Proximity between elastic computations for structural damage estimation

GdR meeting - La Rochelle

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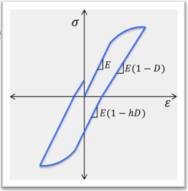
Continuum Damage Mechanics

Continuum Damage Mechanics

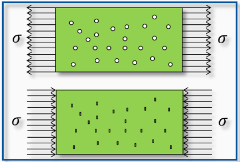
- Evolution of micro-cracks and micro-voids decrease of elastic modulus



Stress-strain diagram with damage

- Decrease of load carrying capacity

- Described as an internal variable, damage, which is non-decreasing in nature



- Growth of micro-defects during tension
- Closure during compression

- Effective modulus of elasticity $\sigma = \tilde{E}\ \varepsilon^e$

- Closure parameter h

$\tilde{E} = E(1-D)$ in tension, $\tilde{E} = E(1-hD)$ in compression

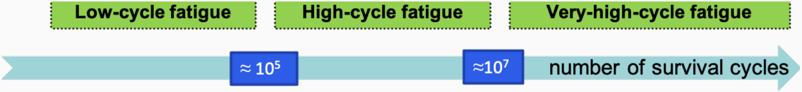
Thermodynamically-consistent material model

- Evolution equations and state laws, satisfy the second law of thermodynamics
  - $\dot{D} = k(Y)$
  - $\dot{\varepsilon}^p = f(\sigma, Z, D)$
  - $\dot{X} = g(\sigma, Z, D)$
  - $Y = p(\sigma, D)$
  - $Z = q(X)$
  - $\sigma = \tilde{E}\ \varepsilon^e$
- Mechanical equilibrium to be solved with boundary conditions
  - $\mathrm{div}(\sigma) + f_V = \rho a$

Challenges in continuum damage mechanics

- Theoretical developments (choice of model, localization, ...)
- Experimental identification
- Numerical estimation

Numerical challenge for fatigue



- If 10 cycles are computed in 5 mins

  10^6 cycles would require approximately **1 year**

- No possible to provide some numerical results for parameter studies e.g.

- Need for some robust and efficient numerical scheme

  quicker than real-time simulations

Large Time Increment (LATIN) mthod

- At each iteration,
  - The approximation of the solution on the **whole discretised time-space domain** is looked for,
  - Two sub-iterations:
    - The **evolution equations**, which are **non-linear**, are solved **locally**.
    - The **balance equation**, which is a **global problem**, is written as **a linear problem**.

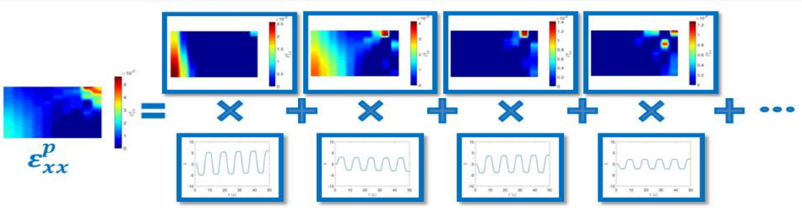
[Rheinboldt, 1986]

Decomposition of the quantity of interest

One Problem defined on a high-dimensional space → **Several Problems defined on one- or small- dimensional spaces**

$$u(x,t) = \sum_{i=1}^{\infty} X_i(x) \cdot T_i(t) \approx \sum_{i=1}^{N} X_i(x) \cdot T_i(t)$$

Space dependency



Time dependency

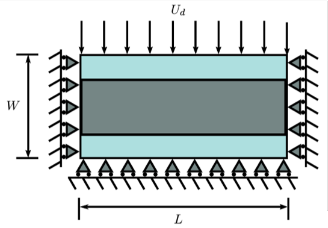
Proper Generalised Decomposition (PGD)

One Problem defined on a high-dimensional space → **Several Problems defined on one- or small- dimensional spaces**

$$u(x,t) = \sum_{i=1}^{\infty} X_i(x)\,.T_i(t) \approx \sum_{i=1}^{N} X_i(x)\,.T_i(t)$$

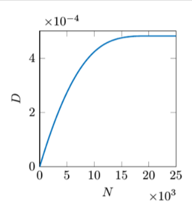
- Solved as a **greedy algorithm** on the fly
- **Number of terms chosen automatically** to satisfy the error criterion
- ++:
  - good convergence properties
  - flexible
  - time and space are decoupled, so the time problem can be specifically tackled
- --:
  - intrusive approach

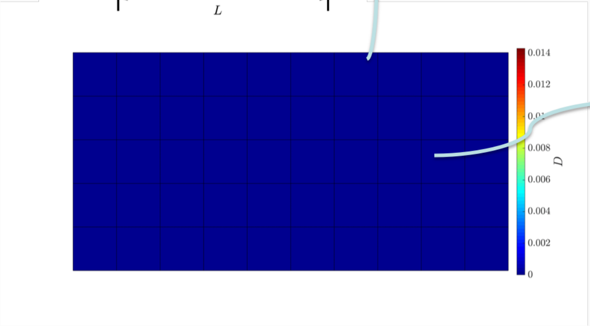
Fatigue damage computation on a sandwich plate

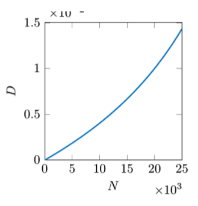


For 25,000 load cycles

Calculation time ≈ 3 h







Numerical challenges for nonlinear structural dynamics

Admissibility - $\mathcal{A}_d$

- Equation of motion
  - $\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{f}_d = \rho\gamma$
  - $\gamma = \ddot{\boldsymbol{u}}$
- Boundary conditions
  - $\boldsymbol{u} = \boldsymbol{u}_d$
  - $\dot{\boldsymbol{u}} = \dot{\boldsymbol{u}}_d$
- Initial conditions
  - $\boldsymbol{u}\big|_{t=0} = \boldsymbol{0}$
  - $\dot{\boldsymbol{u}}\big|_{t=0} = \boldsymbol{0}$

➔ Global (linear) equations in frequency domain

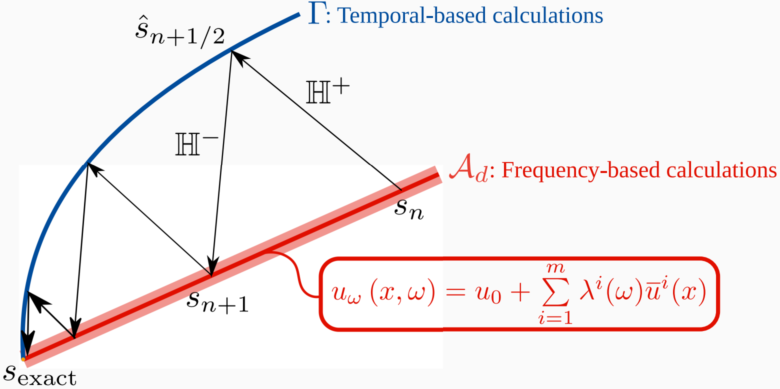
- Numerical benefits
- Frequency-dependent models

Nonlinear behaviour - Γ

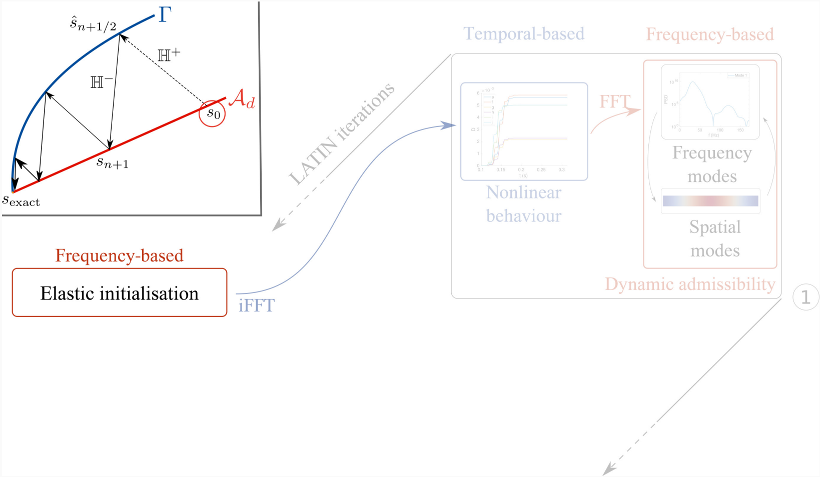
- Plasticity model
  - Linear isotropic and kinematic hardening
- Continuous damage model governed by plasticity [Lemaitre,1992]
- Effective stress $\tilde{\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}_D}{1-D} + \left[\frac{\langle\sigma_H\rangle}{1-D} - \langle-\sigma_H\rangle\right]\boldsymbol{1}$

➔ Local (nonlinear) equations

Temporal-frequency hybrid scheme



Hybrid frequency-temporal ROM for nonlinear dynamics



[Daby-Seesaram et al., Nonlinear dynamics, 2023]

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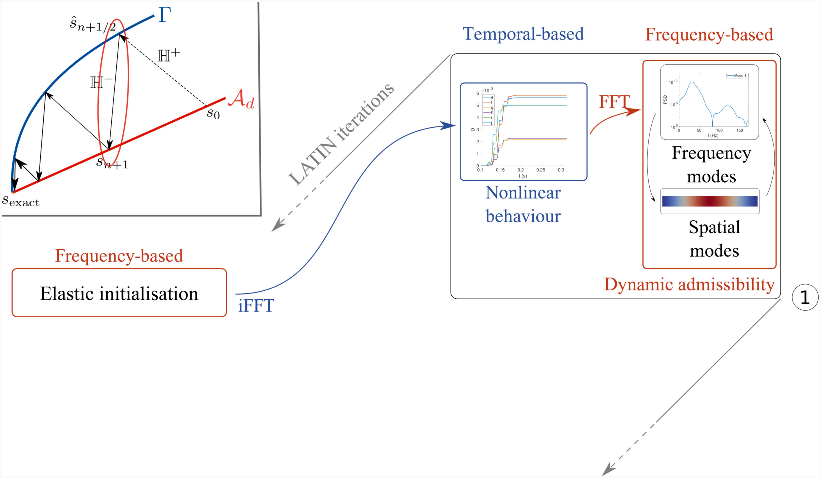

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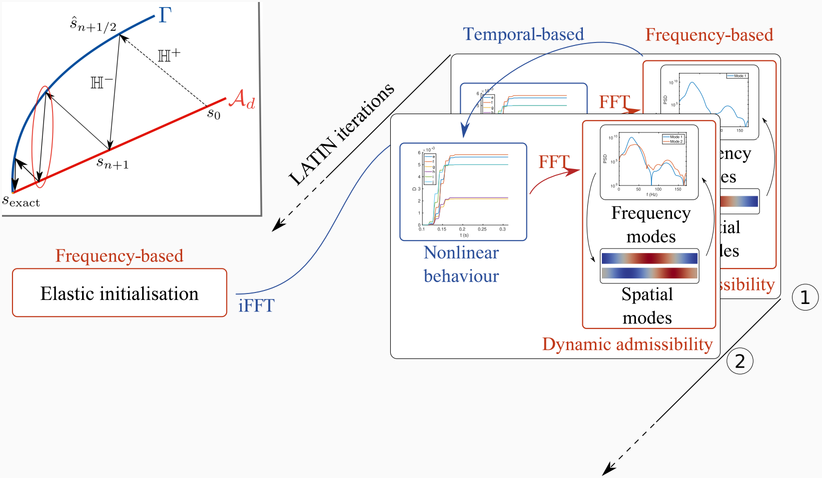

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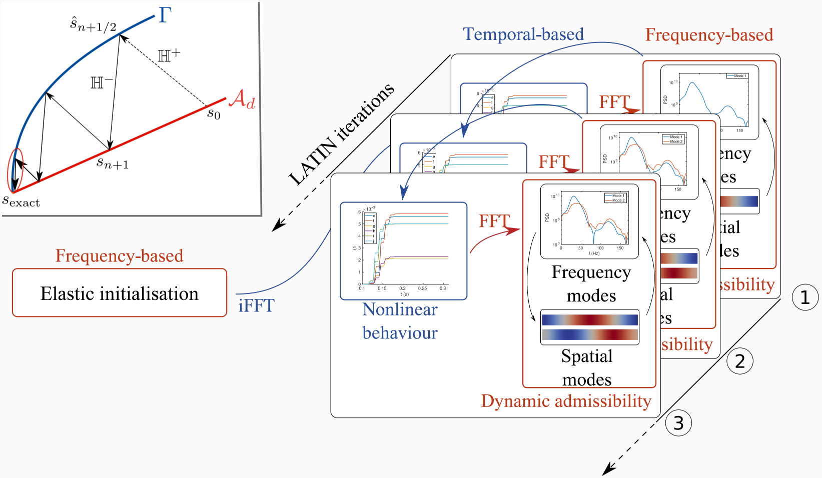

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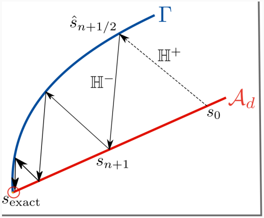
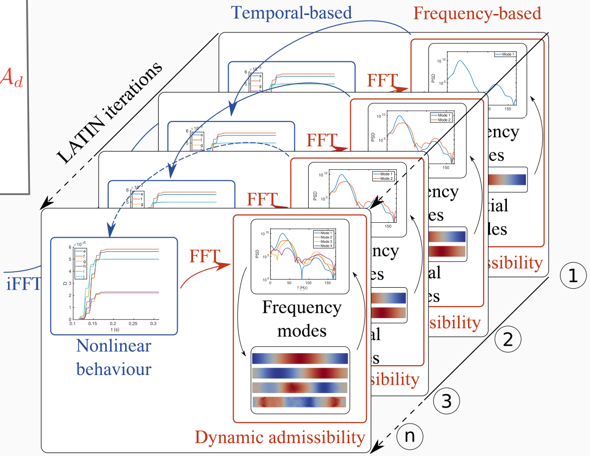


Frequency-based

Elastic initialisation



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Application to a 3D Pipe

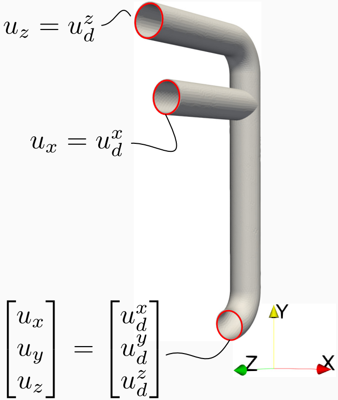


Figure 1: Boundary conditions

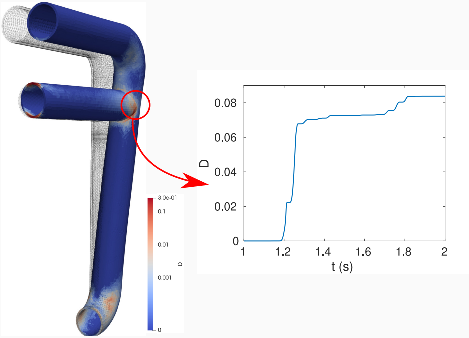


Figure 2: Damage evolution

Plasticity and damage quantities of interest available over the whole space-time domain

Numerical results

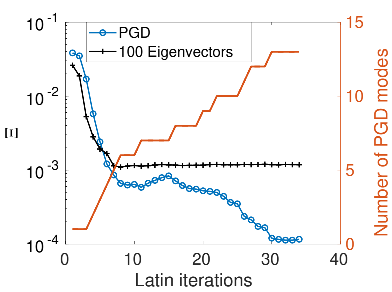


Figure 3: Convergence

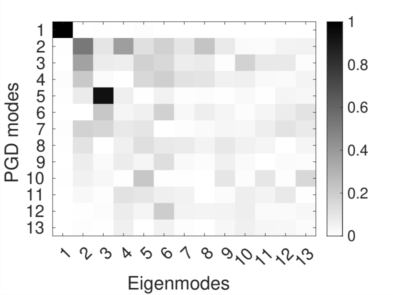
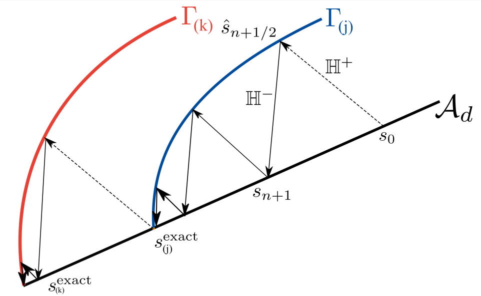


Figure 4: Modes comparison

$$u_\omega(x,\omega) = u_0 + \sum_{i=1}^{m} \lambda^i(\omega)\bar{u}^i(x)$$

Multi-query problem

Parametric work for material variability



Variability in plastic behaviour [Heyberger et al. 2013]

- Change in the Γ space
- The different solutions share the same admissibility

Parametric test: multiple loading for nonlinear dynamics

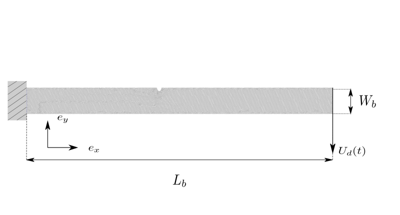


Figure 5: 2D-geometry

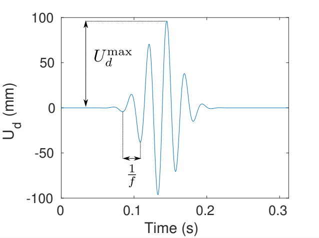
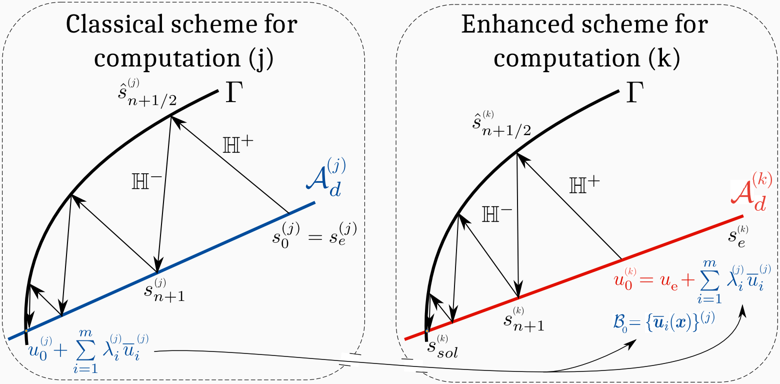


Figure 6: End loading

Testing case

- 2-parameters loading
  - $f \in [5\,\text{Hz}, 90\,\text{Hz}]$
  - $U_d^{\max} \in [0\,\text{mm}, 60\,\text{mm}]$
- Set of 600 parameters pairs

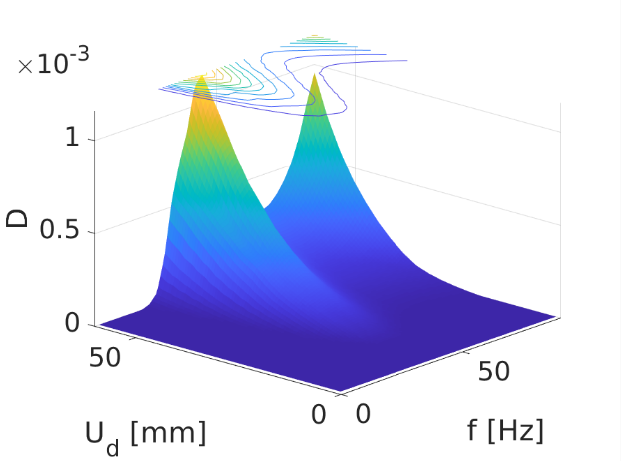
Standard and enhanced nonlinear LATIN-PGD schemes



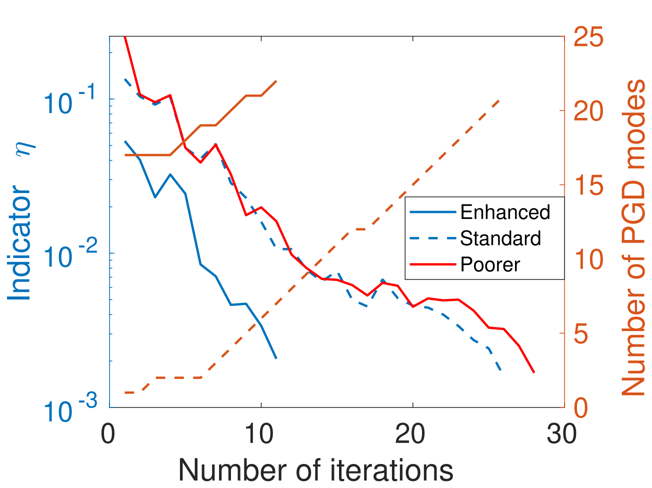
➔ Initialisation of the reduced basis

➔ Initialisation of the displacement field

Maximum damage within the structure

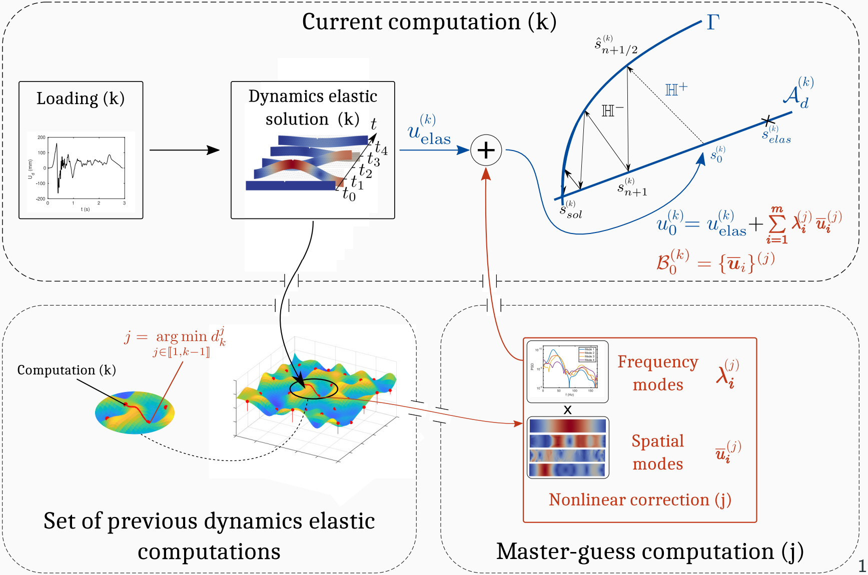


Enhancing the new computation



➔ Discriminating wise master-guess for initialisation

Enhanced multi-query strategy



Proximity indicator

Singular value decomposition of each elastic solution:

$$\begin{cases} \boldsymbol{U}_e^{(k)} \approx \boldsymbol{S}_\Omega^{(k)} \, \boldsymbol{\Sigma}^{(k)} \, {\boldsymbol{S}_t^{(k)}}^T \\ \boldsymbol{U}_e^{(j)} \approx \boldsymbol{S}_\Omega^{(j)} \, \boldsymbol{\Sigma}^{(j)} \, {\boldsymbol{S}_t^{(j)}}^T, \end{cases}$$

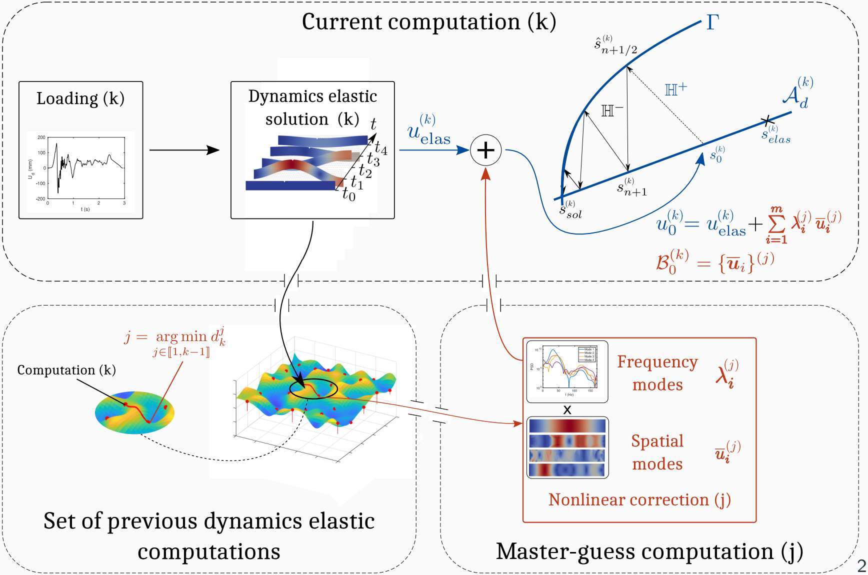
Proximity between elastic computations based on SVD

Complex space analysis:

$$\| \, |\boldsymbol{\Sigma}^{(j)} \mathrm{e}^{i\boldsymbol{\Theta}} - \boldsymbol{\Sigma}^{(k)}| \, \|_2$$

Grassmann distance $\boldsymbol{\Theta}$ between subspaces spanned by $\boldsymbol{S}_\Omega^{(k)}$ and $\boldsymbol{S}_\Omega^{(j)}$

Enhanced multi-query strategy



Finding an optimal path

- Ensuring some close simulations are included in the parent set
- Finding the shortest path to perform all computations once and only once
    - Travelling Salesman Problem (TSP)
    - Genetic algorithm with specific crossover operators [Zhang et al. 2022] $\Rightarrow$ good convergence of the algorithm

Global strategy

1. Compute all the elastic responses $U_e^{(i)},\ \forall i \in [1, n]$

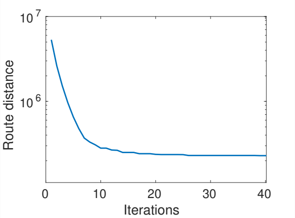
2. Evaluate proximity between all scenarios
   $\|\, |\boldsymbol{\Sigma}^{(j)} \mathrm{e}^{i\boldsymbol{\Theta}} - \boldsymbol{\Sigma}^{(k)}|\, \|_2,\ \forall (j, k) \in [1, n]^2$

3. Minimize the total path $\rightarrow$ optimal order for running the simulations

4. Run the first two simulations

5. From the third simulation until the last one:
   - Look for the closer elastic solution
   - Enhanced simulation

Optimal path



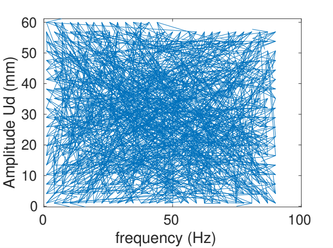


Figure 7: Initial route

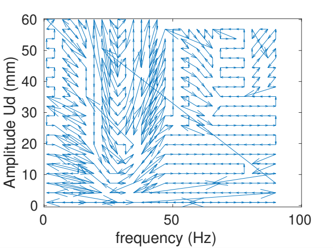
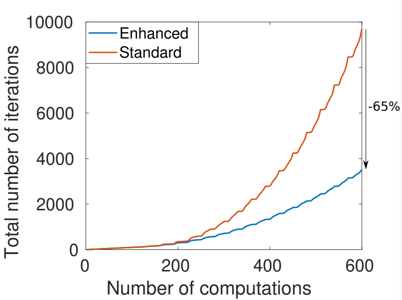
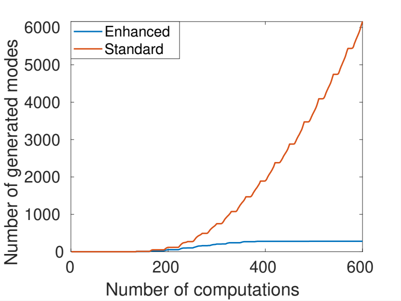


Figure 8: Final converged route

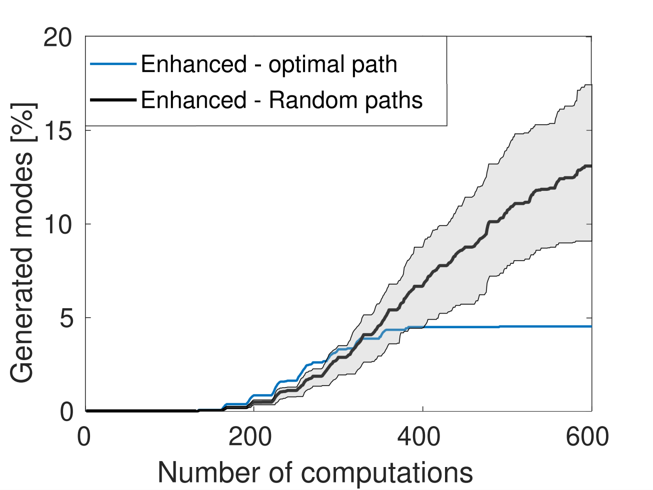
Negligible numerical cost compared to nonlinear computations

Results for the multi-query enhanced strategy





Influence of the order of the computations



Conclusion

- Nonlinear solver
    - Takes advantage of the separation of variables
    - Reduced-order modelling for the linear problem
    - Multi-query framework
        - Takes advantage of redundancy
        - Importance of the choice of the upstream simulation

- Parametrised problem
    - Combine several upstream computations
    - Use the proximity indicator to take advantage of the multi-fidelity solver
    - Decrease the number of computations (metamodels)

Thank you for your attention

Do you have any questions?

Grassmann distance