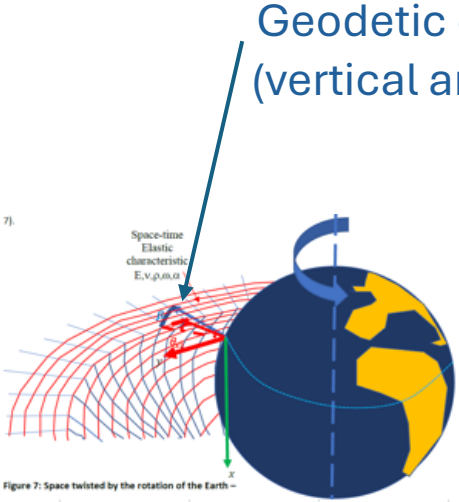
Witch span L of the membrane considered?

Case 4: Geodetic effect Numerical application – look for R_{impact} -Earth

Determination of R_{Earth impact} of gravity to obtain the gravity prob B curvature at x = 6371 + 642 km

Corelation between β and the membrane deflection f

Data	Value	Unit
Thickness sheet (Planck)	1,62E-35	m
Mass of the Earth MT	5,97E+24	kg
Poisson's ratio	1	
Radius of the Earth RT	6371	km
Diameter d of the Earth	12742	km
Gravity Prob B (space) β	6606,10	milliarcsec/Y
Gravity Prob B (space) β	1,02E-12	rad/s
Gravitational contant G	6,67E-11	m ³ /kgs ²
Metric perturbation h ₀₀	1,39E-09	
Deflection f of the membrane = ΔR	0,001477	m



f=ΔR obtained in GR

2) Calculus of the angle β' associated at this span

$$\frac{dy}{dx} = f \left(\frac{2(r_{Earth+642km})}{r_{Earth\ impact}^2} \right) = \beta' \quad 1,0349E-12 \text{ rad}$$

3) Conversion of the gravity prob B angle in radiant

6606,10 milliarc second/yr for Prob B

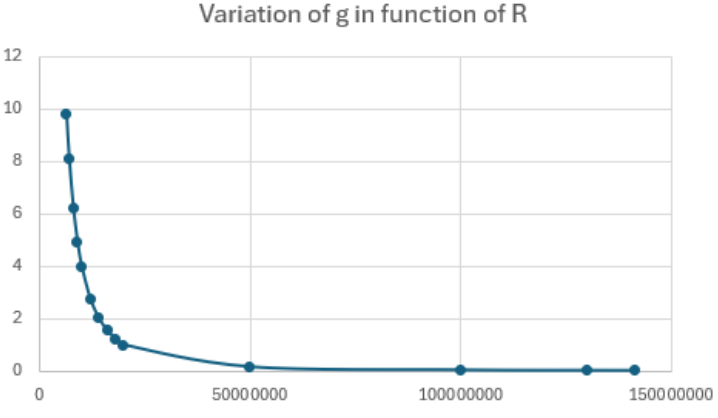
1 milliarc second = 4,85E-09 rad

0,000209478 milliarc second /s = 1,02E-12 rad/s

So we have the good R Earth impact radius the curvature at the R=6371+642 km is equal to β=β'

1) Calculus of the equivalent span of the membrane :look for gravitation of the Earth quasi nul

R(m)	G m3/kgs ²	Earth mass (kg)	g =GM/R ² (m/s ²)	R(m)	g =GM/R ² (m/s ²)
6371000	6,67E-11	5,97E+24	9,819973426	6371000	9,819973426
7013000	6,67E-11	5,97E+24	8,104343588	7013000	8,104343588
8000000	6,67E-11	5,97E+24	6,227956188	8000000	6,227956188
9000000	6,67E-11	5,97E+24	4,920854272	9000000	4,920854272
10000000	6,67E-11	5,97E+24	3,98589196	10000000	3,98589196
12000000	6,67E-11	5,97E+24	2,767980528	12000000	2,767980528
14000000	6,67E-11	5,97E+24	2,033618347	14000000	2,033618347
16000000	6,67E-11	5,97E+24	1,556989047	16000000	1,556989047
18000000	6,67E-11	5,97E+24	1,230213568	18000000	1,230213568
20000000	6,67E-11	5,97E+24	0,99647299	20000000	0,99647299
50000000	6,67E-11	5,97E+24	0,159435678	50000000	0,159435678
100000000	6,67E-11	5,97E+24	0,03985892	100000000	0,03985892
130000000	6,67E-11	5,97E+24	0,02358516	130000000	0,02358516
r Earth impact 141483883,6	6,67E-11	5,97E+24	0,019911848	141483883,6	0,019911848



Corelation between the membrane deflection f and the Young's modulus of the membrane

Case 4: Geodetic effect Numerical application – study of the equivalent rectangular membrane in tension – repartition of the Earth mass on πR_{earth}^2

4) Calculation of the number of Planck sheet concerned by the earth

Assimilating the Sphere at an equivalent cylinder of same volume

$$n = \frac{\frac{4}{3}r}{\ell_p} = 5,24E+41 \text{ Planck sheets}$$

5) Calculus of the mass of the Earth associated at each sheet

$$m = \frac{\text{Mass Earth}}{n} = 1,14E-17 \text{ kg/sheet}$$

6) Calculus of the weight/m² applied

$$p/\text{sheet} = \frac{mg}{\pi r^2} = 8,77E-31 \text{ N/m}^2/\text{sheet}$$

7) Calculus of the load by m of width of sheet (rectangular sheet of span R Earth impact)

$$\frac{q}{\text{sheet}} = p \times d = 1,12E-23 \text{ N/m/sheet}$$

8) Calculus of the vertical reaction of the support

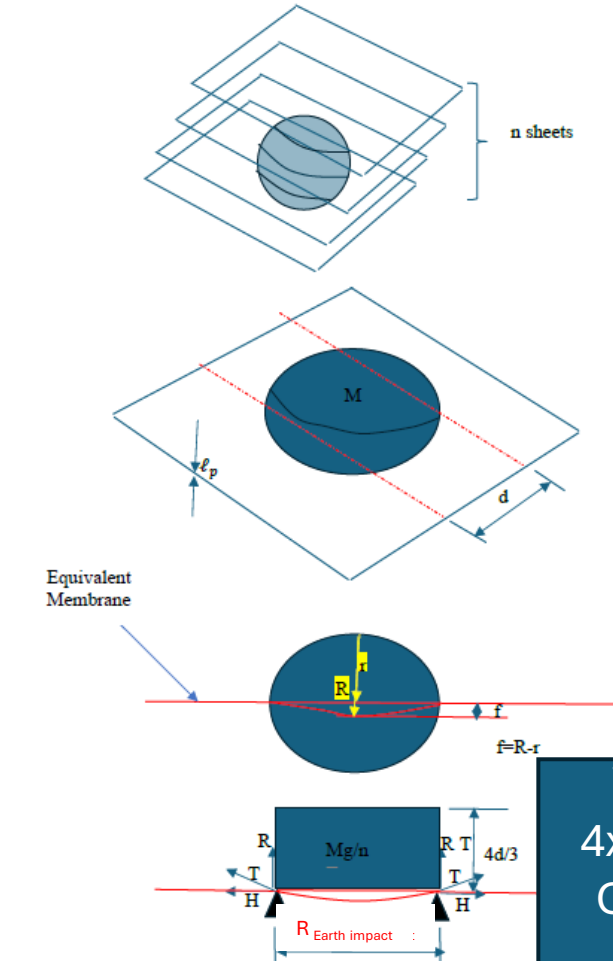
We suppose (defavorable approach) that all the sheet is uniformly loded)

$$R = \frac{qR_{\text{Earth-impact}}}{2} = 7,91E-16 \text{ N}$$

9) Calculus of the Horizontal reaction of the support ($f = \Delta R$ due to space time curvature)

$$H = \frac{qR_{\text{Earth-impact}}^2}{8f} \quad f = \Delta R \text{ obtained in GR} = 1,89E-05 \text{ N}$$

The model of membrane used is described at the figure 128 below:



David Izabel GDR GDM 28 06 2024

10) Calculus of the Tension in the membrane

$$T = \sqrt{R^2 + H^2} = 1,89E-05 \text{ N}$$

11) Calculus of the stress in the membrane

$$\sigma = \frac{T}{d \times \ell_p} = 9,17E+22 \text{ Pa}$$

12) Calculus of the tension strain in the membrane

$$\frac{\Delta R}{R} = \varepsilon = \frac{h_{00}}{6} = 2,32E-10$$

13) Extraction of the associated Young's modulus in the membrane

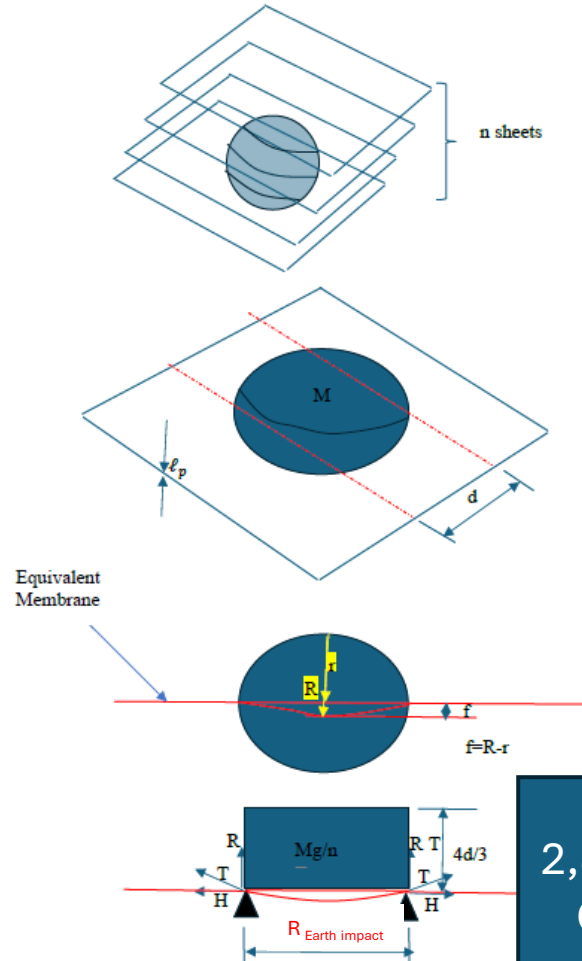
$$Y = \frac{\sigma}{\varepsilon} = 3,95E+32 \text{ Pa}$$

We obtain so a Young's modulus of 4×10^{32} Pa so $< 3 \times 10^{44}$ Pa obtained with GW and frame dragging (same order that R Weiss proposal Nobel Price lecture 10^{20} Y steel)

Case 4: Geodetic effect Numerical application – study of the equivalent rectangular membrane in tension – repartition of the Earth mass on $R_{\text{Earth impact}} \times d$

4) Calculation of the number of Planck sheet concerned by the earth	
Assimilating the Sphere at an equivalent cylinder of same volume	
$n = \frac{4}{3} \frac{r}{\ell_p}$	5,24E+41 Planck sheets
5) Calculus of the mass of the Earth associated at each sheet	
$m = \frac{\text{Mass Earth}}{n}$	1,14E-17 kg/sheet
6) Calculus of the weight/m² applied	
$p/\text{sheet} = \frac{mg}{R_{\text{impact}} \times \text{Earth} \times d}$	6,20E-32 N/m ² /sheet
7) Calculus of the load by m of width of sheet (rectangular sheet of span R Earth impact)	
$\frac{q}{\text{sheet}} = p \times d$	7,90E-25 N/m/sheet
8) Calculus of the vertical reaction of the support	
We suppose (defavorable approach) that all the sheet is uniformly loaded	
$R = \frac{q R_{\text{Earth-impact}}}{2}$	5,59E-17 N
9) Calculus of the Horizontal reaction of the support (f = ΔR due to space time curvature)	
$H = \frac{q R_{\text{Earth-impact}}^2}{8f}$	1,34E-06 N

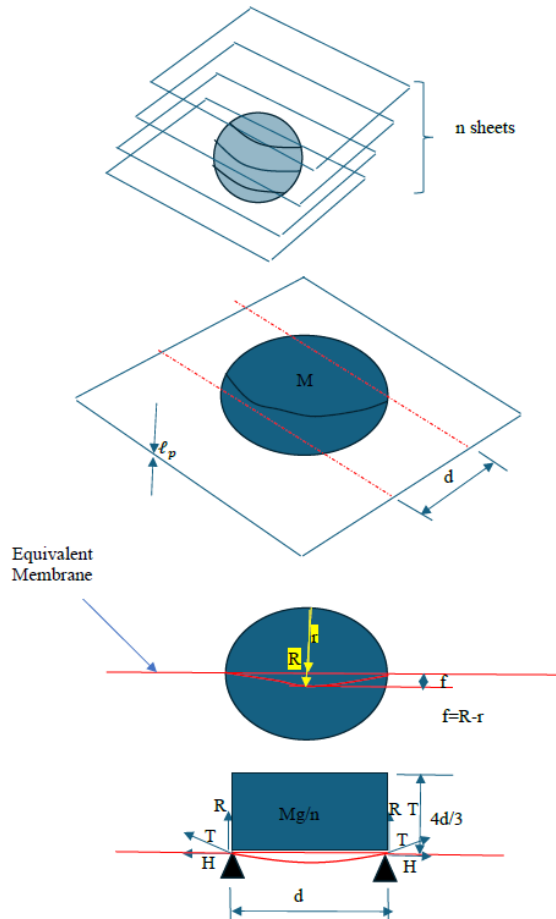
The model of membrane used is described at the figure 128 below:



10) Calculus of the Tension in the membrane	
$T = \sqrt{R^2 + H^2}$	1,34E-06 N
11) Calculus of the stress in the membrane	
$\sigma = \frac{T}{d \times \ell_p}$	6,49E+21 Pa
12) Calculus of the tension strain in the membrane	
$\frac{\Delta R}{R} = \varepsilon = \frac{h_{00}}{6}$	2,32E-10
13) Extraction of the associated Young's modulus in the membrane	
$Y = \frac{\sigma}{\varepsilon}$	Y = 2,80E+31 Pa
	Y (RWeiss) = 2,1E+31 Pa

We obtain so a Young's modulus of $2,8 \times 10^{31}$ Pa so $< 3 \times 10^{44}$ Pa obtained with GW and frame dragging (same order that R Weiss proposal Nobel Price lecture 10^{20} Y steel)

Case 5: Earth gravitation as curvature of the space time



Principle: The variation of Radius of the Earth ΔR due to gravity, associated at the space time curvature is transformed in variation of curvature of an equivalent membrane of span corresponding at the quasi nul gravity at each extremity. From the deflexion f of the membrane it is possible to come back at the tension of the membrane and its Young's modulus of the space time fabric

Data	Value	Unit
Thickness sheet (Planck)	1,62E-35	m
Mass of the Earth M_T	5,97E+24	kg
Poisson's ratio	1	
Radius of the Earth R_T	6371	km
Diameter d of the Earth	12742	km
Gravitational constant G	6,67E-11	m^3/kgs^2
Metric perturbation h_{00}	1,39E-09	
Deflection f of the membrane = ΔR	0,001477	m

Case 5: Earth gravitation as curvature of the space time in weak field

1) Calculus of the gravity g

12742000	6,67E-11	5,97E+24	9,82E+00 m/s ²
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1) Calculation of the number of Planck sheet concerned by the earth

Assimilating the Sphere at an equivalent cylinder of same volume

$$n = \frac{4}{3} \frac{r}{\ell_p}$$

5,24E+41 Planck sheets

2) Calculus of the mass of the Earth associated at each sheet

$$m = \frac{\text{Mass Earth}}{n}$$

1,14E-17 kg/sheet

3) Calculus of the weight/m² applied

$$p/\text{sheet} = \frac{mg}{\pi \cdot 2}$$

8,77E-31 N/m²/sheet

4) Calculus of the load by m of width of sheet (rectangular sheet of span R Earth impact)

$$\frac{q}{\text{sheet}} = p \times d$$

1,12E-23 N/m/sheet

We obtain so a Young's modulus of 3,28x10³⁰ Pa so <3x10⁴⁴ Pa obtained with GW and frame dragging (same order that R Weiss proposal Nobel Price lecture 10²⁰ Y steel)

5) Calculus of the vertical reaction of the support

We suppose (defavorable approach) that all the sheet is uniformly loded)

$$R = \frac{qd}{2}$$

7,12E-17 N

6) Calculus of the Horizontal reaction of the support (f=ΔR due to space time curvature)

$$H = \frac{qd^2}{8f}$$

1,54E-07 N

7) Calculus of the Tension in the membrane

$$T = \sqrt{R^2 + H^2}$$

1,54E-07 N

8) Calculus of the stress in the membrane

$$\sigma = \frac{T}{d \times \ell_p}$$

7,44E+20 Pa

9) Calculus of the tension strain in the membrane

$$\frac{\Delta R}{R} = \varepsilon = \frac{h_{00}}{6}$$

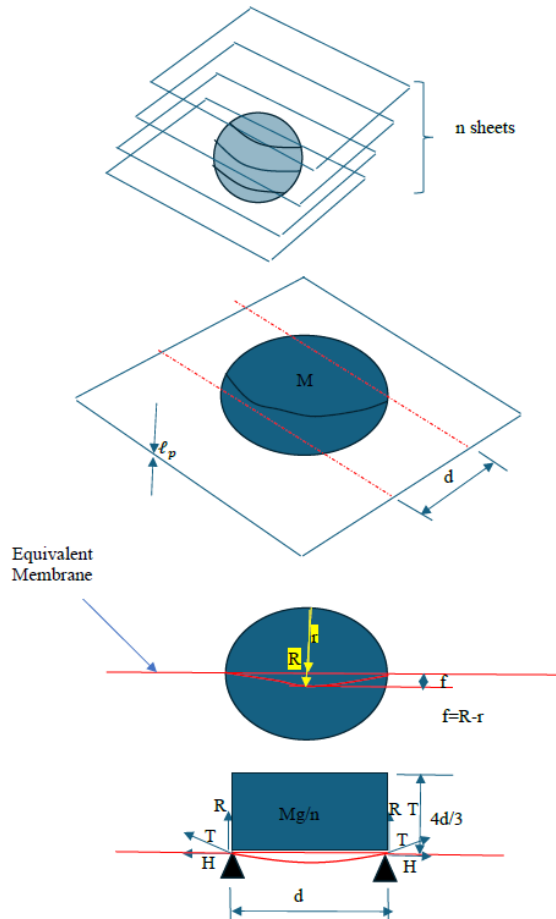
2,32E-10

10) Extraction of the associated Young's modulus in the membrane

$$Y = \frac{\sigma}{\varepsilon}$$

Y = 3,21E+30 Pa
Y (R Weiss) = 2,1E+31 Pa

Case 6: Sun gravitation as curvature of the space time



Principle: The variation of Radius of the Sun ΔR due to gravity, associated at the space time curvature is transformed in variation of curvature of an equivalent membrane of span corresponding at the quasi nul gravity at each extremity. From the deflexion f of the membrane it is possible to come back at the tension of the membrane and its Young's modulus of the space time fabric

Data	Value	Unit
Thickness sheet (Planck)	1,62E-35	m
Mass of the Sun M_S	1,98E+30	kg
Poisson's ratio	1	
Radius of the Sun R_S	696342	km
Diameter d of the Sun	1392684	km
Gravitational constant G	6,67E-11	m^3/kgs^2
Metric perturbation h_{00}	4,24E-06	
Deflection f of the membrane = ΔR	492	m

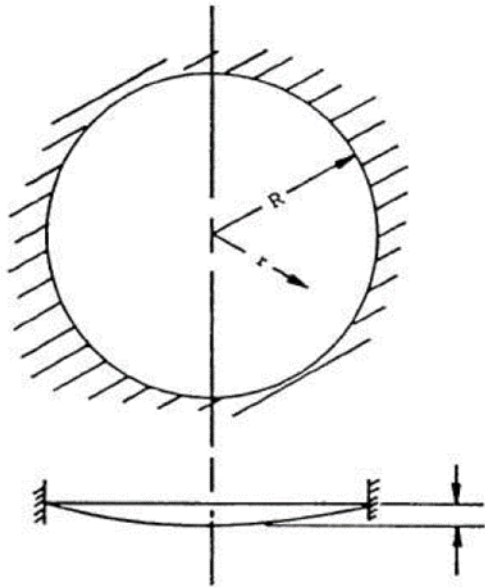
Case 6: Sun gravitation as curvature of the space time in weak field

1) Calculus of the gravity g			
1392684	6,67E-11	1,98E+30	2,73E+08 m/s ²
1) Calculation of the number of Planck sheet concerned by the earth			
Assimilating the Sphere at an equivalent cylinder of same volume			
$n = \frac{4}{3} \frac{r}{\ell_p}$		5,73E+43	Planck sheets
2) Calculus of the mass of the Earth associated at each sheet			
$m = \frac{\text{Mass Sun}}{n}$		3,45E-14	kg/sheet
3) Calculus of the weight/m² applied			
$p/\text{sheet} = \frac{mg}{\pi \cdot r^2}$		6,18E-24	N/m ² /sheet
4) Calculus of the load by m of width of sheet (rectangular sheet of span R Earth impact)			
$\frac{q}{\text{sheet}} = p \times d$		8,61E-15	N/m/sheet

We obtain so a Young's modulus of 2,66x10²⁶ Pa so <3x10⁴⁴ Pa obtained with GW and frame dragging (same order that R Weiss proposal Nobel Price lecture 10²⁰ Y steel)

5) Calculus of the vertical reaction of the support			
We suppose (defavorable approach) that all the sheet is uniformly loaded)			
$R = \frac{qd}{2}$		5,99E-09	N
6) Calculus of the Horizontal reaction of the support (f=ΔR due to space time curvature)			
$H = \frac{qd^2}{8f}$		4,24E-06	N
7) Calculus of the Tension in the membrane			
$T = \sqrt{R^2 + H^2}$		4,24E-06	N
8) Calculus of the stress in the membrane			
$\sigma = \frac{T}{d \times \ell_p}$		1,88E+20	Pa
9) Calculus of the tension strain in the membrane			
$\frac{\Delta R}{R} = \varepsilon = \frac{h_{00}}{6}$		7,07E-07	
10) Extraction of the associated Young's modulus in the membrane			
$Y = \frac{\sigma}{\varepsilon}$		Y = 2,66E+26 Pa	
		Y (R Weiss) = 2,1E+31 Pa	

Case 7 : Gravitation for the Earth with a circular membrane



Über den Spannungszustand in kreisrunden Platten mit verschwindender Biegesteifigkeit.

Von Dr. Ing. H. HENCKY in Darmstadt.

Principle: The variation of Radius of the Earth ΔR due to gravity, associated at the space time curvature is transformed in variation of curvature of an equivalent membrane of span corresponding at the quasi nul gravity at each extremity. From the deflexion f of the membrane it is possible to come back at the tension of the membrane and its Young's modulus of the space time fabric

$$f = 0,662 a \sqrt[3]{\frac{p a}{E h}} \quad (\text{Durchbiegung in Plattenmitte}).$$

Hypothesis 1 : R = 6371 km

Data	Value	Unit
Thickness sheet (Planck)	1,62E-35	m
Mass of the Earth MT	5,97E+24	kg
Poisson's ratio	1	
Radius of the Earth RT	6371	km
Diameter d of the Earth	12742	km
Gravitational contant G	6,67E-11	m ³ /kgs ²
Metric perturbation h ₀₀	1,39E-09	
Deflection f of the membrane = $\Delta R = \delta_c$	0,001477	m

Hypothesis 2 : R = 25000 km

Data	Value	Unit
Thickness sheet (Planck)	1,62E-35	m
Mass of the Earth MT	5,97E+24	kg
Poisson's ratio	1	
Radius of impact of the Earth	25000	km
Diameter d of the Earth	12742	km
Gravitational contant G	6,67E-11	m ³ /kgs ²
Metric perturbation h ₀₀	1,39E-09	
Deflection f of the membrane = $\Delta R = \delta_c$	0,001477	m

Case 7 : Gravitation for the Earth with a circular membrane

Hypothesis 1

Hypothesis 2

1) Calculus of the gravity g	12742000	6,67E-11	5,97E+24	9,82E+00 m/s ²
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1) Calculation of the number of Planck sheet concerned by the earth

Assimilating the Sphere at an equivalent cylinder of same volume

$$n = \frac{\frac{4}{3}r}{\ell_p}$$

5,24E+41 Planck sheets

1) Calculation of the number of Planck sheet concerned by the earth

Assimilating the Sphere at an equivalent cylinder of same volume

$$n = \frac{\frac{4}{3}r}{\ell_p}$$

2,06E+42 Planck sheets

2) Calculus of the mass of the Earth associated at each sheet

$$m = \frac{\text{Mass Earth}}{n}$$

1,14E-17 kg/sheet

2) Calculus of the mass of the Earth associated at each sheet

$$m = \frac{\text{Mass Earth}}{n}$$

2,90E-18 kg/sheet

3) Calculus of the weight/m² applied

$$p/\text{sheet} = \frac{mg}{\pi r^2}$$

8,77E-31 N/m²/sheet

3) Calculus of the weight/m² applied

$$p/\text{sheet} = \frac{mg}{\pi r^2}$$

1,45E-32 N/m²/sheet

4) Calculus of the Young Modulus

$$\delta_c = 0.662R \sqrt[3]{\frac{pR}{Yt}}$$

$$Y = 0.29011753 \frac{pR^4}{t(\delta_c)^3}$$

$$Y = 0.29011753 \frac{8.77 \times 10^{-31} (6371000)^4}{1.62 \times 10^{-35} (0.00177)^3}$$

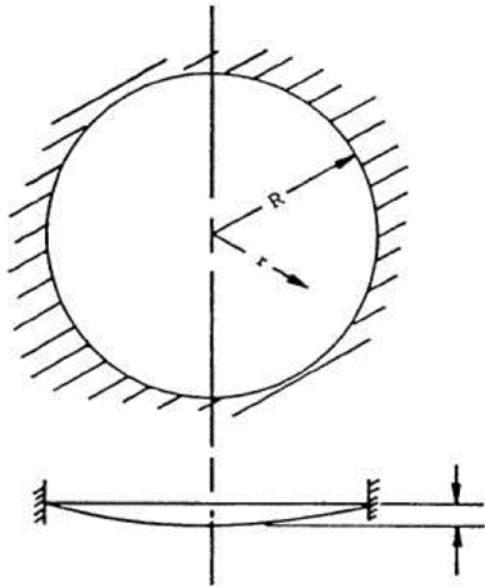
8,03E+39 N/m²

We obtain so a Young's modulus of $8,039 \times 10^{39}$ Pa $< Y < 3,15 \times 10^{40}$ so $< 3 \times 10^{44}$ Pa obtained with GW and frame dragging (same order that R Weiss proposal Nobel Price lecture 10^{20} Y steel)

$$0.29011753 \frac{1.45 \times 10^{-32} (25000000)^4}{1.62 \times 10^{-35} (0.001477)^3}$$

3,15E+40 N/m²

Case 8 : Gravitation for the Sun with a circular membrane



Über den Spannungszustand in kreisrunden Platten mit verschwindender Biegesteifigkeit.

Von Dr. Ing. H. HENCKY in Darmstadt.

Principle: The variation of Radius of the Sun ΔR due to gravity, associated at the space time curvature is transformed in variation of curvature of an equivalent membrane of span corresponding at the quasi nul gravity at each extremity. From the deflexion f of the membrane it is possible to come back at the tension of the membrane and its Young's modulus of the space time fabric

$$f = 0,662 a \sqrt[3]{\frac{p a}{E h}} \quad (\text{Durchbiegung in Plattenmitte}).$$

Hypothesis 1 : R = 696342 km

Data	Value	Unit
Thickness sheet (Planck)	1,62E-35	m
Mass of the Sun MS	1,98E+30	kg
Poisson's ratio	1	
Radius of the Sun RS	696342	km
Diameter d of the Sun	1392684	km
Gravitational contant G	6,67E-11	m ³ /kgs ²
Metric perturbation h ₀₀	4,24E-06	
Deflection f of the membrane = Δ	492	m

Hypothesis 2 : R = 10 000 000 km

Data	Value	Unit
Thickness sheet (Planck)	1,62E-35	m
Mass of the Sun MS	1,98E+30	kg
Poisson's ratio	1	
Radius of impact of the sun	10000000	km
Diameter d of the Sun	1392684	km
Gravitational contant G	6,67E-11	m ³ /kgs ²
Metric perturbation h ₀₀	4,24E-06	
Deflection f of the membrane = Δ	492	m

Case 8 : Gravitation for the Sun with a circular membrane

Hypothesis 1

1) Calculation of the number of Planck sheet concerned by the earth

Assimilating the Sphere at an equivalent cylinder of same volume

$$n = \frac{4}{3} \frac{r}{\ell_p} \quad 5,73E+43 \text{ Planck sheets}$$

2) Calculus of the mass of the Earth associated at each sheet

$$m = \frac{\text{Mass Sun}}{n} \quad 3,45E-14 \text{ kg/sheet}$$

3) Calculus of the weight/m² applied

$$p/\text{sheet} = \frac{mg}{\pi r^2} \quad 6,18E-24 \text{ N/m}^2/\text{sheet}$$

4) Calculus of the Young Modulus

$$\delta_c = 0.662R \sqrt[3]{\frac{pR}{Yt}}$$

$$Y = 0.29011753 \frac{pR^4}{t(\delta_c)^3}$$

$$Y = 0.29011753 \frac{6.18 \times 10^{-24} (696342000)^4}{1.62 \times 10^{-35} (492)^3}$$

$$2,19E+38 \text{ N/m}^2$$

Hypothesis 2

1392684 6,67E-11 1,98E+30 2,73E+08 m/s²

1) Calculation of the number of Planck sheet concerned by the earth

Assimilating the Sphere at an equivalent cylinder of same volume

$$n = \frac{4}{3} \frac{r}{\ell_p} \quad 8,23E+44 \text{ Planck sheets}$$

2) Calculus of the mass of the Earth associated at each sheet

$$m = \frac{\text{Mass Sun}}{n} \quad 2,41E-15 \text{ kg/sheet}$$

3) Calculus of the weight/m² applied

$$p/\text{sheet} = \frac{mg}{\pi r^2} \quad 2,09E-27 \text{ N/m}^2/\text{sheet}$$

We obtain so a Young's modulus of $2,9 \times 10^{38} \text{ Pa}$ $Y < 3,14 \times 10^{39}$ so $< 3 \times 10^{44} \text{ Pa}$ obtained with GW and frame dragging (same order that R Weiss proposal Nobel Price lecture 10^{20} Y steel)

$$Y = 0.29011753 \frac{2.09 \times 10^{-27} (10000000000)^4}{1.62 \times 10^{-35} (492)^3}$$

$$3,14E+39 \text{ N/m}^2$$

4.3) Spatial models

Case 9: Deflection of light rays/ gravitation of the sun from the curvature of space-time

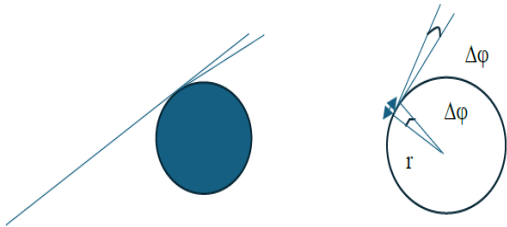
Variation angle for a light beam passing near the earth following the general relativity (see figure 127 below)

$$\Delta\phi_{\text{exact beam light}} = \frac{4GM}{rc^2} = 2h_{00} = 2.784 \times 10^{-9} \text{rad}$$

We obtain from the Schwarzschild approach:

$$\Delta\phi_{\text{approach schwarzschild}} = \frac{h_{00}}{6} = 2.32 \times 10^{-10} \text{rad}$$

$$\Delta s = r \Delta\phi = \frac{4GM}{rc^2} = 2h_{00}$$



Principle: The variation of Radius of the Sun ΔR due to gravity, associated at the space time curvature is transformed in variation of length of a sphere of space time fabric with an internal pression. From the deflexion displacement u_r of the sphere it is possible to come back at Young's modulus of the space time fabric

Data	Value	Unit
Thickness sheet (Planck)	1,62E-35	m
Mass of the Sun MS	1,98E+30	kg
Poisson's ratio	1	
Radius of the Sun a = r	696342	km
Diameter d of the Sun	1392684	km
Gravitational constant G	6,67E-11	m ³ /kgs ²
Metric perturbation h_{00}	4,24E-06	
Internal pression gravitation	6,00E+14	Pa
Displacement u_r	3,09E+03	m
Deflection f of the membrane = ΔR	492	m

Calculations on space-time curvature within the Earth and Sun

Wm. Robert Johnston

last updated 3 November 2008

$$\begin{cases} \phi = \frac{GM}{r} \\ g_{00} = \eta_{00} + h_{00} = 1 + \frac{2\phi}{c^2} \end{cases}$$

$$h_{00} \approx 2 \frac{GM}{rc^2} = \frac{2\phi}{c^2}$$

General relativity result in weak field for the sun

$$h_{00} = 2 \times \frac{6.67430 \times 10^{-11} \times 1.9891 \times 10^{30}}{695990000 \times 299792458 \times 299792458} = 4.244 \times 10^{-6}$$

$$\Delta R = \frac{1}{6} R_s = \frac{h_{00} R c^2}{6 c^2} = \frac{h_{00} R}{6} = \frac{4.244 \times 10^{-6} \times 695990000}{6} = 492 \text{m}$$

$$\frac{\Delta R}{R} = \varepsilon = \frac{h_{00}}{6} = \frac{4.244 \times 10^{-6}}{6} = 7.0733 \times 10^{-7}$$

$$u_r = \Delta L = 2\pi R \frac{h_{00}}{6} = \pi \frac{h_{00}}{3} R = 3093.19 \text{m}$$

Case 9: Deflection of light rays by the gravitation of the sun from the curvature of space-time

VIII.5.7.2 Approach by the elastic sphere theory

The data about the sphere with an internal pressure is given at the figure 129.

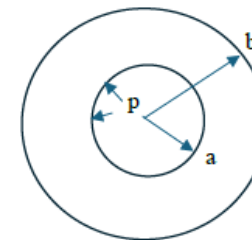


Figure 129: Notation for a sphere loading by an internal pressure

In elasticity, we have the differential equation [308]:

$$\frac{d^2 u_r}{dr^2} + \frac{2}{r} \frac{du_r}{dr} = \frac{2}{r^2} u_r$$

So, the beginning of the equation follow the form of:

$$\Delta \phi = \frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \times \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \times \frac{\partial^2 \phi}{\partial \theta^2} + \frac{1}{r^2 \tan \theta} \times \frac{\partial \phi}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \times \frac{\partial^2 \phi}{\partial \varphi^2}$$

$$\Delta u_r = \frac{2}{r^2} u_r$$

So, it's a development of the Poisson's equation that is this modified by a distribution f that is not constant. The solution is on the form:

$$u_r = C_1 r + \frac{C_2}{r^2}$$

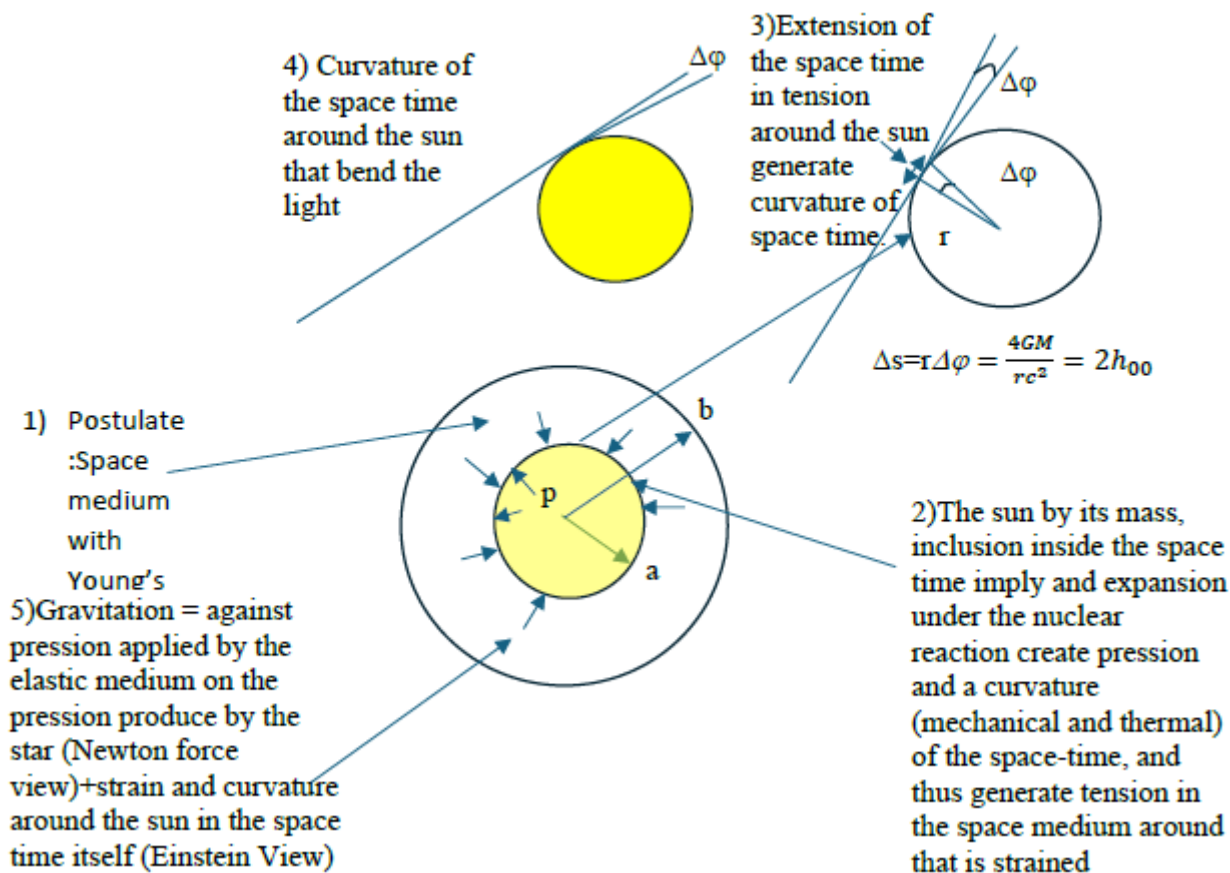
With for the 2 constants:

$$C_2 = \frac{1 + \nu}{2(1 - 2\nu)} b^3 C_1$$

$$C_1 = \frac{(1 - 2\nu)}{E} \frac{a^3}{b^3 - a^3} p$$

We know the displacement u_r , so we can extract the Young's modulus $E = Y$:

$$E = Y = \frac{a^3 p}{u_r (b^3 - a^3)} \left[(1 - 2\nu)r + (1 + \nu) \frac{b^3}{2r^2} \right]$$



Case 9: Deflection of light rays by the gravitation of the sun from the curvature of space-time

1) Research of the gravity influence of the sun on the space time (Equivalent thickness of the sphere)

	R (m)	G	Ms	g
	696342000	6,67E-11	1,98E+30	272,54
	1000000000	6,67E-11	1,98E+30	132,15
	2000000000	6,67E-11	1,98E+30	33,04
	3000000000	6,67E-11	1,98E+30	14,68
	4000000000	6,67E-11	1,98E+30	8,26
	5000000000	6,67E-11	1,98E+30	5,29
	6000000000	6,67E-11	1,98E+30	3,67
	7000000000	6,67E-11	1,98E+30	2,70
	8000000000	6,67E-11	1,98E+30	2,06
	9000000000	6,67E-11	1,98E+30	1,63
b =	10000000000	6,67E-11	1,98E+30	1,32

2) Pression exerced by the gravitation

$$P = \frac{gM}{4\pi r^2}$$

8,86E+13 Pa

pression data

6,00E+14 Pa

max

6,00E+14 Pa

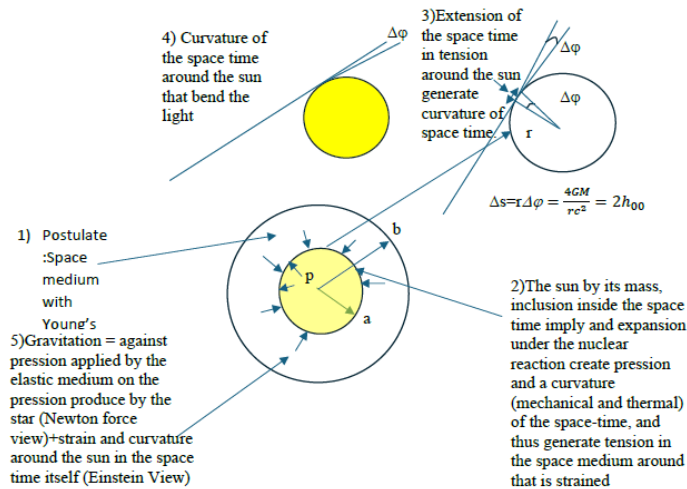
2) Calculus of the Young's modulus

$$E = Y = \frac{a^3 P}{u_r (b^3 - a^3)} \left[(1 - 2\nu)r + (1 + \nu) \frac{b^3}{2r^2} \right]$$

Y = 1,35073E+20 Pa

We obtain so a Young's modulus of $1,35 \times 10^{20}$ Pa so $\lll 3 \times 10^{44}$ Pa obtained with GW and frame dragging (same order that R Weiss proposal Nobel Price lecture 10^{20} Y steel)

Case 10 : Gravitation of the Earth from the curvature of space-time



Principle: The variation of Radius of the Earth ΔR due to gravity, associated at the space time curvature is transformed in variation of length of a sphere of space time fabric with an internal pressure. From the deflexion displacement u_r of the sphere it is possible to come back at Young's modulus of the space time fabric

Data	Value	Unit
Thickness sheet (Planck)	1,62E-35	m
Mass of the Earth M_S	5,97E+24	kg
Poisson's ratio	1	
Radius of the Earth $a = r$	6371	km
Diameter d of the Earth	12742	km
Gravitational constant G	6,67E-11	m^3/kgs^2
Metric perturbation h_{00}	1,39E-09	
Internal pressure gravitation	3,60E+11	Pa
Displacement u_r	9,28E-03	m
Deflection f of the membrane = ΔR	0,00147792	m

$$\epsilon = \frac{(2\pi R + 2\pi\Delta R) - 2\pi R}{2\pi R} = \frac{(\Delta R)}{R} = \frac{h_{00}}{6}$$

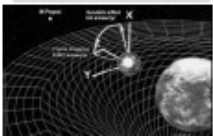
$$\frac{\Delta R}{R} = \epsilon = \frac{h_{00}}{6} = \frac{1.39222 \times 10^{-9}}{6} = 2.320 \times 10^{-10}$$

$$u_r = \Delta L = 2\pi R \frac{h_{00}}{6} = \pi \frac{h_{00}}{3} R = 0.00928m$$

Case 10 : Gravitation of the Earth from the curvature of space-time

1) Research of the gravity influence of the sun on the space time (Equivalent thickness of the sphere)				2) Pression exerted by the gravitation	
	R (m)	G	Ms	g	
	6371000	6,67E-11	5,97E+24	9,82	$P = \frac{gM}{4\pi r^2}$
	7000000	6,67E-11	5,97E+24	8,13	1,15E+11 Pa
	8000000	6,67E-11	5,97E+24	6,23	
	9000000	6,67E-11	5,97E+24	4,92	pression data
	10000000	6,67E-11	5,97E+24	3,99	3,60E+11 Pa
	12000000	6,67E-11	5,97E+24	2,77	max
	14000000	6,67E-11	5,97E+24	2,03	3,60E+11 Pa
	16000000	6,67E-11	5,97E+24	1,56	
	18000000	6,67E-11	5,97E+24	1,23	2) Calculus of the Young's modulus
	20000000	6,67E-11	5,97E+24	1,00	$E = Y = \frac{a^3 P}{u_r (b^3 - a^3)} \left[(1 - 2\nu)r + (1 + \nu) \frac{b^3}{2r^2} \right]$
b =	25000000	6,67E-11	5,97E+24	0,64	Y = 2,47151E+20 Pa

We obtain so a Young's modulus of $2,47 \times 10^{20}$ Pa so $\lll 3 \times 10^{44}$ Pa obtained with GW and frame dragging (same order that R Weiss proposal Nobel Price lecture 10^{20} Y steel)

Case of general relativity	Type of parameter measured or calculated following general relativity	Theoretical results of the general relativity	Measured results	Mechanical model of the Planck sheet associated with an elastic truss	Young's modulus used for the calculation (Pa) (time aspect)
Calculation on space-time curvature within the Earth (Weak Gravitational field)	Augmentation of the Earth radius due to curvature $\Delta R = 1.477 \text{ mm}$	$\Delta R = \frac{1}{6} R_S = \frac{h_{00} R}{6} = \frac{GM}{3c^2}$	Not relevant	Membrane loaded perpendicularly at its plane	3.21×10^{30}
Calculation on space-time curvature within the Sun (Weak Gravitational field)	Augmentation of the Sun radius due to curvature $\Delta R = 492 \text{ m}$ $\Delta\phi_{\text{beam light measured}} = 2.784 \times 10^{-9} \text{ rad}$ $\Delta\phi_{\text{schwarchild}} = 2.32 \times 10^{-10} \text{ rad}$	$\Delta R = \frac{1}{6} R_S = \frac{h_{00} R}{6} = \frac{GM}{3c^2}$ $\Delta\phi_{\text{exact beam light}} = \frac{4GM}{rc^2} = 2h_{00}$ $\Delta\phi_{\text{approach schwarchild}} = \frac{h_{00}}{6}$	Deviation of the sun beam light	Membrane loaded perpendicularly at its plane	2.658×10^{28}
Calculation on space-time curvature within the Earth (Weak Gravitational field)	Calculation on space-time curvature within the Earth $\epsilon = 2.320 \times 10^{-10}$ $u_r = 0.00928 \text{ m}$	$\epsilon = \frac{h_{00}}{6}$ $u_r = \Delta L = 2\pi R \frac{h_{00}}{6} = \pi \frac{h_{00}}{3} R$	Not relevant	Sphere with inside pression	2.471×10^{29}
Calculation on space-time curvature within the Sun	Calculation on space-time curvature within the Sun $\epsilon = 7.073 \times 10^{-7}$ $u_r = 3093.19 \text{ m}$	$\epsilon = \frac{h_{00}}{6}$ $u_r = \Delta L = 2\pi R \frac{h_{00}}{6} = \pi \frac{h_{00}}{3} R$	Not relevant	Sphere with inside pression	1.35×10^{29}
Geodetic effect created by the Earth on the space-time (Weak Gravitational field)	Geodetic angle measured on Earth at $r = 6700 \text{ km}$ by gravity prob B 	$\Omega = \frac{3GM}{2c^2 R^3} (R \times v) + \frac{GI}{c^2 R^3} \left[\frac{3R}{R^2} (\omega \cdot R) - d \right]$ calculated with an equivalent membrane of deflection f and span of gravity Earth influence: $f = \Delta R = \frac{1}{6} R_S = \frac{h_{00} R c^2}{6c^2} = \frac{h_{00} R}{6} = 0.001477 \text{ m}$	6600 milliarc second/year (space estimation)	6600 milliarcsecond/year Model membrane loaded perpendicularly at its plane Influence areal 41000 km (0.02g)	3.96×10^{32}

Can we find an explanation at the different values of the Young's modulus?

- We have two families of values for the Young's modulus of the space time
- In the plane: associated at h_{ij} (space) component of the metric perturbation

$$10^{38} < Y < 10^{44} \text{ Pa}$$

$$10^{-20} < \varepsilon \text{ compression/traction} < 10^{-21}$$

- Out of the plane: associated at h_{00} (time) component of the metric perturbation

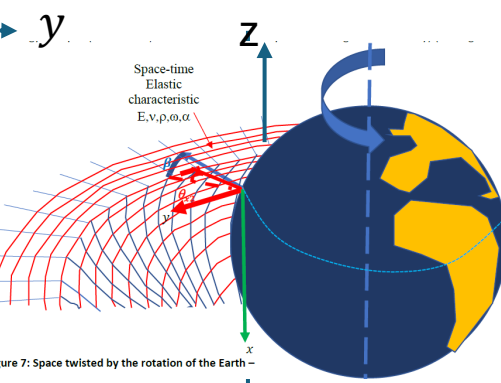
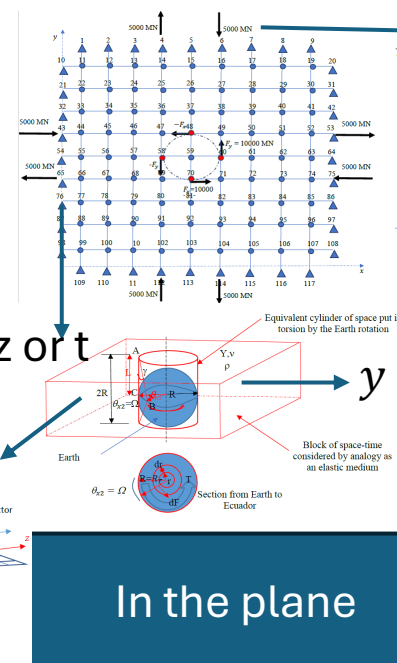
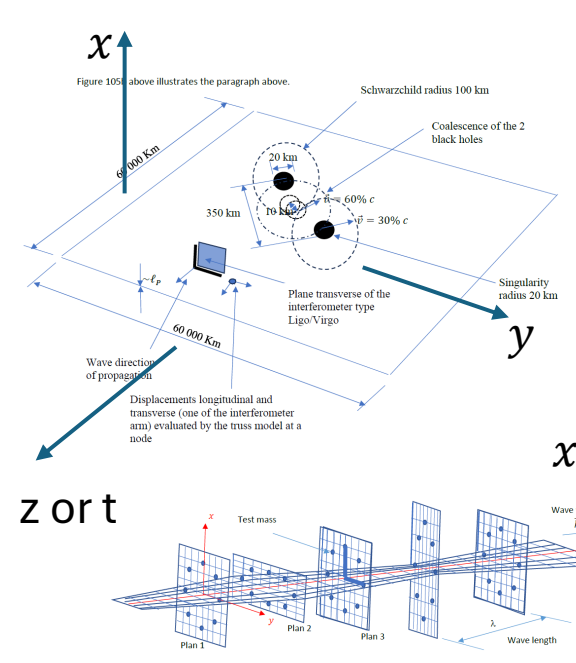
$$10^{20} < Y < 10^{40} \text{ Pa}$$

$$10^{-7} < \varepsilon \text{ compression/traction} < 10^{-10}$$

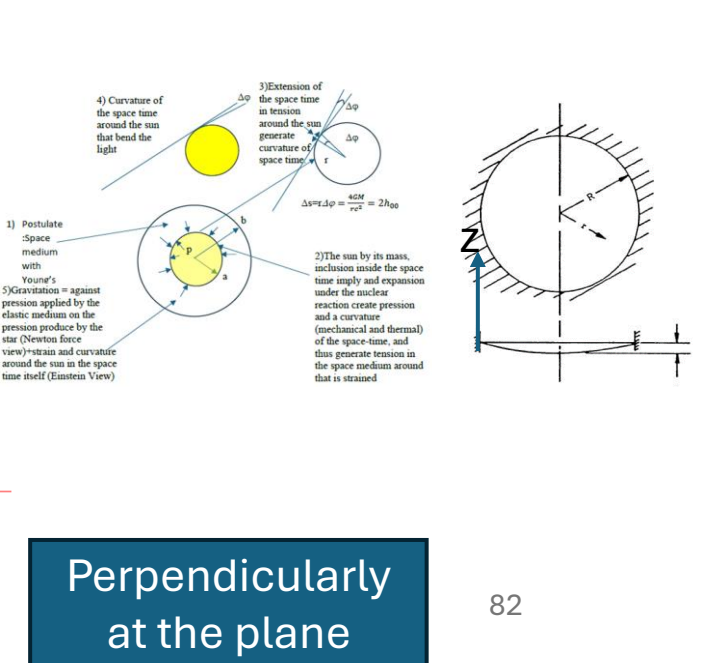
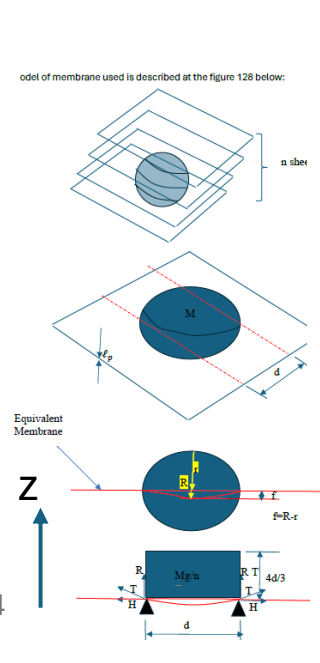
Synthesis of the model's data and results

The range of the Young's modulus is clearly out of the 10^{113} Pa !
 Our values are near R Weiss proposal 10^{31} Pa

General Relativity event	Gravitation	Case studied	Strain	Type	Strain values	Unit	Mechanical model	Type of loading	Y (Pa)	Direction
GW150914	Weak	Black hole coalescence 1	$h_{ij}(x,y)$	ϵ	1,00E-21	-	Truss in torsion	in plane	1,00E+44	x or y
GW150914	Weak	Black hole coalescence 2	$h_{ij}(x,y)$	ϵ	1,00E-21	-	Truss in torsion	in plane	1,00E+44	x or y
GW170817	Weak	Neutron star coalescence	$h_{ij}(x,y)$	ϵ	1,00E-20	-	Truss in torsion	in plane	1,00E+44	x or y
NASA example	Strong	Frame dragging Neutron star	$h_{0i};h_{j0}$	θ	6,37E-10	rad/s	Cylinder in torsion	in plane	7,70E+44	t, x or y or z
Gravity prob B	Weak	Frame dragging Earth	$h_{0i};h_{j0}$	θ	4,00E-15	rad/s	Cylinder in torsion	in plane	4,73E+38	t, x or y or z
Gravity prob B	Weak	Frame dragging Earth	$h_{0i};h_{j0}$	θ	4,00E-15	rad/s	Truss in torsion	in plane	3,00E+44	t, x or y or z
Gravity prob B	Weak	Geodetic Earth	$h_{0i};h_{j0}$	β	1,00E-12	rad/s	Rectangular membrane uniformly loaded (max)	Perpendicular at the plane	3,95E+32	t, or y or z
Gravity prob B	Weak	Geodetic Earth	$h_{0i};h_{j0}$	β	1,00E-12	rad/s	Rectangular membrane uniformly loaded (repartition load on all the membrane)	Perpendicular at the plane	2,80E+31	t, or y or z
Newton/GR	Weak	Earth Gravitation	h_{00}	ϵ	2,32E-10	-	Rectangular membrane uniformly loaded (max)	Perpendicular at the plane	3,21E+30	tt
Eddington eclipse	Weak	Sun Gravitation	h_{00}	ϵ	7,07E-07	-	Rectangular membrane uniformly loaded (max)	Perpendicular at the plane	2,66E+26	tt
Newton/GR	Weak	Earth Gravitation	h_{00}	ϵ	2,32E-10	-	Sphere with internal pression	Perpendicular at the plane	2,47E+20	tt
Eddington eclipse	Weak	Sun Gravitation	h_{00}	ϵ	7,07E-07	-	Sphere with internal pression	Perpendicular at the plane	1,35E+20	tt
Newton/GR	Weak	Earth Gravitation	h_{00}	ϵ	2,32E-10	-	Circular membrane (R=R Earth)	Perpendicular at the plane	8,03E+39	tt
Eddington eclipse	Weak	Sun Gravitation	h_{00}	ϵ	7,07E-07	-	Circular membrane (R=R Sun)	Perpendicular at the plane	2,19E+38	tt
Newton/GR	Weak	Earth Gravitation	h_{00}	ϵ	2,32E-10	-	Circular membrane (R=R Earth impact)	Perpendicular at the plane	3,15E+40	tt
Eddington eclipse	Weak	Sun Gravitation	h_{00}	ϵ	7,07E-07	-	Circular membrane (R=R Sun impact)	Perpendicular at the plane	3,14E+39	tt



Mixte in the plane and out of the plane



In the plane

Perpendicularly at the plane

5) Possible unification of all the models in the plane and perpendicular at the plane via the interval ds^2 in quasi flat metric

5.1 Interval of special relativity

Considering that:

- We are in a weak gravitational field
- The perturbation of the metric $h_{\mu\nu}$ is very small
- The gravitational wave speed is exactly the speed of light (see GW180817)
- The behaviour of space can be disconnected in xy and z (plane and out of plane)

Postulate:

We can consider the interval of Minkowski in first approach (quasi flat metric)

The type is light $ds^2=0 = c^2 dt^2 = dx^2 + dy^2 + dz^2$

Same type of hypothesis that :

Gravitation in the surface tension model of spacetime

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5.2 Mechanic transposition

4. Hypersurface Continuum Mechanics

It would be useful to apply traditional continuum mechanics to describe hypersurface geometry with surface tension. Continuum mechanics offers tools based in Riemannian geometry for relating stress energy to reconfiguration and surface evolution that satisfy conditions of covariance and uniqueness [8]. However, one cannot apply these tools without a change of coordinates. Spacetime geometry is pseudo-Riemannian and, in fact, hyperbolic. According to special relativity,

$$-c^2 d\tau^2 = -c^2 dt^2 + dx^{1^2} + dx^{2^2} + dx^{3^2}$$

where dx^j are spatial coordinates with index, j , running from 1 to 3, and t is the coordinate of time. This pseudo-Riemannian geometry can be transformed to a Riemannian geometry by rewriting the equation above in complex coordinates, as given by

$$d\tau'^2 = dx^{0^2} + dx^{1^2} + dx^{2^2} + dx^{3^2}$$

where

$$x^0 = ict$$

$$\tau' = ic\tau$$

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Mechanical transposition of the interval

Flat metric

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

Principle n°5

$$Y = \rho c^2$$
$$\mu = \frac{Y}{2(1 + \nu)} = \rho c^2$$

Mechanical transposition of the interval

$$ds^2 = - \left(\frac{Y_t}{\rho} \right) dt^2 + dx^2 + dy^2 + dz^2$$

As these quantities should be equivalent, it should be possible to transpose in terms of Y_s / ρ these terms

Mechanical transposition of the interval

$$\varepsilon_{xx} = \frac{u(x+dx) - u(x)}{dx} = \frac{du}{dx}$$

The interval then becomes:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right) dt^2 + \left(\frac{du_x}{\varepsilon_{xx}}\right)^2 + \left(\frac{du_y}{\varepsilon_{yy}}\right)^2 + \left(\frac{du_z}{\varepsilon_{zz}}\right)^2$$

Considering Hooke's law:

$$\sigma_{xx} = \varepsilon_{xx} Y_x$$

By replacing the strains with their expressions as a function of the stresses, the interval becomes:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right) dt^2 + \left(\frac{du_x}{\sigma_{xx}} Y_x\right)^2 + \left(\frac{du_y}{\sigma_{yy}} Y_y\right)^2 + \left(\frac{du_z}{\sigma_{zz}} Y_z\right)^2$$

We have shown in [26] and [94] that the stress tensor and thus the normal stresses can be expressed as a function of velocities v_i and v_j as follows:

$$\sigma_{ij} = \rho v_i v_j$$

By substituting the normal stresses for their density and velocity expressions in the interval ρ , we obtain:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right) dt^2 + \left(\frac{du_x}{\rho v_x^2} Y_x\right)^2 + \left(\frac{du_y}{\rho v_y^2} Y_y\right)^2 + \left(\frac{du_z}{\rho v_z^2} Y_z\right)^2$$

Substituting one of the velocities in each term $\frac{du_i}{dt}$ for the interval, we obtain for the interval:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right) dt^2 + \left(\frac{du_x}{\rho dx v_x} Y_x dt\right)^2 + \left(\frac{du_y}{\rho dy v_y} Y_y dt\right)^2 + \left(\frac{du_z}{\rho dz v_z} Y_z dt\right)^2$$

To have an expression similar to the one we have for the time component, we factor the ratio $\frac{v_i}{\rho} dt^2$ and replace $\frac{du_i}{dx}$ by ε_{ii} in the interval:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right) dt^2 + \left(\frac{\varepsilon_{xx}^2}{\rho v_x^2} Y_x\right) \left(\frac{Y_x}{\rho}\right) dt^2 + \left(\frac{\varepsilon_{yy}^2}{\rho v_y^2} Y_y\right) \left(\frac{Y_y}{\rho}\right) dt^2 + \left(\frac{\varepsilon_{zz}^2}{\rho v_z^2} Y_z\right) \left(\frac{Y_z}{\rho}\right) dt^2$$

Using Hooke's law again:

$$\sigma_{xx} = \varepsilon_{xx} Y_x$$

We obtain so:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right) dt^2 + \left(\frac{\sigma_{xx}}{\rho v_x^2} \varepsilon_{xx}\right) \left(\frac{Y_x}{\rho}\right) dt^2 + \left(\frac{\sigma_{yy}}{\rho v_y^2} \varepsilon_{yy}\right) \left(\frac{Y_y}{\rho}\right) dt^2 + \left(\frac{\sigma_{zz}}{\rho v_z^2} \varepsilon_{zz}\right) \left(\frac{Y_z}{\rho}\right) dt^2$$

Again using the relationship between Young's modulus and density and velocities.

$$ds^2 = -\left(\frac{Y_t}{\rho}\right) dt^2 + \left(\frac{\sigma_{xx}}{Y_x} \varepsilon_{xx}\right) \left(\frac{Y_x}{\rho}\right) dt^2 + \left(\frac{\sigma_{yy}}{Y_y} \varepsilon_{yy}\right) \left(\frac{Y_y}{\rho}\right) dt^2 + \left(\frac{\sigma_{zz}}{Y_z} \varepsilon_{zz}\right) \left(\frac{Y_z}{\rho}\right) dt^2$$

Using Hooke's law again $\varepsilon_{ii} = \frac{\sigma_{ii}}{Y_i}$:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right) dt^2 + \varepsilon_{xx}^2 \left(\frac{Y_x}{\rho}\right) dt^2 + \varepsilon_{yy}^2 \left(\frac{Y_y}{\rho}\right) dt^2 + \varepsilon_{zz}^2 \left(\frac{Y_z}{\rho}\right) dt^2$$

The equation to the dimensions is also checked:

$$\frac{\frac{kgm}{m^2 s^2}}{\frac{kg}{m^3}} s^2 = m^2$$

Or:

$$\rho \left(\frac{ds}{dt}\right)^2 = \rho(v)^2 = -Y_t + \varepsilon_{xx}^2 Y_x + \varepsilon_{yy}^2 Y_y + \varepsilon_{zz}^2 Y_z$$

Taking into account the approach of T Tenev [47] recalled at the beginning of the chapter on the one hand, and to be consistent with spatial terms, there is no reason why there should not be time-related distortions.

We therefore postulate a term in front of the temporal term ε_{tt} .

$$\rho \left(\frac{ds}{dt}\right)^2 = \rho(v)^2 = -\varepsilon_{tt}^2 Y_t + \varepsilon_{xx}^2 Y_x + \varepsilon_{yy}^2 Y_y + \varepsilon_{zz}^2 Y_z$$

Law in strain²

We obtain so a law that link the several strains issued of space and time with the several Young's modulus Y_t and Y_s

Mechanical transposition of the interval

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<https://doi.org/10.1007/s12043-020-01954-5>

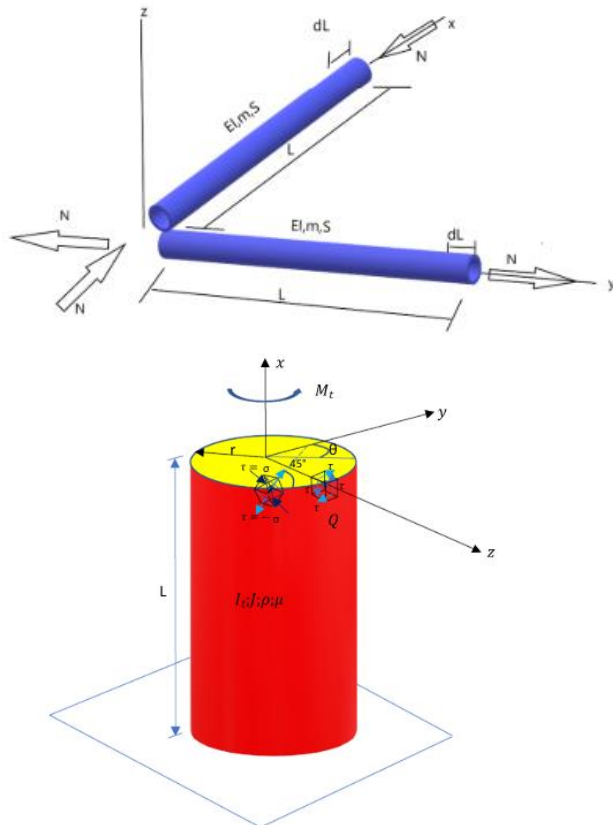
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Mechanical conversion of the gravitational Einstein's constant κ

IZABEL DAVID

Pramana – J. Phys. (2020) 94:119



This expression is therefore in a way the law of elasticity of special relativity for a flat (or near-flat) space.

Note 1

This expression is in squared deformations as in the case of the linearized version of the Einstein field that we obtained from the analysis of the interferometers or the torsional cylinder [26] respectively:

$$\begin{bmatrix} \frac{1}{L^2} & 0 \\ 0 & \frac{1}{L^2} \end{bmatrix} \begin{bmatrix} (\varepsilon_{xx})^2 & 0 \\ 0 & (\varepsilon_{yy})^2 \end{bmatrix} = \frac{8\pi G}{c^4} \begin{bmatrix} T_{xx} & 0 \\ 0 & T_{yy} \end{bmatrix}$$

$$\frac{1}{L^2} \gamma^2 = \frac{16\pi G}{c^4} T$$

Law in strain²

Note 2

So, we have a relationship between Young's time-related modules and space-related modules.

$$\rho \left(\frac{ds}{dt} \right)^2 = Y_{space-time} = \rho(c)^2 = -\varepsilon_{tt}^2 Y_t + \varepsilon_{xx}^2 Y_x + \varepsilon_{yy}^2 Y_y + \varepsilon_{zz}^2 Y_z$$

We recall the expression of an energy in the mechanics of continuous media:

$$U = \frac{1}{2} \sigma^{ij} \varepsilon_{ij} = \frac{1}{2} C^{ijkl} \varepsilon_{ij} \varepsilon_{kl}$$

This law is a generalized strain energy of the space fabric

So, we have in a plane system with low deformation.

$$U = \frac{1}{2} \rho(c)^2 = -\frac{1}{2} \varepsilon_{tt}^2 Y_t + \frac{1}{2} \varepsilon_{xx}^2 Y_x + \frac{1}{2} \varepsilon_{yy}^2 Y_y + \frac{1}{2} \varepsilon_{zz}^2 Y_z$$

So, Young's modulus equation above is really just a kind of energy equation of a space-time virtual lattice.

5.3 Case of the light type interval and associated equation

For the space, If we are in flat (or quasi flat) space $h_{\mu\nu}$ is very small. In the case of gravitational wave $h=10^{-21}$.

$$g_{ij} = \eta_{ij} + h_{ij} \approx 1 + h$$

With $\phi = \frac{GM}{R}$

So, around the sun h_{00} is proportional at $\frac{2\phi}{c^2} \approx 10^{-6}$ or around the Earth 10^{-9} . So, we have

$$g_{\mu\nu} \cong \eta_{\mu\nu}$$

So, Pythagoras applied, and we have:

$$ds_{space}^2 \approx dx^2 + dy^2 + dz^2$$

In addition, we know with [44] that the gravitational wave travels at the speed of light. Indeed in [44] the gravitational wave and the electromagnetic wave travels through space at c . They arrive at the same time on the Earth.

So, we have:

$$c^2 dt^2 = ds_{space}^2 \approx dx^2 + dy^2 + dz^2$$

So, if we consider a particular light-like space-time (gravitational waves move at the speed of light and deformations materialize in space are very small (sun, Earth). The Pythagoras length is so equal at the time traveled by the light in this space quasi flat

If we consider that we are in a plane receiving distortions and, in a direction x , we have interval:

$$ds^2 = 0$$

So, we have in the direction x (it would be the same in the direction y):

$$0 = -c^2 dt^2 + dx^2$$

$$\text{So: } c^2 dt^2 = dx^2$$

In [205] the author does the same approach to study the membrane. He confirms that is possible in weak field and quasi flat metric.

And therefore, the content of the expression in interval mechanics:

$$0 = -\epsilon_{tt}^2 Y_t + \epsilon_{xx}^2 Y_x$$

In this case, there is symmetry between the spatial and temporal distortions.

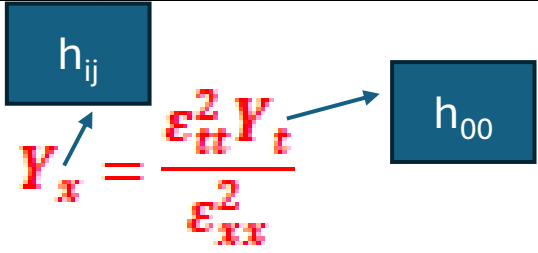
Let be the following relationship between the time-related Young's modulus and the space-related Young's modulus:

$$Y_x = \frac{\epsilon_{tt}^2 Y_t}{\epsilon_{xx}^2}$$

We obtain so what we want: a law that allows to connect the space and time Young's modulus

Test of the law about Young's modulus following space and time - Application of the Young's modulus connection formula

										$Y_x = \frac{\epsilon_{tt}^2 Y_t}{\epsilon_{xx}^2}$
Sun Data										
Eddington eclipse	Weak	Sun Gravitation	h00	ϵ	7,07E-07	-	Rectangular membrane uniformly loaded (max)	Perpendicular at the plane	2,66E+26	1,33E+56
Eddington eclipse	Weak	Sun Gravitation	h00	ϵ	7,07E-07	-	Sphere with internal pression	Perpendicular at the plane	1,35E+20	6,75E+49
Eddington eclipse	Weak	Sun Gravitation	h00	ϵ	7,07E-07	-	Circular membrane (R=R Sun)	Perpendicular at the plane	2,19E+38	1,09E+68
Eddington eclipse	Weak	Sun Gravitation	h00	ϵ	7,07E-07	-	Circular membrane (R=R Sun impact)	Perpendicular at the plane	3,14E+39	1,57E+69
Earth Data										$Y_x = \frac{\epsilon_{tt}^2 Y_t}{\epsilon_{xx}^2}$
Newton/GR	Weak	Earth Gravitation	h00	ϵ	2,32E-10	-	Rectangular membrane uniformly loaded (max)	Perpendicular at the plane	3,21E+30	1,73E+53
Newton/GR	Weak	Earth Gravitation	h00	ϵ	2,32E-10	-	Sphere with internal pression	Perpendicular at the plane	2,47E+20	1,33E+43
Newton/GR	Weak	Earth Gravitation	h00	ϵ	2,32E-10	-	Circular membrane (R=R Earth)	Perpendicular at the plane	8,03E+39	4,32E+62
Newton/GR	Weak	Earth Gravitation	h00	ϵ	2,32E-10	-	Circular membrane (R=R Earth impact)	Perpendicular at the plane	3,15E+40	1,70E+63



Only the sphere model (spatial approach) respect the precedent equation even if the other models gives an order of magnitude in the range of 10^{20} 10^{40} Pa

$$Y_y = \frac{\epsilon_{tt}^2 Y_t}{\epsilon_{yy}}$$

GR event	Gravitation	Case studied	Strain	Type	Strain values	Unit	Mechanical model	Type of loading	Y (Pa)	Direction
GW150914	Weak	Black hole coalescence 1	hij	ϵ	1,00E-21	-	Truss in torsion	in plane	1,00E+44	xory
GW150914	Weak	Black hole coalescence 2	hij	ϵ	1,00E-21	-	Truss in torsion	in plane	1,00E+44	xory
GW170817	Weak	Neutron star coalescence	hij	ϵ	1,00E-20	-	Truss in torsion	in plane	1,00E+44	xory
NASA example	Strong	Frame dragging Neutron star	hij	θ	6,37E-10	rad/s	Cylinder in torsion	in plane	7,70E+44	xory
Gravity prob B	Weak	Frame dragging Earth	hij	θ	4,00E-15	rad/s	Cylinder in torsion	in plane	4,73E+38	xory
Gravity prob B	Weak	Frame dragging Earth	hij	θ	4,00E-15	rad/s	Truss in torsion	in plane	3,00E+44	xory

We therefore use the formulation demonstrated in the previous chapter:

$$Y_x = \frac{\epsilon_{tt}^2 Y_t}{\epsilon_{xx}^2}$$

So with ϵ_{tt} and Y_t issued of table 23 and ϵ_{xx} issued of table 22 we obtain:

With data from the Earth and gravitational waves via our Chapter VIII elastic models, we get:

$$Y_{x,space} = \frac{(2.32 \times 10^{-10})^2 \times 2.471 \times 10^{20}}{(1.33 \times 10^{-19})^2} = 7.51 \times 10^{38} Pa$$

$$Y_{x,space} = \frac{(2.32 \times 10^{-10})^2 \times 2.471 \times 10^{20}}{(3.32 \times 10^{-20})^2} = 1.20 \times 10^{40} Pa$$

With the data from the Earth and the gravitational waves measured, we get:

$$Y_{x,space} = \frac{(2.32 \times 10^{-10})^2 \times 2.471 \times 10^{20}}{(3.32 \times 10^{-21})^2} = 1.20 \times 10^{42} Pa$$

With the data of the sun and gravitational waves from the elastic models of Chapter VIII we obtain:

$$Y_{x,space} = \frac{(7.073 \times 10^{-7})^2 \times 1.35 \times 10^{20}}{(1.33 \times 10^{-19})^2} = 3.81 \times 10^{45} Pa$$

$$Y_{x,space} = \frac{(7.073 \times 10^{-7})^2 \times 1.35 \times 10^{20}}{(3.32 \times 10^{-20})^2} = 6.12 \times 10^{46} Pa$$

With the data from the sun and the gravitational waves measured, we get:

$$Y_{x,space} = \frac{(7.073 \times 10^{-7})^2 \times 1.35 \times 10^{20}}{(1 \times 10^{-21})^2} = 6.75 \times 10^{49} Pa$$

And we obtained empirically in Chapter VIII for Young's modulus of space:

$$Y_{x,space} = Y_{y,space} = 3 \times 10^{44} Pa$$

We have therefore found a mechanical expression that allows us to connect the different Young's moduli characteristic of the transverse anisotropy of the elastic spatio-temporal medium.

5.4 Test of the equation basing on the previous model unifying the different Young's modulus obtained

The sphere spatial model is the most adapted to satisfy these equations

- In the plane: associated at h_{ij} (space) component of the metric perturbation

$$10^{38} < Y < 10^{44} Pa$$

$$10^{-20} < \epsilon \text{ compression/traction} < 10^{-21}$$

- Out of the plane: associated at h_{00} (time) component of the metric perturbation

$$10^{20} < Y < 10^{40} Pa$$

$$10^{-7} < \epsilon \text{ compression/traction} < 10^{-10}$$

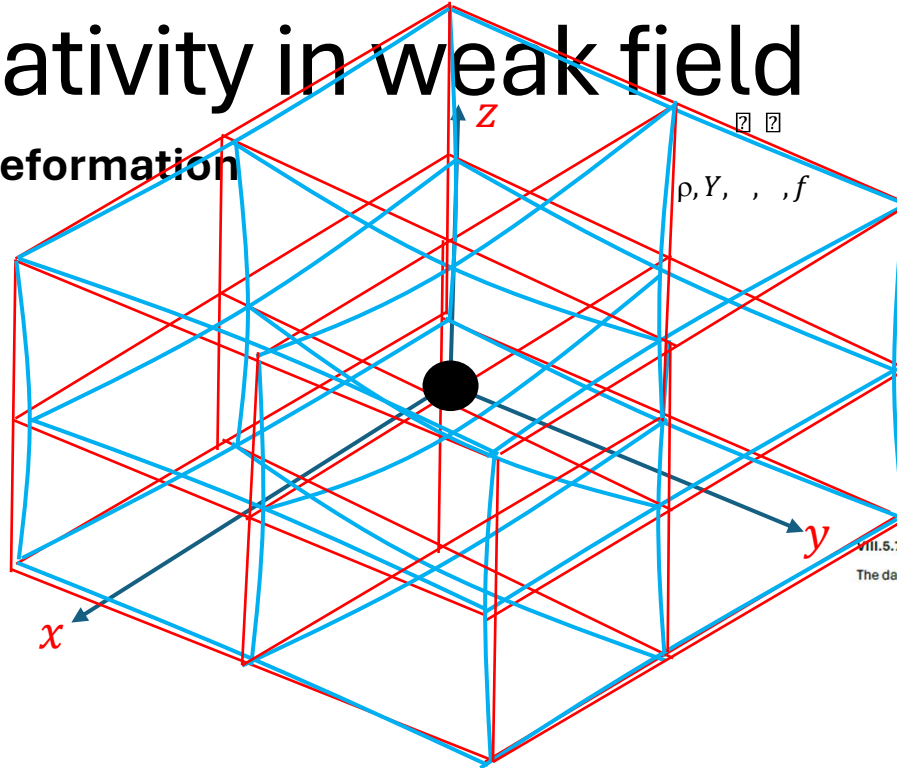
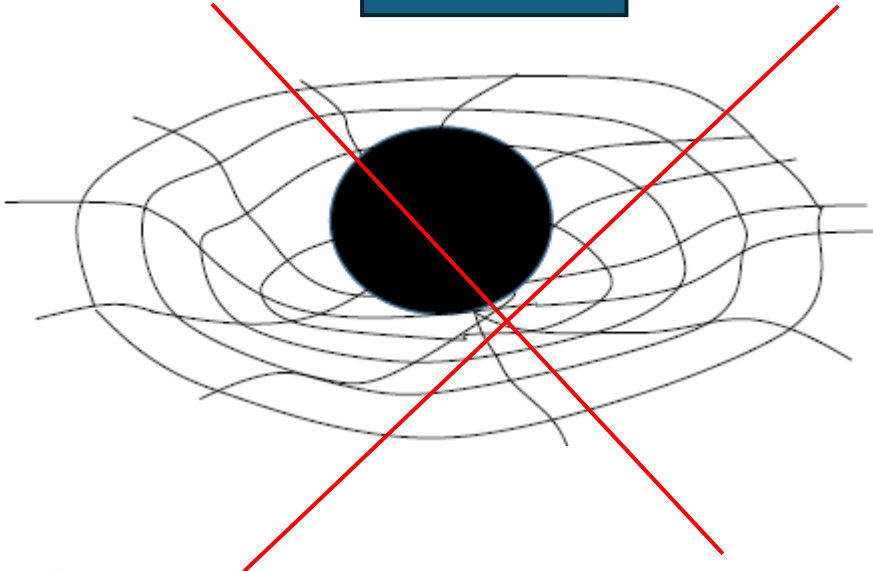
$$Y_x = \frac{\epsilon_{tt}^2 Y_t}{\epsilon_{xx}^2} \quad Y_y = \frac{\epsilon_{tt}^2 Y_t}{\epsilon_{yy}^2}$$

6) Come back of the analogy in
direction of physics – potential
consequences

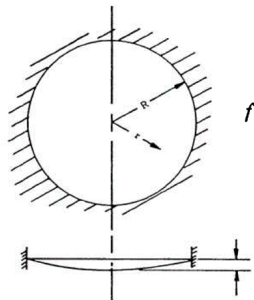
6.1 Didactic explanation proposed by the elastic analogy of general relativity in weak field

- The gravitation is well a space and time deformation

NO



ρ, Y, \dots, f



$$f = 0,662 a \sqrt[3]{\frac{p a}{E h}} \quad (\text{Durchbiegung in Plattenmitte}).$$

Yes

$$Y_x = \frac{\epsilon_{tt}^2 Y_t}{\epsilon_{xx}^2} \quad Y_y = \frac{\epsilon_{tt}^2 Y_t}{\epsilon_{yy}} \quad Y_z = \frac{\epsilon_{tt}^2 Y_t}{\epsilon_{zz}}$$

VIII.5.7.2 Approach by the elastic sphere theory
The data about the sphere with an internal pression is given at the figure 129.

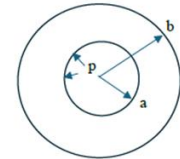


Figure 129: Notation for a sphere loading by an internal pression

In elasticity, we have the differential equation [308]:

$$\frac{d^2 u_r}{dr^2} + \frac{2}{r} \frac{du_r}{dr} = \frac{2}{r^2} u_r$$

So, the beginning of the equation follow the form of:

$$\Delta \phi = \frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \times \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \times \frac{\partial^2 \phi}{\partial \theta^2} + \frac{1}{r^2 \tan \theta} \times \frac{\partial \phi}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \times \frac{\partial^2 \phi}{\partial \varphi^2}$$

$$\Delta u_r = \frac{2}{r^2} u_r$$

So, it's a development of the Poisson's equation that is this modified by a distribution f that is not constant. The solution is on the form:

$$u_r = C_1 r + \frac{C_2}{r^2}$$

With for the 2 constants:

$$C_2 = \frac{1 + \nu}{2(1 - 2\nu)} b^3 C_1$$

$$C_1 = \frac{(1 - 2\nu)}{E} \frac{a^3}{b^3 - a^3} p$$

We know the displacement u_r , so we can extract the Young's modulus $E = Y$:

$$E = Y = \frac{a^3 p}{u_r (b^3 - a^3)} \left[(1 - 2\nu) r + (1 + \nu) \frac{b^3}{2r^2} \right]$$

6.1 Didactic explanations proposed by the elastic analogy of general relativity in weak field

- **Number of polarisations of the gravitational waves**
- 2 GW polarisations compatible with 2 expressions of the strain tensor in case of pure torsion

Pramana - J. Phys. (2020) 94:119
https://doi.org/10.1007/s12043-020-01954-5

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Check for updates

Mechanical conversion of the gravitational Einstein's constant κ

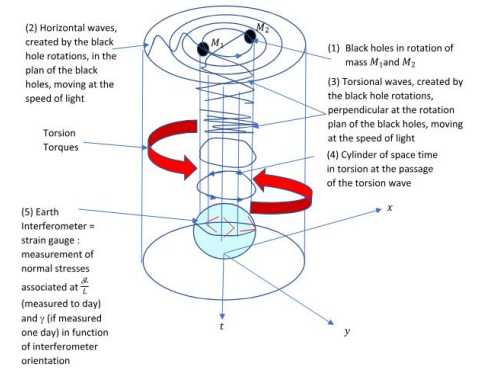
IZABEL DAVID

$$\varepsilon_{xy}(A_+) = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Compression/traction

$$\varepsilon_{xy}(A_x) = \begin{bmatrix} 0 & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Shear



- Why c ?

By the Principe 5, c becomes effectively a fundamental characteristic of the space fabric as a ratio between the Young's modulus Y and the density ρ of the medium constituting the space fabric

$$Y = \rho c^2$$

$$\mu = \frac{Y}{2(1 + \nu)} = \rho c^2$$

- Effect of the time on the mechanic relativist

All is always in dynamic in the space fabric. When we measure a strain a part of strain is not yet arrived, all the measurement are as blurred

6.2 Didactic explanations proposed by the elastic analogy of general relativity in weak field

The Einstein's Constant and the gravitational constant can be developed basing on mechanical parameter of the space fabric

Pramana - J. Phys. (2020) 94:119
<https://doi.org/10.1007/s12043-020-01954-5> © Indian Academy of Sciences

Check for updates

Mechanical conversion of the gravitational Einstein's constant κ

IZABEL DAVID

$$G = \pi f^2 \frac{1}{\rho}$$

$$\kappa = 2\rho \left(\frac{\omega}{E}\right)^2$$

$$\kappa = 2\rho \left(\frac{\omega}{\mu}\right)^2$$

$$Y = 6c^7 / 2\pi\hbar G^2, \quad \nu = 1$$

David Izabel GDR GDM 28 06 2024

Foundations of Physics manuscript No.
 (will be inserted by the editor)

The Mechanics of Spacetime – A Solid Mechanics Perspective on the Theory of General Relativity

T G Tenev · M F Horstemeyer

Table 1 Comparison between the General Relativity and Solid Mechanics Perspectives.

General Relativity Perspective	Solid Mechanics Perspective
Physical space	Mid-hypersurface of a hyperplate called "cosmic fabric".
Spacetime	The world volume of the cosmic fabric's mid-hypersurface
Intrinsic curvature of physical space	Intrinsic curvature of the fabric's mid-hypersurface
Intrinsic curvature of spacetime	Intrinsic curvature of the fabric's world volume
Gravitational potential Φ	Volumetric strain ϵ^{3D} , such that $\epsilon^{3D} = -\Phi/c^2$
Gravitational waves	Shear waves traveling at the speed of light
Matter curves spacetime	Matter induces prescribed strain causing the fabric to bend
Action integral in free space, $S = \frac{1}{2\kappa} \int R\sqrt{ g } d^4x$	Action integral outside of inclusions, $S = \frac{L^2 Y}{24} \int R\sqrt{ g } d^4x$
Constants of Nature: G, \hbar, c	Elastic constants: $Y = 6c^7 / 2\pi\hbar G^2, \nu = 1$

6.2 Predictive consequences of the elastic analogy of the general relativity in weak field

a) About vacuum energy density / Value of the Young's modulus

if we read principle 5 in the other direction, the Young moduli are to be considered as energy densities of the vacuum. Consequently, as we have two families of Young's moduli, there will not be one but 2 vacuum energy values. One associated with space distortions and one associated with time distortion!

We have so 3 possibles sources for the vacuum energy:

- Quantum Field Theory : $E_{\text{vacuum}} = 1 \times 10^{113} \text{ kg m}^2/\text{s}^2/\text{m}^3$
- Cosmological constant Λ $T_{\text{vacuum}} = 8.987551787 \times 10^{-10} \text{ kg m}^2/\text{s}^2/\text{m}^3$
- Stain energy of the cosmic fabric as an elastic medium in weak Field :

Compatible with R
Weiss approach 10^{20} Y
steel 10^{31} Pa

$$\begin{array}{c}
 Y_{\text{longitudinal}} = \rho c^2 \\
 \longleftrightarrow \\
 U_{\text{vacuum}} = \begin{cases} (time) 10^{20} \text{ (spheric)} \text{ at } 10^{40} \text{ (membrane)} \\ (space) 10^{38} \text{ at } 10^{40} \text{ (truss and cylinder)} \end{cases}
 \end{array}$$

³⁸ (spheric) at ⁴⁰ (membrane)
³⁸ at ⁴⁰ (truss and cylinder)

6.2 Predictive consequences of the elastic analogy of the general relativity in weak field

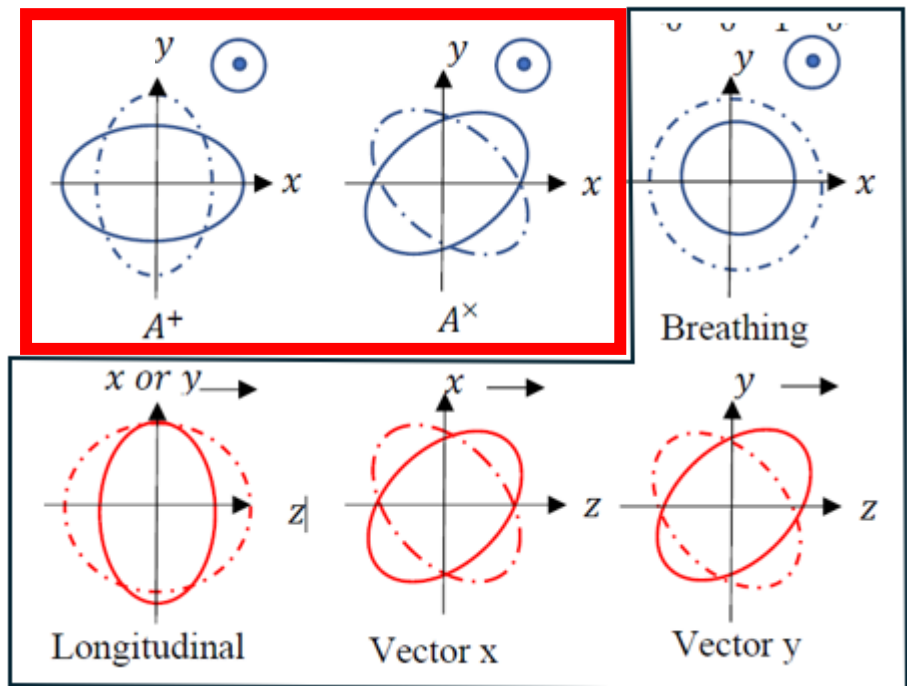
b) About the structure of the fabric

We found the strain deformation of the space medium only if we consider Planck sheet ! As it is the case in quantum gravity or string theory.

7) How to check all that ?

7.1) Measure complementary polarisations and study of their shapes

- To have a medium in 3 dimensions, the geometric Torsion is necessary in general relativity. The consequences is 4 complementary polarisations. Their measurement could validate so this approach.



$$\epsilon_{xy(A_+)} = \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & -\epsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \epsilon_{xy(A_x)} = \begin{bmatrix} 0 & \epsilon_{xy} & 0 \\ \epsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\epsilon^{(i)(j)} = \frac{1}{2} \begin{pmatrix} 0 & 0 & \frac{\sqrt{2}J}{\rho A} \sin\Phi \\ 0 & 0 & -\frac{\sqrt{2}J}{\rho A} \cos\Phi \\ \frac{\sqrt{2}J}{\rho A} \sin\Phi & -\frac{\sqrt{2}J}{\rho A} \cos\Phi & H/A \end{pmatrix}$$

7.2) Measure lateral motions of the interferometer in 3D

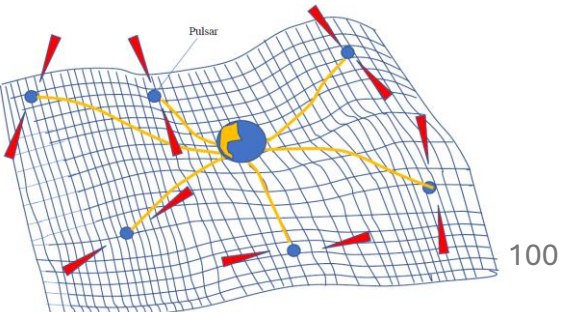
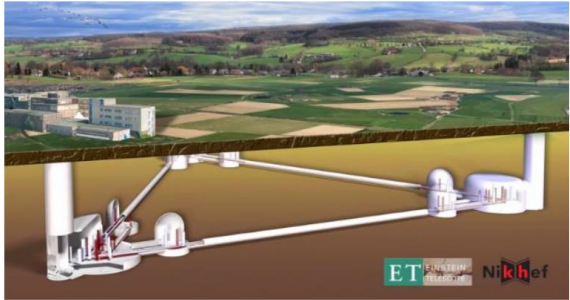
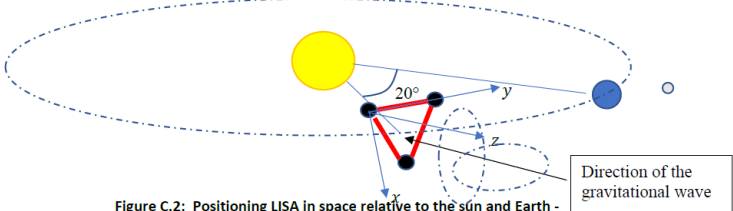
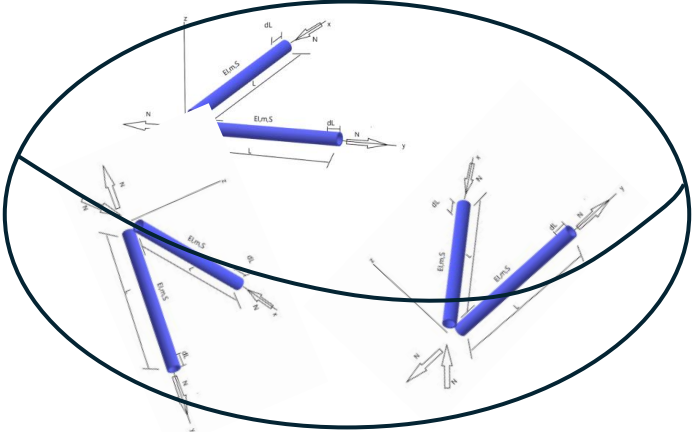
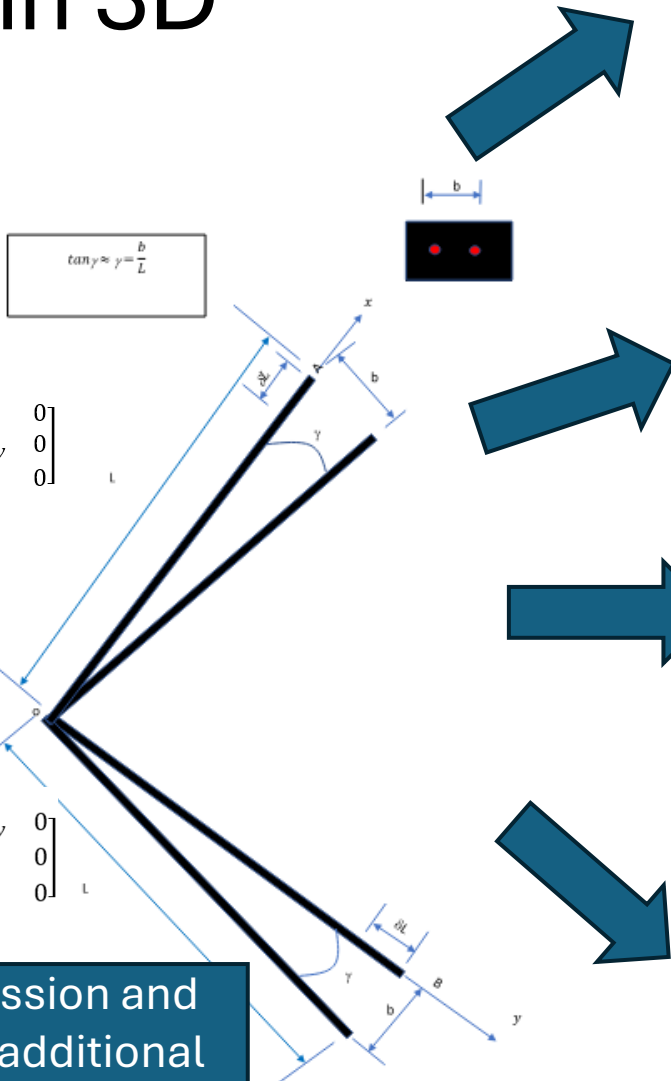
Actually, measured by Ligo/Virgo

$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \varepsilon_{xy}(A_+) = \frac{1}{2} A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

That could be measured by the interaction between the Earth interferometer Lisa, Einstein telescope or Pulsar telescope

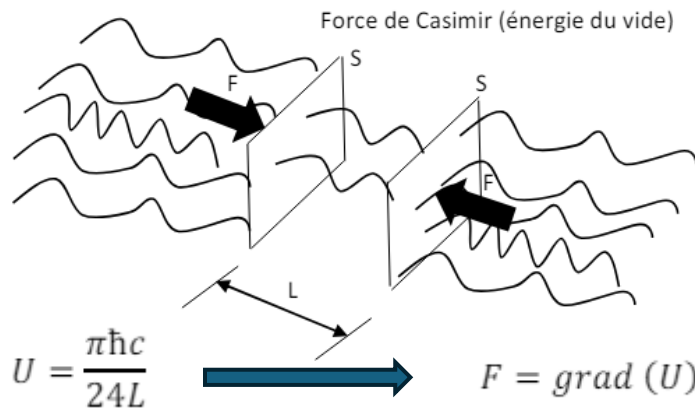
$$h_{\mu\nu} = A_x \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \varepsilon_{xy}(A_x) = \frac{1}{2} A_x \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So, if we measure additional strains of compression and shear, by the equivalence principle 1 there are additional perturbations and thus polarisations. The geometrical torsion become mandatory in general relativity



7.3) Measure of the Casimir strains and forces to have a realistic value of the space Young's modulus and strain elastic energy

- A possible direct measure of the Young's modulus of the vacuum can be done via the Casimir effect considered as an equivalent compression test of the medium.
- Actually, the force is measured, and the displacement calculated, if the forces and the displacement was measured it will be possible to have a direct value of the vacuum energy



$$F = -\frac{\pi \hbar c}{24L^2}$$

Casimir effect from a scattering approach

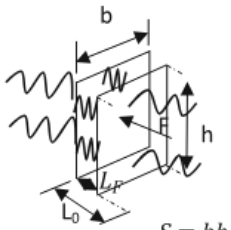
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 (Dated: April 29, 2014)*

The Casimir force is a spectacular consequence of the existence of vacuum fluctuations and thus deserves a place in courses on quantum theory. We argue that the scattering approach within a one-dimensional field theory is well suited to discuss the Casimir effect. It avoids in a transparent way divergences appearing in the evaluation of the vacuum energy. Furthermore, the scattering approach connects in a natural manner to the standard discussion of one-dimensional scattering problems in a quantum theory course. Finally, it allows to introduce students to the methods employed in the current research literature to determine the Casimir force in real-world systems.

Noting that the exponential in the numerator of (30) cuts off the integrand, we may obtain the limit of perfect reflectors $r_1 = r_2 = -1$. The integral can be evaluated by first expressing the integrand in terms of a geometric series and performing a resummation after having integrated each term. We thus arrive at the Casimir force for perfect reflectors in one dimension

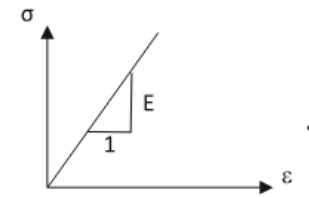
$$F_{1D} = -\frac{\hbar c \pi}{24L^2}$$



$$S = bh$$

$$\sigma = \frac{F}{S} = \epsilon E$$

$$\epsilon = \frac{L_F - L_0}{L_0}$$



DeLLight experiment : vacuum behaviour as a material that diffract light under huge magnetic field

The Young's modulus and the Poisson's ratio of the equivalent material medium are then obtained from the following formulas, which depend on the longitudinal and transverse velocities measured by varying the angle of incidence θ of the emitted wave on the front face of the sample under test:

Snell-Descartes law.

$$\frac{\sin \theta_1}{\sin \theta_2} = n_{21} = \frac{n_2}{n_1} = \frac{v_2}{v_1}$$

Quantum mechanics results : Under intensive magnetic field, vacuum diffract light as an elastic medium

$$Y = E = \frac{\rho C_T^2 (3C_L^2 - 4C_T^2)}{C_L^2 - C_T^2}$$

$$\nu = \frac{(C_L^2 - 2C_T^2)}{2(C_L^2 - C_T^2)}$$

nonlinear optics

$$n(I) = n_0 + n_2 I$$



$$n(I) = n_0 + n_2(I) = \frac{C_{vacuum}}{C_{Vacuum+Strong Magnetic Field}} = \frac{\sin \theta}{\sin r}$$

Euler-Heisenberg model derived from QED

$$n_{2,max} = n_{2,QED} = \frac{56}{45} \alpha^2 \frac{\hbar^3}{m_e^4 c^6} = 1.55 \times 10^{-33} \text{ cm}^2 / \text{W}$$

Variation of the speed of light
=> diffraction => equivalent elastic medium

n_2 is a nonlinear index related to the variable part of the refractive index n . This variable part depends on the light intensity. It is not the same as the n_2 of the Snell-Descartes law, For a low luminous intensity I $n(I)$ can then be equal to n_1 or n_2 of the Snell-Descartes law.

n_0 is the linear refraction index under low light intensity (that of Snell Descartes).

[335] Scott Robertson (2019) « Optical Kerr effect in vacuum »

[336] Scott Robertson, Aurélie Mailliet, Xavier Sarazin, Francois Couchot, Elsa Baynard, Julien Demailly, Moana Pittman, Arache Djannati-Ata, Sophie Kazamias, and Marcel Urban (2021) «The DeLLight experiment to observe an optically-induced change of the vacuum index»

8) Beyond the analogy – some speculations

8.1) In Strong field? Example of theoretical frame dragging for a neutron star (1/2)

Theoretical value of the frame dragging for a neutron star (source NASA)

Problem 2 - A neutron star is the compressed nuclear core of a massive star after it has become a supernova. Suppose the mass of a neutron star is equal to our sun, its radius is 12 kilometers, a gyroscope orbits the neutron star at a distance from its center of $r = 6,000$ kilometers, and its orbit period is $T = 8$ seconds. To two significant figures, what is Ω for such a dense, compact system in degrees/year?

$$R = \frac{2(6.67 \times 10^{-11})(2.0 \times 10^{30})}{(300,000,000)^2} = 2,964 \text{ meters} \quad a = \frac{2(12,000)^2}{5(300,000,000)} \left(\frac{2(3.141)}{8.0} \right) = 0.15 \text{ meters}$$

then

$$\Omega = \frac{(2964)(0.15)(3 \times 10^8)}{(6.0 \times 10^6)^3 + (0.15)^2 (6.0 \times 10^6) + (4150)(0.15)^2} \left(\frac{360}{2(3.141)} \right)$$

Angle distortion Ω of space in a Neutron star field

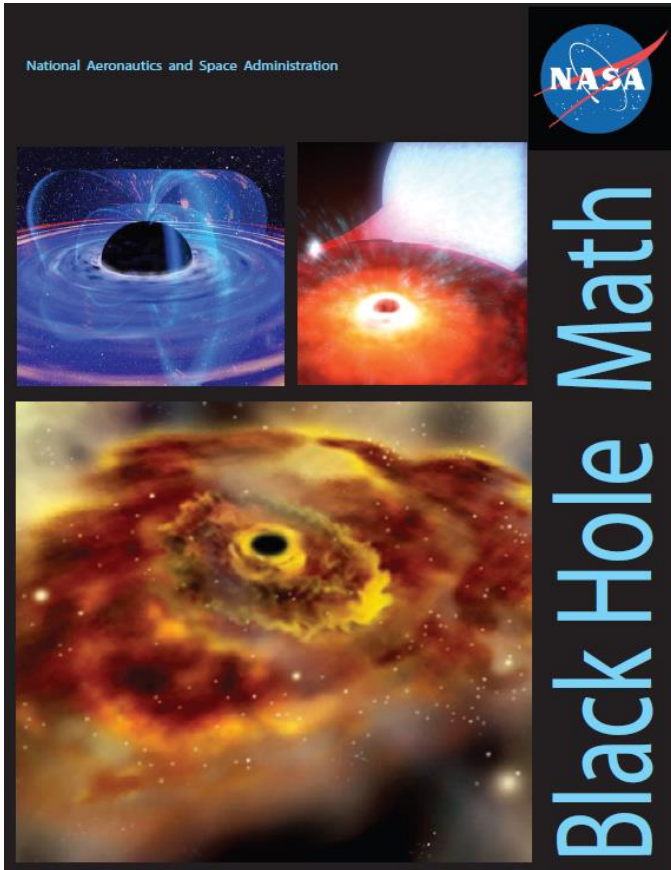
$$\Omega = \frac{(1.33 \times 10^{11})}{(2.16 \times 10^{20}) + (1.35 \times 10^5) + (93.4)} \left(\frac{360}{(6.242)} \right) = 3.65 \times 10^{-8} \text{ degrees/sec}$$

$$\Omega = 3.65 \times 10^{-8} \text{ deg/sec} \times (365 \text{d}/1\text{yr}) \times (24 \text{h}/1\text{day}) \times (3600 \text{ s}/1 \text{hr}) = 1.1 \text{ deg/yr}$$

Note this is nearly 100,000 times the corresponding Lens-Thirring rate near Earth.



So, we are well in strong Field



8.1) In Strong field? Example of the theoretical frame dragging for a neutron star (2/2)

This example used the results and calculation of [306].

We consider a neutron star of diameter 12 km, mass 1.9891×10^{30} kg, orbital period 8s.

The calculus of the frame dragging angle at $r = 6$ km following [306] p65 66 $\Omega = 3.6510^{-8}$ deg/s.

The kinetic energy of the Neutron star in rotation is :

$$E_{Kinetic,T} = \frac{1}{2} \times J \times (\omega_{NS})^2$$

In the case of a rotating ball the moment of kinetic inertia is:

$$J = \frac{2}{5} M_{NS} R_{NS}^2$$

We can define the angular velocity in rad/s by the expression above function of the time taken by the neutron star to do a complete tour in 8s:

$$\omega_{NS} = \frac{1 \text{ tour}}{8} \times \frac{2\pi \text{ rad}}{1 \text{ tour}} = \frac{\pi}{4} \text{ rad/s}$$

We obtain the following expression of the Kinetic energy:

$$E_{Kinetic,T} = \frac{1}{2} \times \frac{2}{5} M_{NS} R_{NS}^2 \times (\omega_{NS})^2$$

$$E_{Kinetic,T} = \frac{1}{2} \times \frac{2}{5} M_{NS} R_{NS}^2 \times \left(\frac{\pi}{4}\right)^2$$

So, the final expression of the Kinetic energy by torsion of the Earth:

$$E_{Kinetic,T} = \frac{\pi^2}{80} M_{NS} R_{NS}^2$$

Dimension is well an energy:

$$\frac{\text{rad}^2}{\text{s}^2} \text{kg} \times \text{m}^2$$

Let's equalize the kinetic and strain energies for the total cylinder encompassing the whole Neutron star (2 half cylinders) (as done in the case of the Earth with prob B experiment):

$$E_{cinetic,T} = \frac{\pi^2}{80} M_{NS} R_{NS}^2 = U = 2 \frac{\pi Y (2R_{NS})^4 \theta_{x2,space}^2}{384(1+\nu)R_{NS}}$$

We extract an expression of Young's modulus Y of spacetime:

$$Y = \frac{\pi^2 \times 384(1+\nu)R_{NS}}{2\pi \times 80(2R_{NS})^4 \theta_{x2,space}^2} M_{NS} R_{NS}^2$$

We check the dimensional equation that is correct:

$$\frac{\text{rad}^2 \times \text{m} \times \text{kg} \times \text{m}^2}{\text{s}^2 \times \text{m}^4 \times \text{rad}^2} = \frac{\text{kg}}{\text{s}^2 \times \text{m}} = \frac{\text{N}}{\text{m}^2}$$

Or after some mathematics:

$$Y = \frac{\pi \times 3(1+\nu)}{20 \times R_{NS} \times \theta_{x2,space}^2} M_{NS}$$

We can now, carry out the numerical application to have an estimation of the Young's modulus Y of the associated elastic medium corresponding to the space:

$\nu = 1$

Mass of the Neutron star:

$$M_{NS} = 1.9891 \times 10^{30} \text{ kg}$$

Radius of the Neutron star:

$$R_{NS} = 6000 \text{ m}$$

Angular distortion (Lense-Thirring effect) via NASA calculation [306]:

$$\theta_{x2,space} = \Omega = 3.65 \times 10^{-8} - \text{degree/s}$$

$$\theta_{x2,space} = \Omega = 6.37045 \times 10^{-10} - \text{rad/s}$$

And we obtain:

$$Y = \frac{\pi \times 3 \times (1+\nu)}{20 \times R_{NS} \times \theta_{x2,space}^2} M_{NS} = \frac{3.14 \times 3 \times 2 \times 1.9891 \times 10^{30}}{20 \times 6000 \times (6.3740 \times 10^{-10})^2}$$

$$E_{space-time}(\nu=1) = 7.68 E^{44} \frac{\text{N}}{\text{m}^2}$$

$$E_{space-time}(\nu=1) = 7.68 E^{38} \text{ MPa}$$

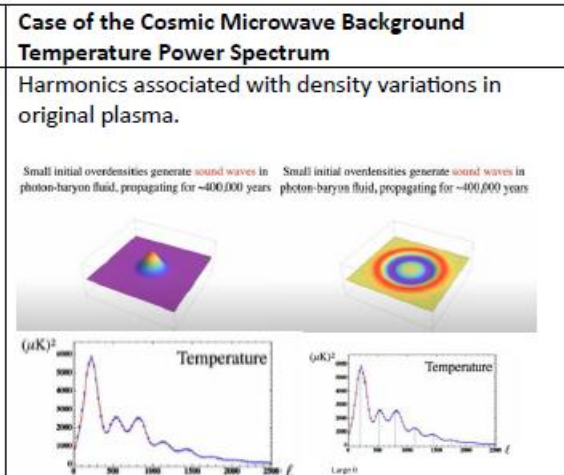
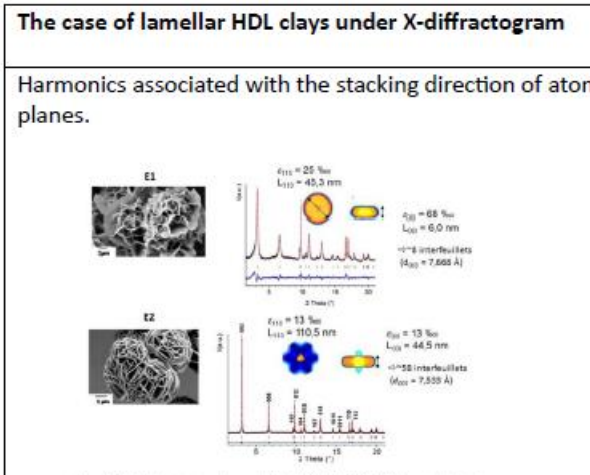
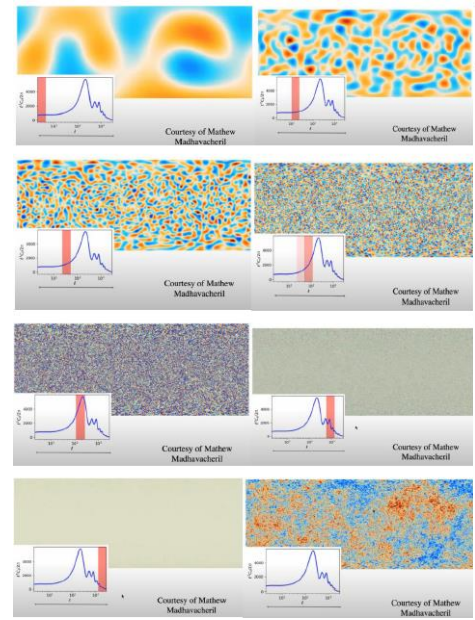
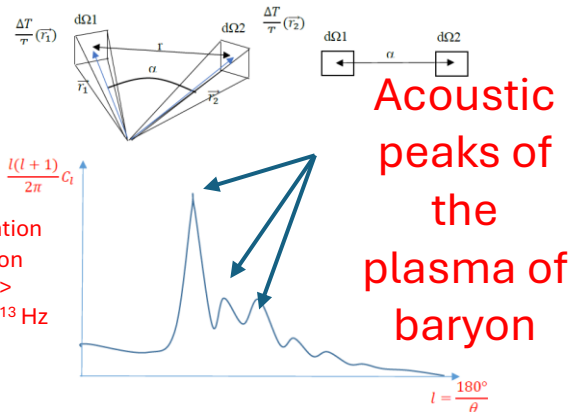
On a theoretical value of the frame dragging around a neutron star we obtain again Y
10⁴⁴ Pa

So, we obtain the same magnitude of Y that in weak field

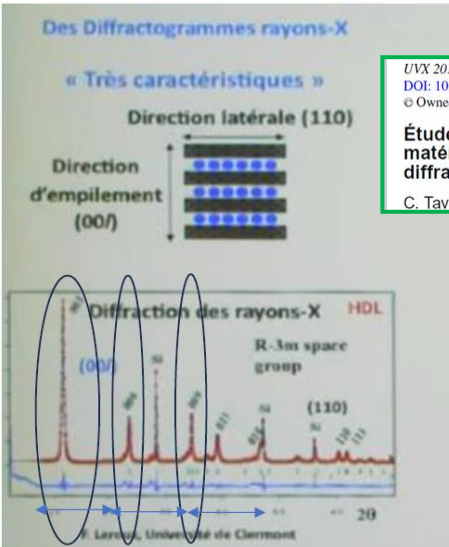
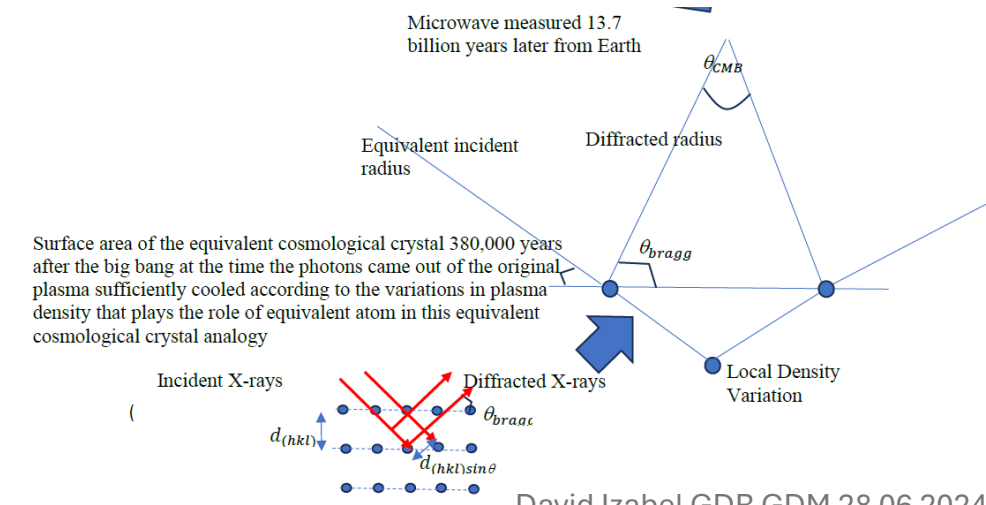
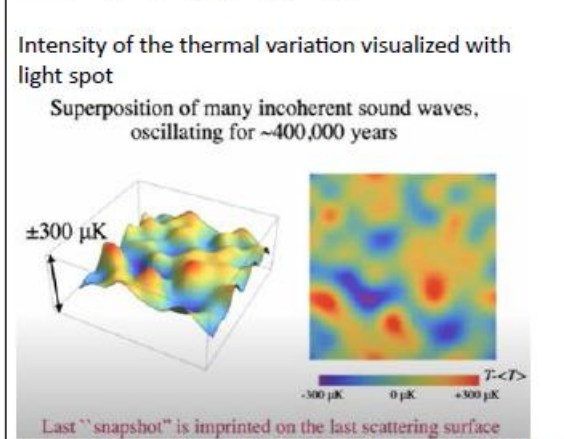
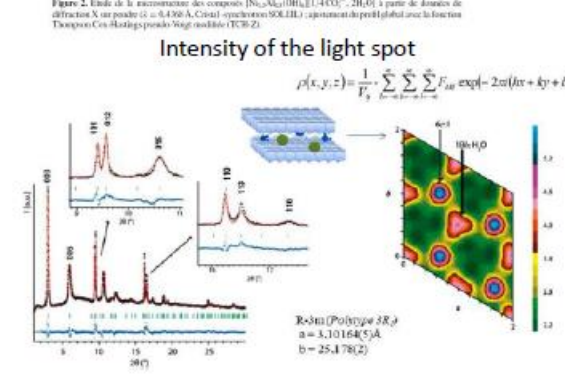
Data	Value	Unit				
Mass of the Neutron star	1,99E+30	kg		39	milliarc second/year	for Prob B
Poisson's ratio	1					
Radius of the Neutron star	6	km		1	milliarc second	4,85E-09 rad
Frame dragging angle (space) θ_{x2}	6,37E-10	rad		1,31E-01	milliarc second /s	6,37E-10 rad/s
				4,14E+06	Milliarc second /year	Neutron star
				39	Milliarc second /year	Earth
				106255.0305	x Earth Frame dragging	
				7,70E+44	Pa	space only

$$Y = \frac{\pi \times 3(1+\nu)}{20 \times R_{NS} \times \theta_{x2,space}^2} M_{NS}$$

8.2) Analyse of the CMB power spectrum as a diffractogram X (eg clay in sheets)



The power spectrum of the cosmic microwave background can be seen by analogy as the equivalent of an X-ray diffractogram of a medium. The regular series of peaks on the one hand and the great width of the peaks on the other hand could be representative of a certain structure of the dark matter of the plasma and of the extreme smallness of the density variation grains on the other hand



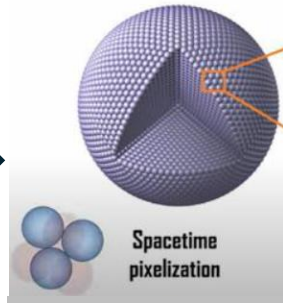
UVX 2012, 01016 (2013)
 DOI: 10.1051/uvx/201301016
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Étude du mécanisme d'échange et de la structure des matériaux hydroxydes doubles lamellaires (HDL) par diffraction et diffusion des rayons X

C. Taviot-Guého, F. Leroux, F. Goujon, P. Malfreyt et R. Mahiou

The Origin of Mass and the Nature of Gravity

Nassim Hamein[†], Cyprien Guermontprez[†], Olivier Alirol[†]



Medium of Planck grain size?

Energy density of a volume filled of PSUs:

$$\frac{Nm_l c^2}{V} = \frac{6}{\pi} \frac{c^7}{G^2 \hbar} = \rho_{vac}$$

106

8.3) Geometrical Torsion in CMB logical to take into account in Einstein-Cartan

- In this paper the author show that the geometric torsion is included in the polarisation B of the cosmic microwave

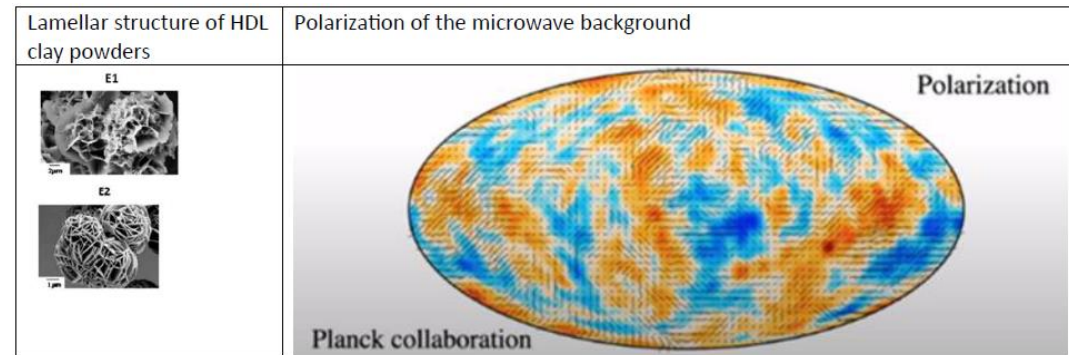
Constraints on background torsion from birefringence of CMB polarization

Moumita Das,^{1,*} Subhendra Mohanty,^{1,†} and A.R.Prasanna^{1,2,‡}

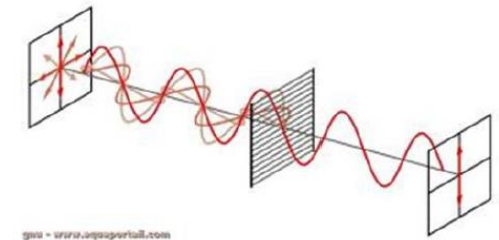
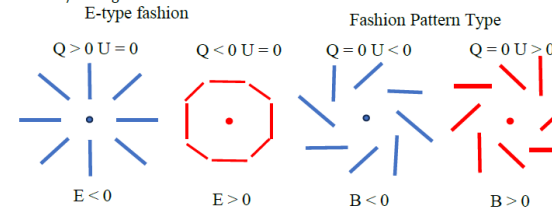
¹Physical Research Laboratory, Ahmedabad 380009, India
²L.J.Institute of Computer Application, Ahmedabad, India

Abstract

We show that a non-minimal coupling of electromagnetism with background torsion can produce birefringence of the electromagnetic waves. This birefringence gives rise to a B-mode polarization of the CMB. From the bounds on B-mode from WMAP and BOOMERanG data, one can put limits on the background torsion at $\xi_1 T_1 = (-3.35 \pm 2.65) \times 10^{-22} \text{ GeV}^{-1}$.



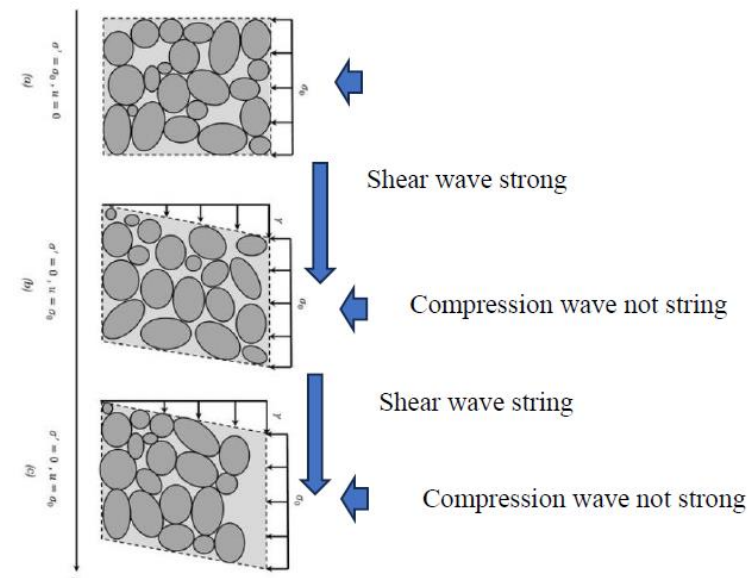
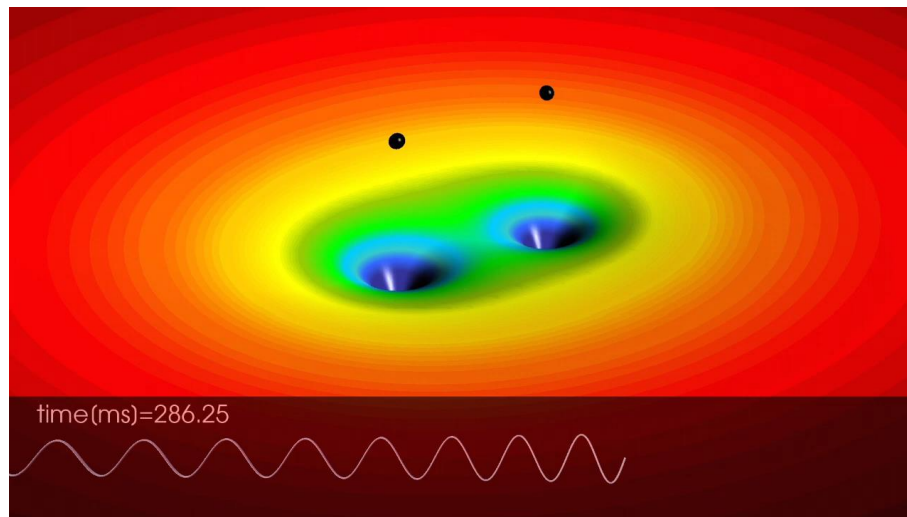
Modes E and B generate polarizations of the following spiral shape (torsion of the medium/rotational of the medium) see Figure 78 below.



So as the universe is growing since 13,7 billions Years it should be normal to find again the geometric torsion in space and so in general relativity. An associated sheet structure or lamella structure become possible

8.4) Self-repair/self-clogging of space after the passage of a rotating black hole, a sign of its great plasticity – Soil liquefaction

- During the coalescence of the black hole, they turn each other. But space doesn't tear itself apart => huge plasticity of space



UIB Binary Black Hole Merger, GW150914-like: lapse + orbital + strain evolution



GRG@UIB - Relativity and Gravitation
1,67 k abonnés

S'abonner

12



Partager

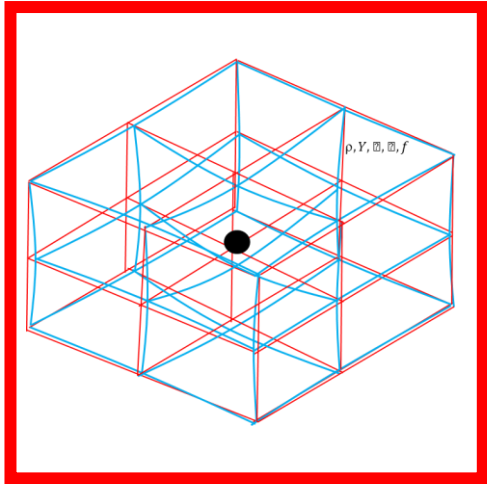
Enregistrer

9) Conclusions

Conclusion about the analogy approach of the general relativity in weak Field

- On a didactic point of view the analogy propose some possible explanations:
 - **Why gravitation is a space and time deformation** (mechanistic relation between Space Young's modulus and Time Young's modulus),
 - **Why there are two polarisations** (two facets of the strain tensor in pure torsion),
 - why $c \Rightarrow (Y/\rho)^{0,5}$ is fundamental, because connected with the mechanical characteristic of the elastic medium,
 - **Why it's well space time and not space that must be taken into account.**(space time is a dynamic object, impossible to measure static strains, always strains arrive with a delay at the measure point),
- On a predictive point of view:
 - **Complementary polarisations should exist** with gravitational wave, thus all the components of the metric perturbation tensor $h_{\mu\nu}$ will be defined,
 - **Lateral motions of the interferometer arms** should be measured in the future LISA/Einstein telescope,
 - **2 energy densities of vacuum should exist** one for space different for one for time in weak field regime,
 - **A microstructure of the Planck size** made of thin sheets should constitute space time at minimum for gravitational wave or frame dragging/geodetic effects
 - **Space-time should have a huge plasticity** capacity compatible with plastic crystallography

Gravitation in weak Field summarize (Overview of the metric perturbation tensor $h_{\mu\nu}$)



$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$$

$$\text{Curvature} = \frac{\text{Angle}}{\text{Surface}} = \frac{8\pi G}{c^4} \times \frac{\text{Energy}}{\text{Volume}}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} = 2\varepsilon_{\mu\nu}$$

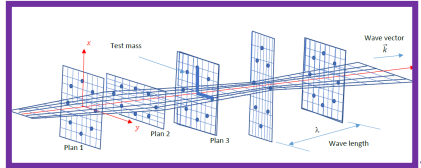
Gravitation

Newton Gravitation

$$\Delta\phi = 4\pi G\rho$$

Einstein in weak Field

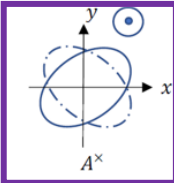
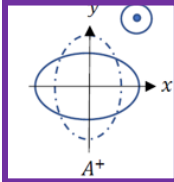
$$\Delta h_{00} = \frac{2}{c^2}\Delta\phi = \frac{8\pi G}{c^4}\rho c^2$$



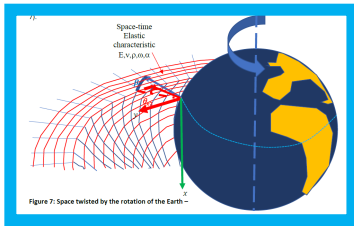
Gravitational wave polarisation A^+

$$\partial^\nu \partial_\nu \bar{h}_{\mu\nu} = \square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} = 0$$

GW



Lense-Thirring



Lense Thirring frame dragging and geodetic effect

$$\frac{\partial^2 x}{\partial t^2} = \frac{GM}{r^2} \frac{\omega l^2}{r} \left[\frac{4x^2 + y^2 - 2z^2}{5} \frac{\partial y}{\partial t} + \frac{12yz}{5r^2} \frac{\partial z}{\partial t} \right] - \frac{GM}{r^2} \frac{x}{r}$$

$$\frac{\partial^2 y}{\partial t^2} = -\frac{GM}{r^2} \frac{\omega l^2}{r} \left[\frac{4x^2 + y^2 - 2z^2}{5} \frac{\partial x}{\partial t} + \frac{12xz}{5r^2} \frac{\partial z}{\partial t} \right] - \frac{GM}{r^2} \frac{y}{r}$$

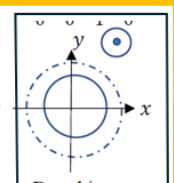
$$\frac{\partial^2 z}{\partial t^2} = \frac{GM}{r^2} \frac{\omega l^2}{r} \left[\frac{12xz}{5r} \frac{\partial y}{\partial t} - y \frac{\partial x}{\partial t} \right]$$

$$h_{\mu\nu} = \begin{pmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ h_{10} & h_{11} & h_{12} & h_{13} \\ h_{20} & h_{21} & h_{22} & h_{23} \\ h_{30} & h_{31} & h_{32} & h_{33} \end{pmatrix}$$

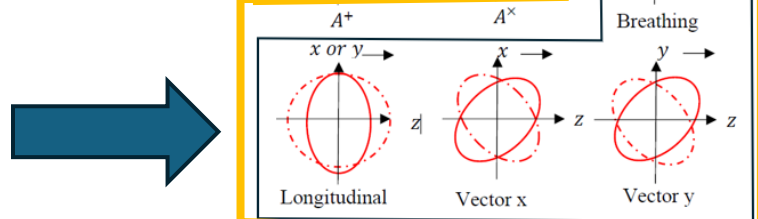
Gravitational wave polarisation A^x

Gravitational wave with possible new polarisations (Einstein-Cartan theory etc)

Hypothetic new polarisation with Einstein-Cartan theory (geometrical torsion)



Remain to measure



«Tensor calculus knows physics better than the physicist himself »
(Paul Langevin)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



$$h_{\mu\nu} = 2\varepsilon_{\mu\nu}$$

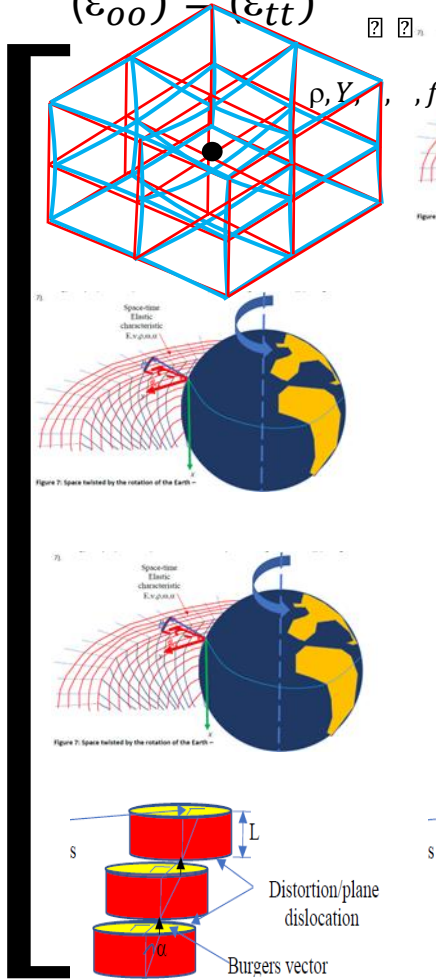


$$h_{\mu\nu} =$$

$$\begin{bmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ h_{10} & h_{11} & h_{12} & h_{13} \\ h_{20} & h_{21} & h_{22} & h_{23} \\ h_{30} & h_{31} & h_{32} & h_{33} \end{bmatrix}$$

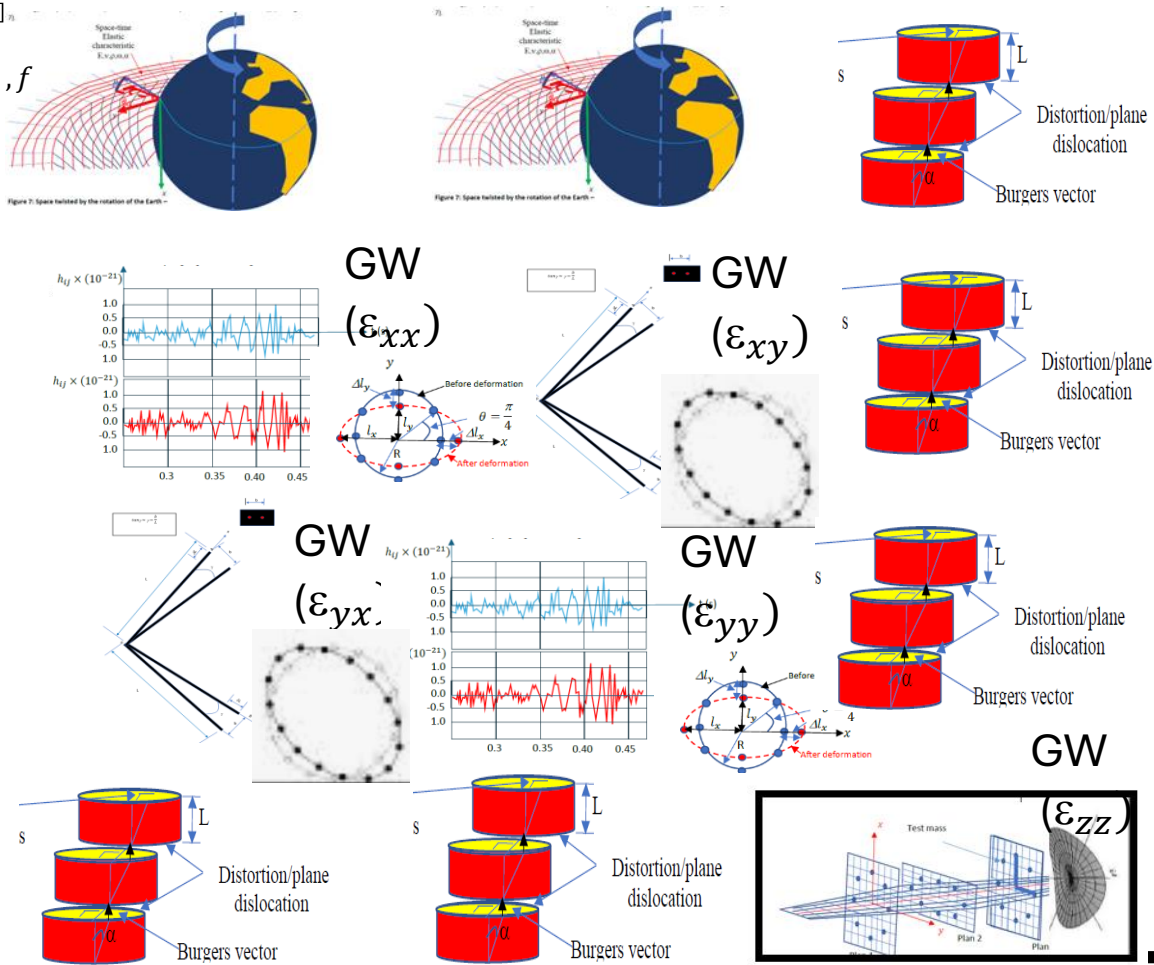
Strains tensor of the space time in weak field

Gravitation g
(ε_{00}) = (ε_{tt})



Angle $\Omega = \varepsilon_{j0}$
Frame dragging and geodetic effect

Angle $\Omega = \varepsilon_{0i}$ Frame dragging and geodetic effect



Geometric torsion/ defect theory /crystallography

Gravito electromagnetism
GR 2eme order

Geometric torsion/ defect theory /crystallography

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



$$h_{\mu\nu} = 2\varepsilon_{\mu\nu}$$

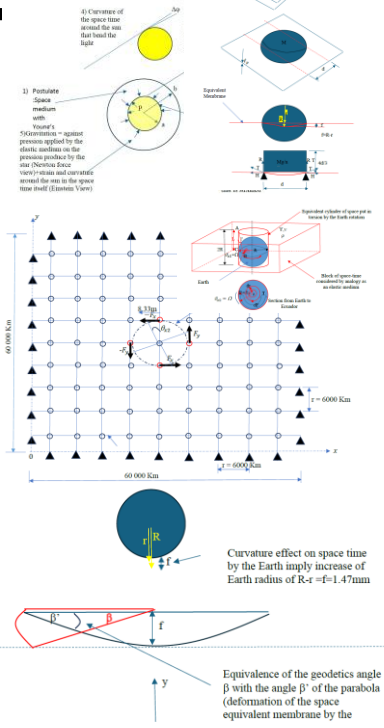


$$h_{\mu\nu} =$$

$$\begin{bmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ h_{10} & h_{11} & h_{12} & h_{13} \\ h_{20} & h_{21} & h_{22} & h_{23} \\ h_{30} & h_{31} & h_{32} & h_{33} \end{bmatrix}$$

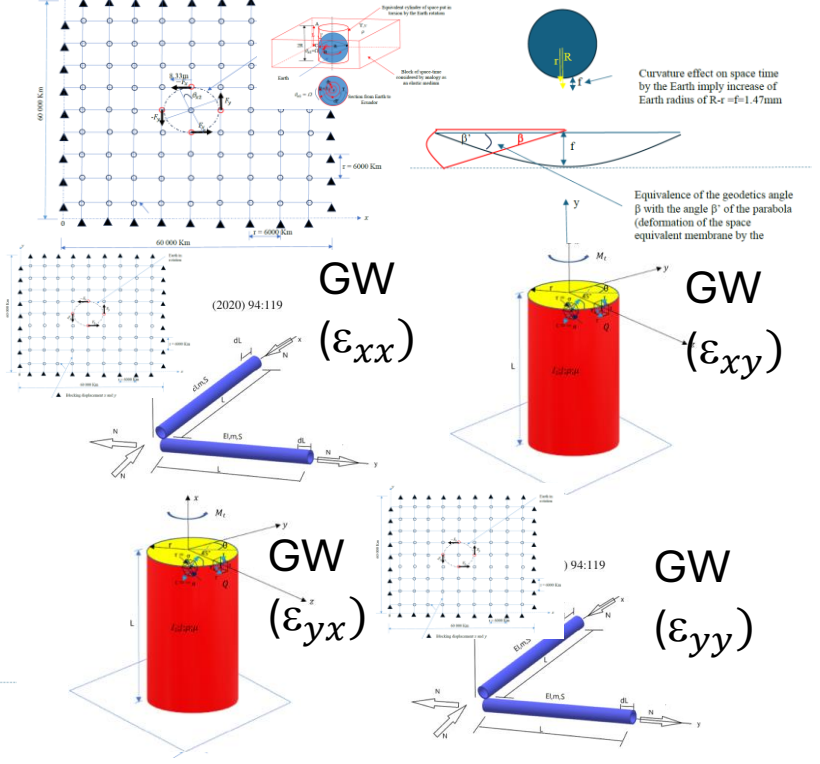
Timoshenko's models that can reproduce the order of magnitude of the space time strains

Gravitation g
(ε_{00}) = (ε_{tt})



Not tested

Angle $\Omega = \varepsilon_{ni}$ Frame dragging and geodetic effect



Not tested

Not tested

GW (ε_{zz})

Not tested

Not tested

Not tested

Geometric torsion/ defect theory /crystallography

Angle $\Omega = \varepsilon_{j0}$
Frame dragging and geodetic effect

Geometric torsion/ defect theory /crystallography

Gravito electromagnetism GR 2nd order

Bibliography and references

Constraints on background torsion from birefringence of CMB polarization

Moumita Das,^{1,*} Subhendra Mohanty,^{1,†} and A.R.Prasanna^{1,2,‡}

The Mechanics of Spacetime – A Solid Mechanics Perspective on the Theory of General Relativity

Pramana – J. Phys. (2020) 94:119
<https://doi.org/10.1007/s12043-020-01954-5>

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Mechanical conversion of the gravitational Einstein's constant κ

A FOUR-DIMENSIONAL HOOKE'S LAW CAN ENCOMPASS LINEAR ELASTICITY AND INERTIA

S. ANTOCI AND L. MIHICH

Einstein-Cartan theory as a theory of defects in space-time

M. L. Ruggiero* and A. Tartaglia[†]
Dip. Fisica, Politecnico and INFN, Torino, Italy, I-10129

Gravitational Waves in ECSK theory: Robustness of mergers as standard sirens and nonvanishing torsion

Emilio Elizalde^{1,*}, Fernando Izaurieta^{2†}, Cristian Riveros^{3‡}, Gonzalo Salgado^{2§} and Omar Valdivia^{1,4,5¶}.

Non-linear plane gravitational waves as space-time defects

F. L. Carneiro^{1,a}, S. C. Ulhoa^{2,3,b}, J. W. Maluf^{1,c}, J. F. da Rocha-Neto^{1,d}

¹ Instituto de Física, Universidade de Brasília, Brasília, DF 70919-970, Brazil

² International Center of Physics, Instituto de Física, Universidade de Brasília, Brasília, DF 70910-900, Brazil

³ Canadian Quantum Research Center, 204-3002 32 Ave, Vernon, BC V1T 2L7, Canada

On the Polarization of Gravitational Waves

Gravitational Wave Polarizations: A test of General Relativity using Binary Black hole mergers

Components of the gravitational force in the field of a gravitational wave

D. Baskaran[†] and L. P. Grishchuk^{‡§}

[†] Department of Physics and Astronomy, Cardiff University, Cardiff CF24 3YB, United Kingdom

[‡] Sternberg Astronomical Institute, Moscow University, Moscow 119899, Russia

Gravitomagnetic induction in the field of a gravitational wave

Matteo Luca Ruggiero*

Rainer Weiss Lecture



Ligo and the Discovery of Gravitational Waves, I

Nobel Lecture, December 8, 2017 by Rainer Weiss
Massachusetts Institute of Technology (MIT), Cambridge, MA, USA.

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Fachgebiet Technische Akustik
Sekt. TA 7 - Einsteinufer 25 - 10587 Berlin

Acoustic analogies with general relativity, quantum fields, and thermodynamics

Drasko Masovic, TU Berlin, 2018 (last update: August 15, 2022)

Introducing surface tension to spacetime

H A Perko¹
¹Koppa Research, Office 11, 140 E. 4th Street, Loveland, CO, USA 80537

Dark matter and dark energy: cosmology of spacetime with surface tension

H A Perko¹
¹Office 14, 140 E. 4th Street, Loveland, CO, USA 80537

Gravitation in the surface tension model of spacetime

H A Perko¹
¹Office 14, 140 E. 4th Street, Loveland, CO, USA 80537

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An Elastic Constitutive Model of Spacetime and its Applications

Tichomir G. Tenev

EPJ Web of Conferences **58**, 01005 (2013)
DOI: 10.1051/epjconf/20135801005
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Time travel, Clock Puzzles and Their Experimental Tests

Calculations on space-time curvature within the Earth and Sun

Wm. Robert Johnston

last updated 3 November 2008

Casimir effect from a scattering approach

Gert-Ludwig Ingold*
Institut für Physik, Universität Augsburg, Universitätsstraße 1, D-86135 Augsburg, Germany

Astrid Lambrecht[†]
*Laboratoire Kastler Brussel, CNRS, ENS, UPMC,
Campus Jussieu Case 74, F-75252 Paris Cedex 05, France*
(Dated: April 29, 2014)

The Casimir force is a spectacular consequence of the existence of vacuum fluctuations and thus deserves a place in courses on quantum theory. We argue that the scattering approach within a one-dimensional field theory is well suited to discuss the Casimir effect. It avoids in a transparent way divergences appearing in the evaluation of the vacuum energy. Furthermore, the scattering approach connects in a natural manner to the standard discussion of one-dimensional scattering problems in a quantum theory course. Finally, it allows to introduce students to the methods employed in the current research literature to determine the Casimir force in real-world systems.

Bibliography and references

Constraints on background torsion from birefringence of CMB polarization

Moumita Das,^{1,*} Subhendra Mohanty,^{1,†} and A.R.Prasanna^{1,2,‡}

Binary black holes, gravitational waves, and numerical relativity

Joan M. Centrella¹, John G. Baker¹, William D. Boggs², Bernard J. Kelly¹, Sean T. McWilliams² and James R. van Meter³

¹ Gravitational Astrophysics Laboratory, NASA Goddard Space Flight Center, 8800 Greenbelt Rd., Greenbelt, MD 20771, USA

² University of Maryland, Department of Physics, College Park, MD 20742, USA

³ Center for Space Science & Technology, University of Maryland Baltimore County, Physics Department, 1000 Hilltop Circle, Baltimore, MD 21250, USA



[Hencky, H., “Über Den Spannungszustand in Kreisrunden Platten Mit Verschwindender Biegeungssteifigkeit,” Zeitschrift für Mathematik und Physik, Vol. 63, 1915, pp. 311–317.](#)

Brazilian Journal of Physics, vol. 35. no. 2A, June, 2005

Emerging Gravity from Defects in World Crystal

H. Kleinert
Institut für Theoretische Physik, Freie Universität Berlin, Arnimallee 14, D14195 Berlin
Received on 25 January, 2005

UVX 2012, 01016 (2013)

DOI: [10.1051/uvx/201301016](https://doi.org/10.1051/uvx/201301016)

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Étude du mécanisme d'échange et de la structure des matériaux hydroxydes doubles lamellaires (HDL) par diffraction et diffusion des rayons X

C. Taviot-Guého, F. Leroux, F. Goujon, P. Malfreyt et R. Mahiou

The Origin of Mass and the Nature of Gravity

Nassim Hamein[†], Cyprien Guermonprez[†], Olivier Alirol[†]

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Y. Zhou et al.: Young's modulus in nanostructured metals

Y. Zhou^a, U. Erb^b, K. T. Aust^a and G. Palumbo^b

^a Department of Materials Science and Engineering, University of Toronto, Toronto, Ontario, Canada

^b Integran Technologies Inc., Toronto, Ontario, Canada

Young's modulus in nanostructured metals