GDR-GDM

Dérivation thermodynamique du modèle d'endommagement à gradient non local Eikonal par le formalisme des milieux micromorphes avec une métrique Riemannienne

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La rochelle, June 2023

Plan

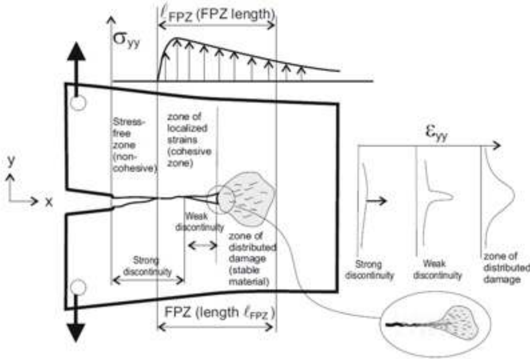
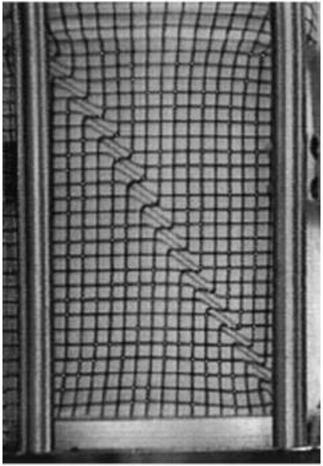


Schéma de la Fracture Process Zone (FPZ) [1]

1. Huespe and Oliver, 2011.

Introduction

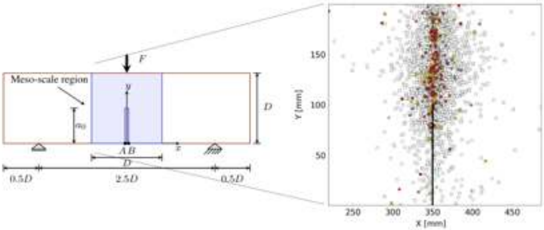
Introduction



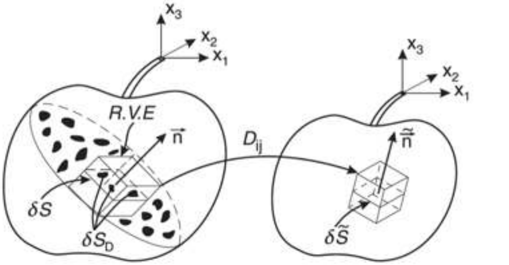
Strain localization in a sand specimen (Alshibli and Sture, 2000)

Introduction

Acoustic events in a bending test (Grégoire et al., 2015)



Illustrative scheme of damaged volume element (Lemaitre and Desmorat, 2005)



Introduction

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- Continuum damage mechanics [Kachanov, 1958; Rabotnov, 1969; Lemaitre, 1971; Marigo, 1981; Mazars, 1984]
  - Localization $\Rightarrow$ **infinity** solutions linearly independent at the bifurcation point
  - Strain field $\rightarrow$ Dirac $\Rightarrow$ Hypothesis of micro-cracks well distributed in the Representative Elementary Volume **is no longer valid**
  - The size of the localization zone **cannot be determined**.
  - **Mesh-dependent** finite element results

Introduction

- Continuum damage mechanics [Kachanov, 1958; Rabotnov, 1969; Lemaitre, 1971; Marigo, 1981; Mazars, 1984]
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  - The size of the localization zone **cannot be determined**.
  - **Mesh-dependent** finite element results
- Classic solutions (regularization methods)
  - Energetic regularization [Hillerborg et al., 1976]
  - **Non-local methods** [Pijaudier-Cabot and Bažant, 1987; Peerlings et al., 1996]
  - Delay-damage in dynamics [Allix and Deü, 1997; Allix et al., 2003; Desmorat et al., 2010]
  - Phase-field [Francfort and Marigo, 1998; Bourdin et al., 2000], Thick Level Set [Moës et al., 2011], Lip-field approach [Moës and Chevaugeon, 2021]

Non-local models

Classic non-local models

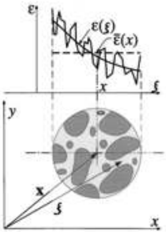
Integral non-local (INL) [2]

$$\bar{e}(\boldsymbol{x}) = \frac{1}{V_r(\boldsymbol{x})} \int_\Omega \phi\left(l_{\boldsymbol{x\xi}}, l_c\right) e(\boldsymbol{\xi}) d\Omega_\xi$$

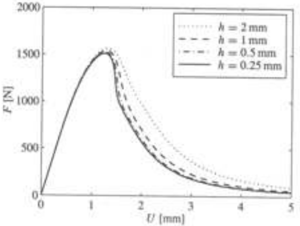
Gradient-enhanced non-local (GNL) [3]

$$\bar{e}(\boldsymbol{x}) - c\nabla^2 \bar{e}(\boldsymbol{x}) = e(\boldsymbol{x})$$

2. Pijaudier-Cabot and Bažant, 1987.
3. Peerlings et al., 1996.



Strain field in the scale of the REV and its average (Bažant and Jirásek, 2002)

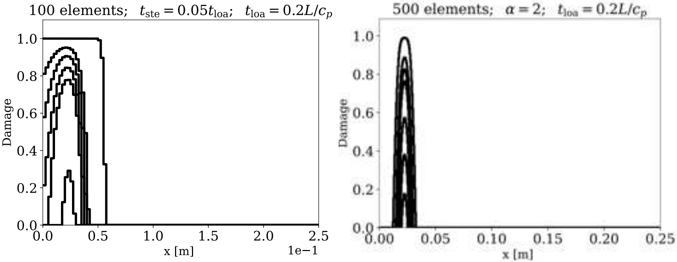


Convergence of the structural response upon mesh refinement (Peerlings, 1999)

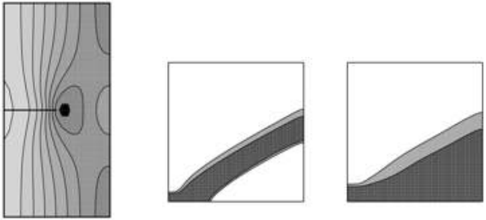
[Geers et al., 1998; Simone et al., 2004; Krayani et al., 2009; Giry et al., 2011; Pijaudier-Cabot and Grégoire, 2014]

Drawbacks

- Issues of standard non-local regularization methods
  - Damage **diffusion** and **attraction**
  - Damage **initiation and shear bands**
  - Spalling distance **cannot be determined**
- Need of **evolving** non-local interactions
  - Interactions : stress ? strain ? **damage** ? ... ?



Source: Ribeiro Nogueira et al., 2022



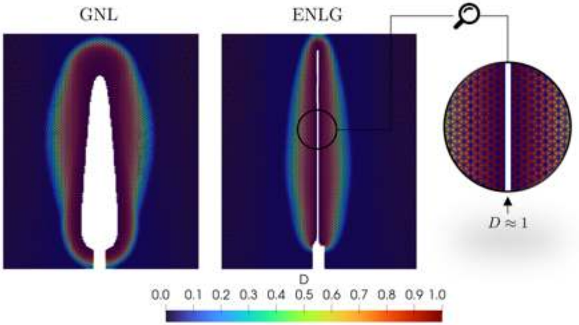
Source: Simone et al., 2004

Eikonal non local integral and gradient (ENLI, ENLG)

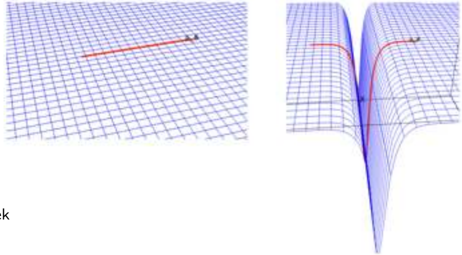
$$\bar{e}(\boldsymbol{x}) = \frac{1}{V_r(\boldsymbol{x})} \int_{\Omega} \phi(\tilde{l}_{\boldsymbol{x}\boldsymbol{\xi}}, l_c) e(\boldsymbol{\xi}) d\Omega_{\xi}$$

$$\bar{e} - c \frac{1}{\sqrt{\det \mathbf{g}}} \nabla \cdot \left(\sqrt{\det \mathbf{g}}\, \mathbf{g}^{-1} \cdot \nabla \bar{e} \right) = e$$

- Notion of interaction distances in function of damage - evolving **Riemannian metric**
- Non-local interactions **vanish** across a pseudo-crack
- Natural modelling of the transition between diffuse damage and **localized** macro-crack



Source: Ribeiro Nogueira et al. 2023 [Forthcoming]



(Desmorat and Gatuingt, 2007; Desmorat et al., 2015; Rastiello et al., 2018; Jirásek and Desmorat, 2019; Thierry, 2021; Marconi, 2022; Ribeiro Nogueira et al., 2022)

Thermodynamics of Eikonal gradient damage

Derivation from the micromorphic framework

Extended virtual power principle (Forest, 2009):

$$\mathcal{P}^*_{\text{int}}(\boldsymbol{v}^*, \dot{\eta}^*) = -\int_\Omega \underbrace{\left(\boldsymbol{\sigma} : \boldsymbol{\varepsilon}(\boldsymbol{v}^*) + a\dot{\eta}^* + \boldsymbol{b} \cdot \nabla\dot{\eta}^*\right)}_{:= p^{(i)}(\boldsymbol{v}^*, \dot{\eta}^*)} dV$$

$$\mathcal{P}^*_{\text{ext}}(\boldsymbol{v}^*, \dot{\eta}^*) = \int_\Omega \left(a^e\dot{\eta}^* + \boldsymbol{b}^e \cdot \nabla\dot{\eta}^*\right) dV + \int_{\partial\Omega} \left(\boldsymbol{t}^d \cdot \boldsymbol{v}^* + a^c\dot{\eta}^*\right) dS$$

$$\begin{aligned} \nabla \cdot \boldsymbol{\sigma} &= \boldsymbol{0} && \text{on } \Omega \\ \nabla \cdot (\boldsymbol{b} - \boldsymbol{b}^e) - a + a^e &= 0 && \text{on } \Omega \\ \boldsymbol{t}^d &= \boldsymbol{\sigma} \cdot \boldsymbol{n} && \text{on } \partial\Omega \\ a^c &= (\boldsymbol{b} - \boldsymbol{b}^e) \cdot \boldsymbol{n} && \text{on } \partial\Omega \end{aligned}$$

Proposition of a free-energy Helmholtz potential from Peerlings et al., 2004:

$$\rho\psi = \rho\psi(\boldsymbol{\varepsilon},\mathbf{D},\bar{e},\tilde{\nabla}\bar{e}) = \rho\psi_0 + \frac{1}{2}h(e-\bar{e})^2 + \frac{1}{2}hc\|\tilde{\nabla}\bar{e}\|^2_{\mathbf{g}}, \qquad \chi = \bar{e} \text{ (micromorphic variable)}$$

The state laws read:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_0 + h(e-\bar{e})\frac{\partial e}{\partial \boldsymbol{\varepsilon}} \qquad \sigma^{ij} = \sigma_0^{ij} + h(e-\bar{e})\left(\frac{\partial e}{\partial \boldsymbol{\varepsilon}}\right)^{ij}$$

$$\mathbf{Y} = \mathbf{Y}_0 + \mathbf{Z} \qquad Y^{ij} = Y_0^{ij} + Z^{ij}$$

$$a = -h(e-\bar{e})$$

$$\boldsymbol{b} = \rho\frac{\partial\psi}{\partial\tilde{\nabla}\bar{e}} = hc\mathbf{g}\cdot\tilde{\nabla}\bar{e} \qquad b_i = hc\mathrm{g}_{ij}(\tilde{\nabla}\bar{e})^j = hc(d\bar{e})_i = hc\partial_i\bar{e}$$

where:

$$\mathbf{Z} = -\frac{hc}{2}\frac{\partial\|d\bar{e}\|^2_{\mathbf{g}}}{\partial\mathbf{g}^{-1}} : \frac{\partial\mathbf{g}^{-1}}{\partial\mathbf{D}} \qquad Z^{ij} = -\frac{hc}{2}\left(\frac{\partial\|d\bar{e}\|^2_{\mathbf{g}}}{\partial\mathbf{g}^{-1}}\right)_{kl}\left(\frac{\partial\mathbf{g}^{-1}}{\partial\mathbf{D}}\right)^{klij}$$

From the balance equation, one has:

$$\tilde{\nabla} \cdot \boldsymbol{b}^\flat - a = 0 \quad \text{with} \quad \boldsymbol{b}^\flat = hc\mathbf{g}^{-1} \cdot \boldsymbol{b} = hc\tilde{\nabla}\bar{e} \qquad (\boldsymbol{b})^{\flat,i} = hc\mathrm{g}^{ij}\partial_j\bar{e}$$

One obtains:

$$\bar{e} - c\tilde{\nabla} \cdot \left(\tilde{\nabla}\bar{e}\right) = e \Rightarrow \bar{e} - \frac{c}{\sqrt{\det \mathbf{g}}}\nabla \cdot (\sqrt{\det \mathbf{g}}\; \mathbf{g}^{-1} \cdot \nabla\bar{e}) = e$$

with the "Laplace-Beltrami operator" defined as:

$$\Delta f := \nabla \cdot \nabla f = \frac{1}{\sqrt{\det(\mathrm{g}_{ij})}}\partial_i \left(\sqrt{\det(\mathrm{g}_{ij})}\mathrm{g}^{ij}\partial_j f\right).$$

Energy dissipation is modified for anisotropic and isotropic damage:

$$\mathcal{D} = \left(\mathbf{Y}_0 + \frac{hc}{2}(\nabla\bar{e} \otimes \nabla\bar{e})\right) : \dot{\mathbf{D}} \geq 0 \qquad \mathcal{D} = Y\dot{D} = \left(-3\rho\frac{\partial\psi_0}{\partial D} + \frac{hc}{2}\nabla\bar{e} \cdot \nabla\bar{e}\right)\dot{D} \geq 0$$

Admissibility spaces

$$\mathcal{U} = \{\boldsymbol{w} \mid \boldsymbol{w} \in H^1(\Omega),\ \boldsymbol{w} = \boldsymbol{u}^d \ \ \forall \boldsymbol{x} \in \partial\Omega_u\}$$
$$\mathcal{U}(\mathbf{0}) = \{\boldsymbol{w} \mid \boldsymbol{w} \in H^1(\Omega),\ \boldsymbol{w} = \mathbf{0} \ \ \forall \boldsymbol{x} \in \partial\Omega_u\}$$
$$\mathcal{V} = \{w \mid w \in H^1(\Omega)\}$$

Variational formulation: Search $\boldsymbol{u} \in \mathcal{U}$ and $\bar{e} \in \mathcal{V}$ such as:

$$\int_\Omega \boldsymbol{\varepsilon}(\boldsymbol{u}) : \tilde{\mathbb{E}}(\mathbf{D}) : \boldsymbol{\varepsilon}(\boldsymbol{v})\, d\Omega = \int_{\partial\Omega_F} \boldsymbol{t}^d \cdot \boldsymbol{v}\, dS \ \ \forall \boldsymbol{v} \in \mathcal{U}(\mathbf{0})$$

$$\int_\Omega \sqrt{\det \mathbf{g}}\,\bar{e}\eta\, d\Omega + \int_\Omega c\sqrt{\det \mathbf{g}}(\mathbf{g}^{-1} \cdot \nabla\bar{e}) \cdot \nabla\eta\, d\Omega = \int_\Omega \sqrt{\det \mathbf{g}}\,e\eta\, d\Omega \ \ \forall \eta \in \mathcal{V}$$

Choice of Riemannian metric (induced anisotropic interactions)

$$\mathbf{g} = (\mathbf{q} - \mathbf{D})^{-1} \qquad g_{ij} = [(\mathbf{q} - \mathbf{D})^{-1}]_{ij}$$

where $\mathbf{q} = \mathbf{I} = \delta_{ij}$.

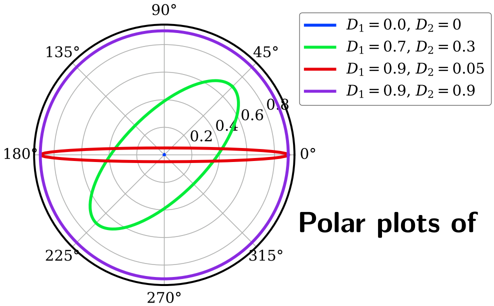
General aspects

Given a second order tensor $\mathbf{S}$, it has an homogeneous associated polynomial in its eigenbasis $\boldsymbol{v}'_i \otimes \boldsymbol{v}'_j$:

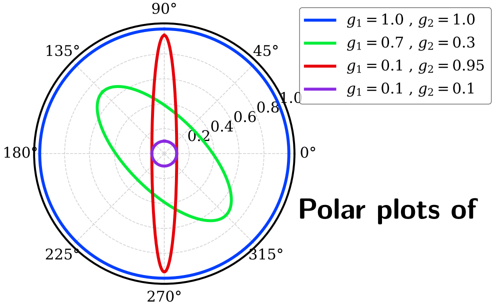
$$\begin{aligned} p(\boldsymbol{x}') &= (\mathbf{S}')^{-2}(\boldsymbol{x}',\boldsymbol{x}') \\ &= (\mathbf{S}')^{-2} : (\boldsymbol{x}' \otimes \boldsymbol{x}') \\ &= \frac{x'^2}{S_1^2} + \frac{y'^2}{S_2^2} + \frac{z'^2}{S_3^2} = 1 \end{aligned}$$

with $\boldsymbol{x}' = [x',y',z']$ a position vector.

The case $p(\boldsymbol{x}') = 1$ defines an ellipse in $\mathbb{R}^3/\mathbb{R}^2$ with the eigenvalues in the major and minor axis.

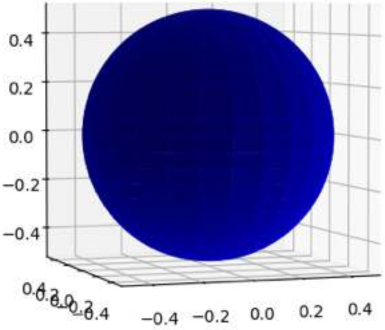


Polar plots of D

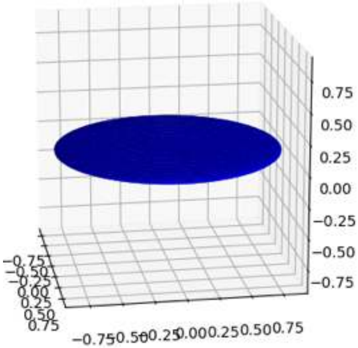


Polar plots of g^{-1}

$$\mathbf{g}^{-1} = \begin{bmatrix} 0.5 & & \\ & 0.5 & \\ & & 0.5 \end{bmatrix}$$



$$\mathbf{g}^{-1} = \begin{bmatrix} 0.9 & & \\ & 0.1 & \\ & & 0.9 \end{bmatrix}$$



Results

Isotropic damage simulations

The case of spherical metric tensor $\mathrm{g}^{-1} = (1 - D)\mathbf{I}$.

$$\rho\psi_0 = \frac{1}{2}(1 - D)\boldsymbol{\varepsilon} : \mathbb{E} : \boldsymbol{\varepsilon}, \qquad D = 1 - \frac{\kappa_0}{\kappa}\left(1 - \alpha + \alpha e^{-B(\kappa - \kappa_0)}\right), \quad \kappa = \max_t(\kappa_0, \bar{e})$$

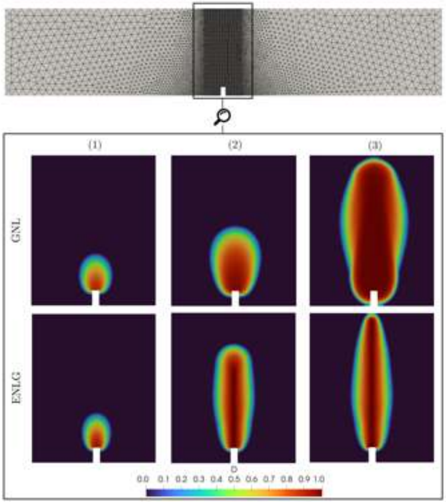
$$\boldsymbol{\sigma}(\boldsymbol{u}, D) = (1 - D)\mathbb{E} : \boldsymbol{\varepsilon}(\boldsymbol{u}) = \tilde{\mathbb{E}} : \boldsymbol{\varepsilon}(\boldsymbol{u})$$

Variational formulation reduced to (very similar to Poh and Sun, 2017):

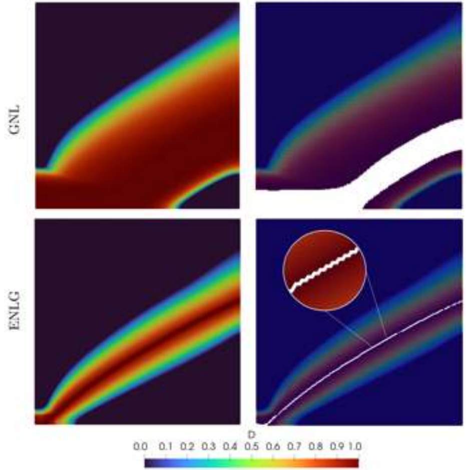
$$\int_{\Omega^h} (1 - D^{h,k})\boldsymbol{\varepsilon}(\boldsymbol{u}^{h,k+1}) : \mathbb{E} : \boldsymbol{\varepsilon}(\boldsymbol{v}^h)\, dV = \int_{\partial\Omega_F^h} \boldsymbol{t}^d \cdot \boldsymbol{v}^h \, dS$$

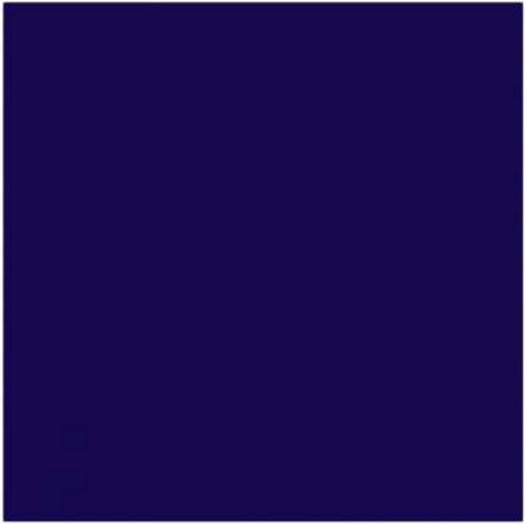
$$\int_{\Omega^h} \frac{1}{1 - D^{h,k}}\, \bar{e}^{h,k+1}\, \eta^h dV + \int_{\Omega^h} c\nabla\bar{e}^{h,k+1} \cdot \nabla\eta^h dV = \int_{\Omega^h} \frac{1}{1 - D^{h,k}}\, e(\boldsymbol{\epsilon}(\boldsymbol{u}^{h,k+1}))\, \eta^h dV$$

Four-point bending



Shear band





Anisotropic damage simulations

$$\rho\psi_0 = \sup_{\sigma}[\boldsymbol{\sigma} : \boldsymbol{\varepsilon} - \rho\psi_0^\star(\boldsymbol{\sigma})]$$

$$\rho\psi_0^\star(\boldsymbol{\sigma}) = \frac{\mathrm{tr}\left(\mathbf{H}\cdot\boldsymbol{\sigma}^D\cdot\mathbf{H}\cdot\boldsymbol{\sigma}^D\right)}{4G} + \frac{1}{18K}\left[\frac{1}{3}\mathrm{tr}\mathbf{H}^2\left\langle\mathrm{tr}\boldsymbol{\sigma}\right\rangle^2 + \left\langle-\mathrm{tr}\boldsymbol{\sigma}\right\rangle^2\right]$$

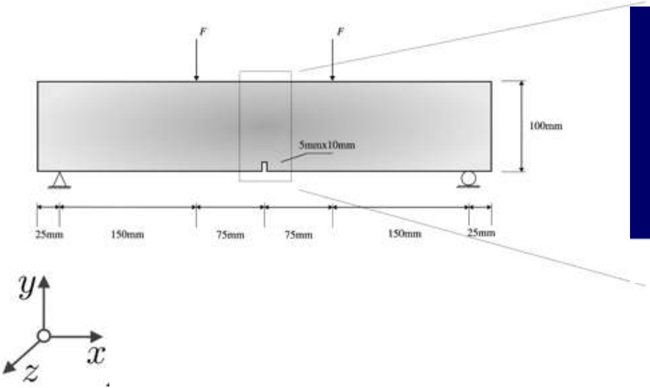
The effective Hooke's tensor is defined as (Desmorat, 2015):

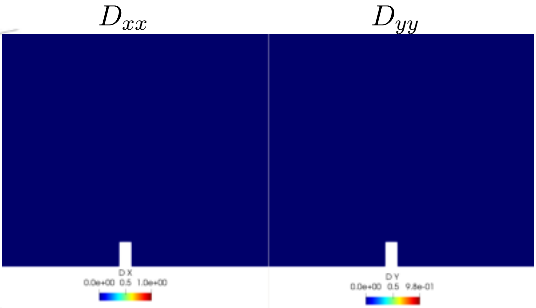
$$\tilde{\mathbb{E}} = 2G\left[\mathbf{H}^{-1}\,\underline{\overline{\otimes}}\,\mathbf{H}^{-1} - \frac{\mathbf{H}^{-2}\otimes\mathbf{H}^{-2}}{\mathrm{tr}\mathbf{H}^{-2}}\right] + \frac{3K}{\mathrm{tr}\mathbf{H}^2}\mathbf{I}\otimes\mathbf{I} \qquad \text{with} \quad \mathbf{H} = (\mathbf{I}-\mathbf{D})^{-\frac{1}{2}}$$

The criterion function and evolution read:

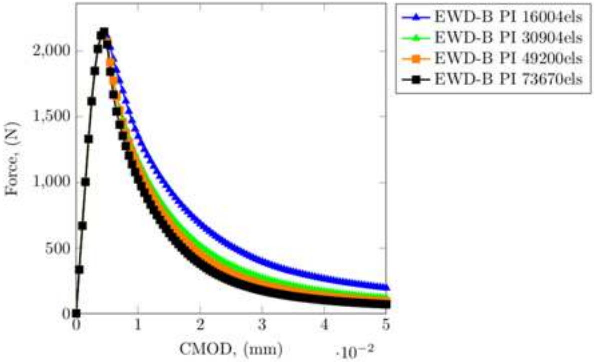
$$f = \bar{e} - \kappa, \quad \kappa = \kappa_0 + SR_v^s(\mathrm{tr}\,\mathbf{H} - 3), \quad \dot{\mathbf{H}} = \dot{\lambda}\langle\tilde{\boldsymbol{\varepsilon}}\rangle, \quad \tilde{\boldsymbol{\varepsilon}} = \mathbb{E}^{-1} : \boldsymbol{\sigma}$$

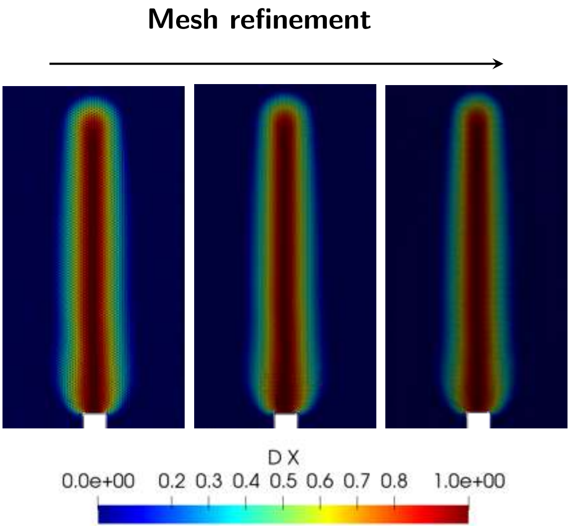
Four-point bending

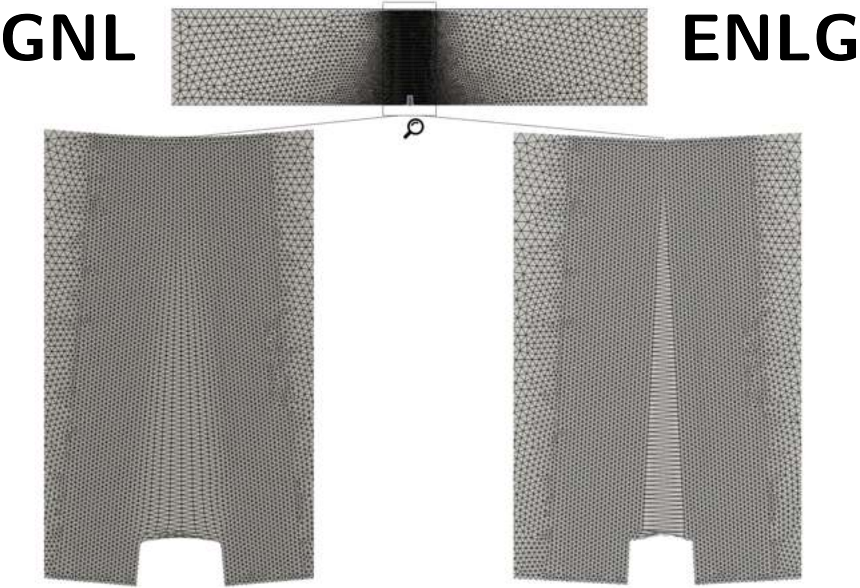




Four-point bending

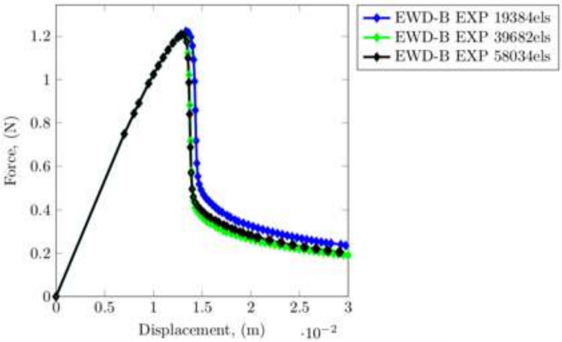


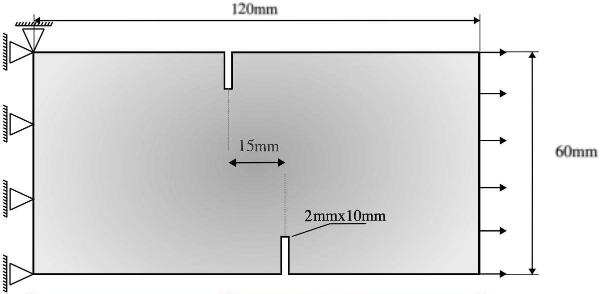


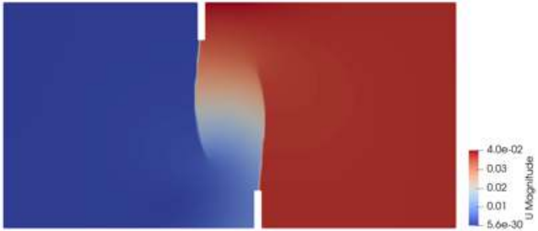

GNL
ENLG

Shi test

[Shi et al., 2000]



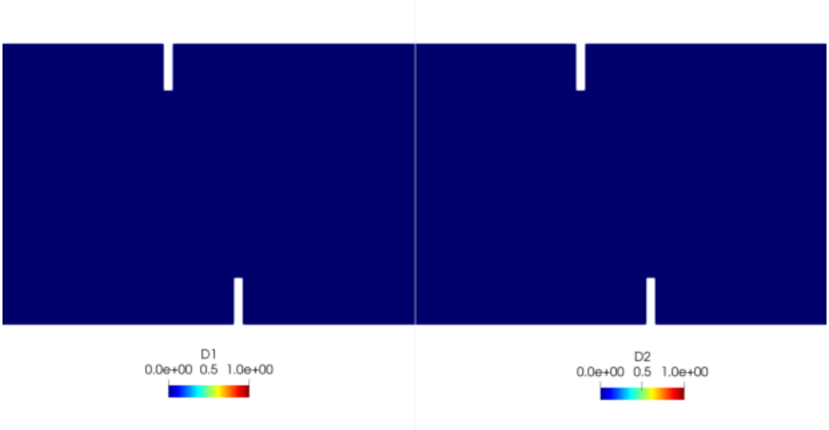




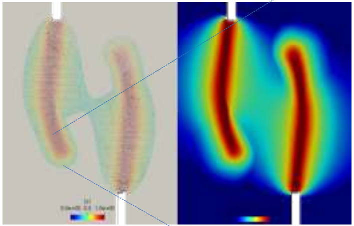
Shi test

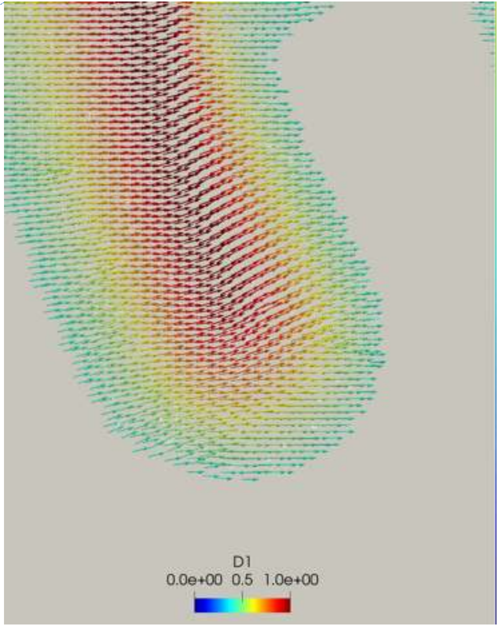
[Shi et al., 2000]

Damage eigenvalues D_1 and D_2

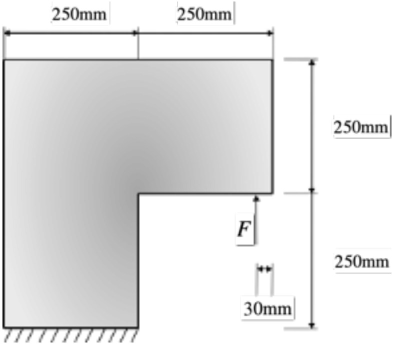


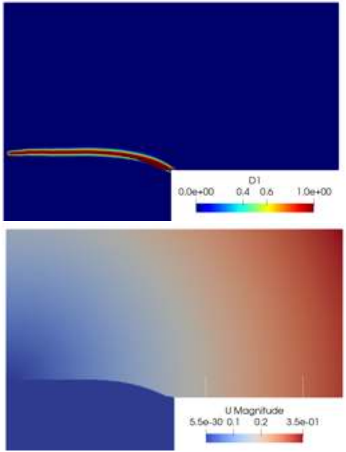
Eigenvectors associated to damage eigenvalues ⇒ crack orientation

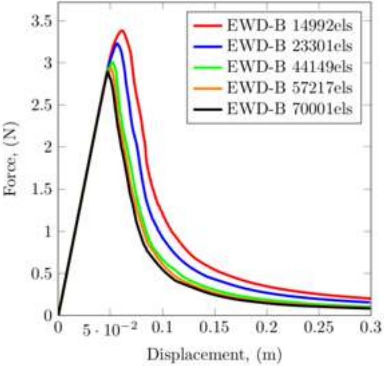




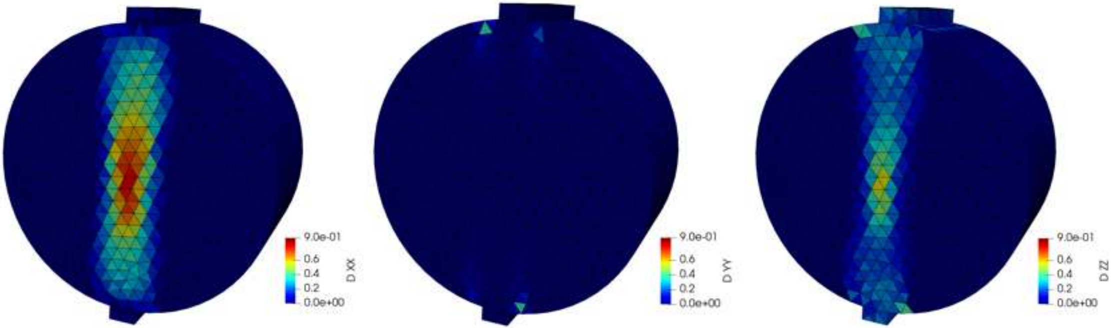
L-shape


250mm
250mm
250mm
F
250mm
30mm


D1
0.0e+00
0.4
0.6
1.0e+00
U Magnitude
5.5e-30
0.1
0.2
3.5e-01

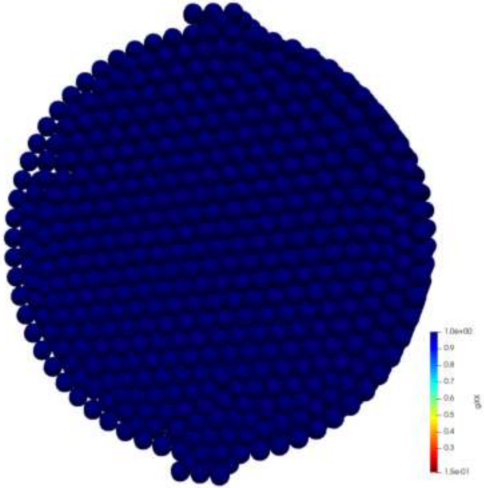

EWD-B 14992els
EWD-B 23301els
EWD-B 44149els
EWD-B 57217els
EWD-B 70001els
Force, (N)
Displacement, (m)
0
0.5
1
1.5
2
2.5
3
3.5
$5 \cdot 10^{-2}$
0.1
0.15
0.2
0.25
0.3

3D simulations and interactions - brazilian test

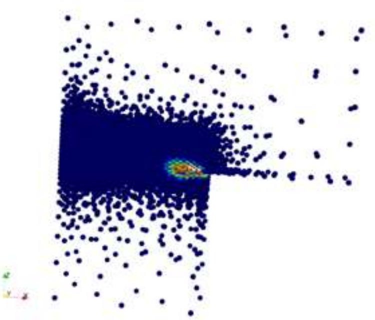


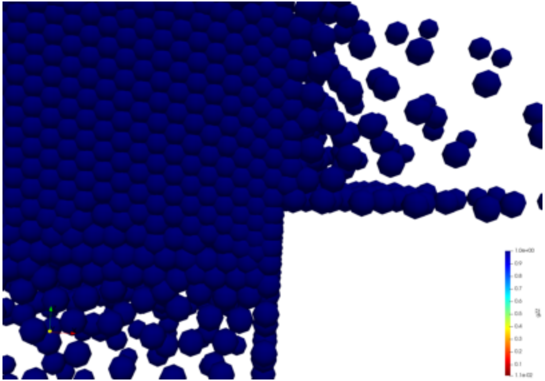
Anisotropic behavior guaranteed by only **one** tensorial damage state variable

Ellipsoids representing evolving anisotropic non-local interactions **collapse upon localization**



3D simulations and interactions - L shape





Ellipsoids representing evolving anisotropic non-local interactions **collapse upon localization**

Conclusion

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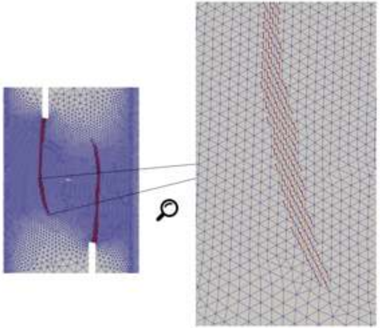
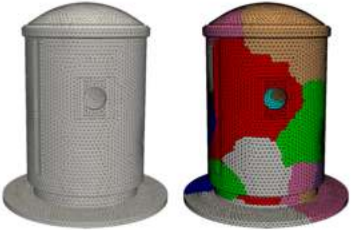
- **Overall remarks** :
  - Classic micromorphic framework **enhanced** by differential geometry concepts
  - Realistic crack paths obtained with **ENLG**
  - Induced anisotropic non-local **interactions**
  - **Relocalization** properties bridging damage and fracture
  - Metric tensor could possibly depend on **other** internal variables or even treat **initial anisotropic** media...
- **Perspectives** :
  - **Large scale** simulations
  - Transition to a **strong discontinuities** approach



Eigenvectors of damage tensor $\mathbf{D}$



Example of nuclear reactor mesh (Benchmark VERCORS)

Thank you for your attention

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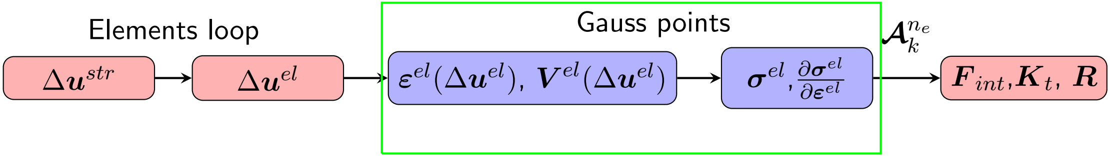
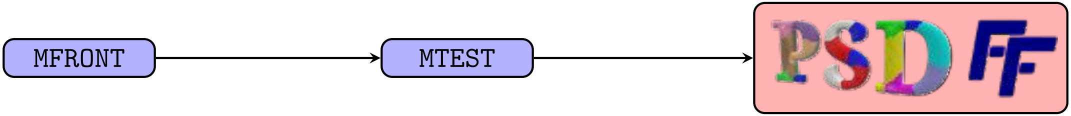
Where Mfront takes action ?

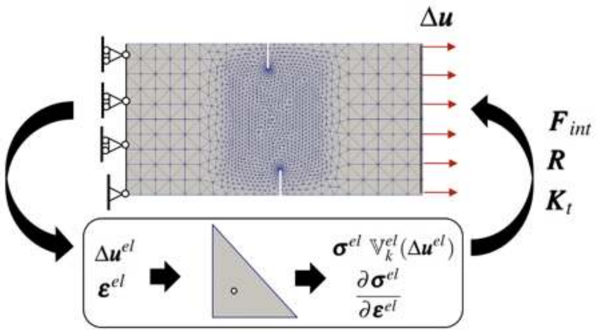


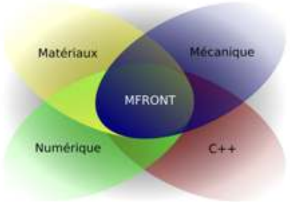



```
@DSL ImplicitII;
@Behaviour desmorat3Dnonlocal;
@Author Breno Ribeiro Nogueira;
if (tr_e > 0) {
 const auto tr_H2 = H | H;
 sig = 2. * G * (symmetric_product_aba(iH, e) − (iH2 | e) * iH2 / tr_iH2) +
        3. * K / tr_H2 * tr_e * Stensor::Id();
}
...
```

```
@ImposedStrain 'EXX' {0:0,1:1e−3};
@NonLinearConstraint<Stress> 'SXX−SYY';
```

Where Mfront takes action ?





Helfer et al., 2015

Récalage de paramètres

- Un exemple de récalage des paramètres pour les deux essais
- Possibilité d'avoir des réponses moins fragiles pour **ENLG**
- Propriétés de rélocalisation sont **indépendantes** des paramètres