Modèle d'endommagement anisotrope basé sur une décomposition du tenseur d'élasticité en termes de covariants

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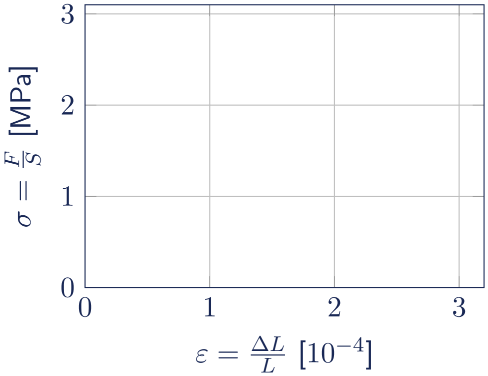
Context – Observations

Tensile test on concrete
(Terrien, 1980)



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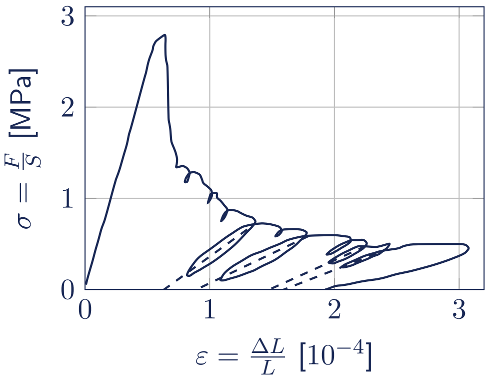

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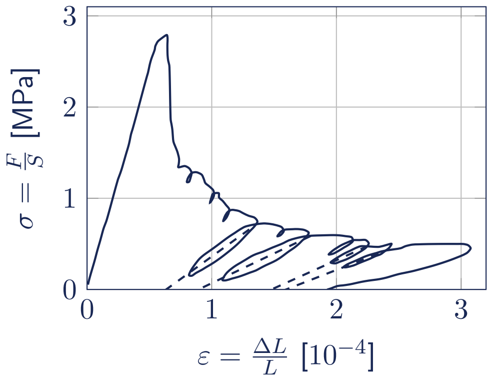
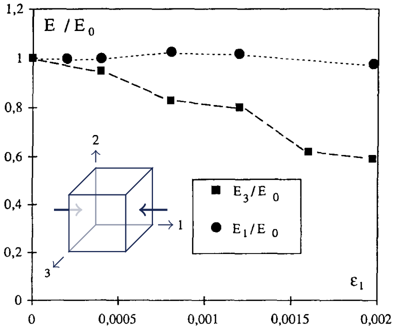


Damage-induced anistropy
(Berthaud, 1991)



Context – Elasticity

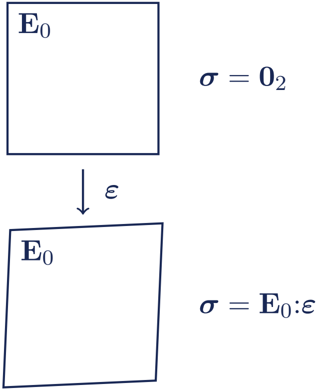
Notations

$$\varepsilon \in S^2(\mathbb{R}^2) \qquad \text{(Strain)}$$
$$\boldsymbol{\sigma} \in S^2(\mathbb{R}^2) \qquad \text{(Stress)}$$
$$\mathbf{E} \in \mathbb{E}\mathrm{la}(\mathbb{R}^2) \qquad \text{(Elasticity tensor)}$$

Let us consider a material with elastic properties $\mathbf{E}_0$.

Applying a strain ε produces a homogeneous stress (reaction)

$$\boldsymbol{\sigma} = \mathbf{E}_0 : \varepsilon$$



Context – Damage

Let us consider a material with initial elastic properties $\mathbf{E}_0$.

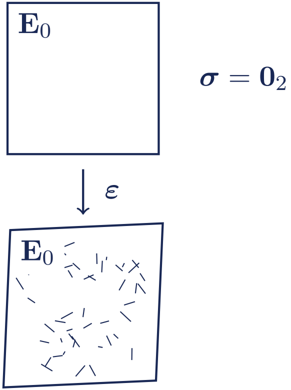
Applying a strain ε produces a change of state quantified by damage $\mathbf{D}$

$$\frac{\mathrm{d}\mathbf{D}}{\mathrm{d}t} = \mathsf{F}(\varepsilon, \mathbf{D})$$

and a reaction

$$\boldsymbol{\sigma} = \mathbf{E}(\mathbf{D}):\varepsilon \neq \mathbf{E}_0:\varepsilon$$

Remark Damage "decreases" $\mathbf{E}$ and its symmetries.



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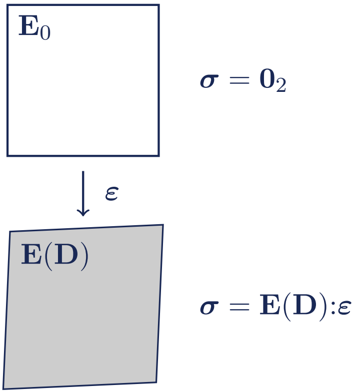

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Objective of the project

Formulating an anisotropic damage model for quasi-brittle materials in 2D

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Formulating an anisotropic damage model for quasi-brittle materials in 2D

Structure of a damage model

$$\mathcal{V} = \{\boldsymbol{\varepsilon}, \mathbf{D}, \ldots\} \qquad \text{(State variables)}$$

$$\boldsymbol{\sigma} = \mathbf{E}(\mathbf{D}):\boldsymbol{\varepsilon} \qquad \text{(Hooke's law)}$$

$$\frac{\mathrm{d}\mathbf{D}}{\mathrm{d}t} = \ldots \qquad \text{(Damage evolution)}$$

Constraints

- $\mathbf{E}(\mathbf{D})$ is positive definite
- Positive dissipation

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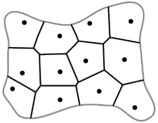
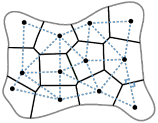
Objectives of the presentation

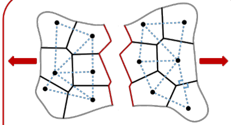
- Generate a dataset of effective elasticity tensors
- Definition of the damage (state) variable
- Formulate a state model $\mathbf{E}(\mathbf{D})$

Context – In the previous episode – Dataset of effective elasticity tensors

Discrete model
(Vassaux et al., 2016)



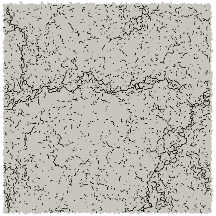




Virtual testing Measure $\mathbb{E}$ for

- 1 material,
- 36 meso-structures,
- 21 (prop and non-prop) loadings,
- 100 time steps,

for a total of $\approx$ 76 000 tensors.



Bi-tension

Shear

Damage variable – Distance of symmetry stratum

Distance to orthotropy in 2D (Antonelli et al., 2022)

Question What tensorial order for the damage variable?

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Tool Relative distance to a symmetry stratum Σ $\Delta_{\Sigma}(\mathbf{E}) = \min_{\mathbf{E}^* \in \Sigma} \frac{\|\mathbf{E} - \mathbf{E}^*\|}{\|\mathbf{E}\|}$

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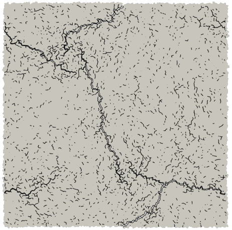
Illustration with periodic bi-tension



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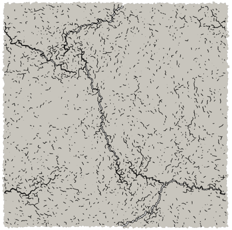

$\mathbf{E}$ [MPa]

$$\begin{bmatrix} 0.93 & -0.38 & -0.50 \\ -0.38 & 1.47 & 0.36 \\ -0.50 & 0.36 & 3.66 \end{bmatrix}$$

—— Hypersurface of $\mathbb{R}^2$ associated to tensor

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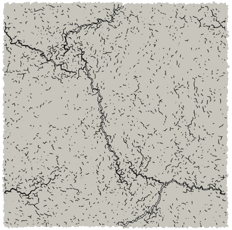

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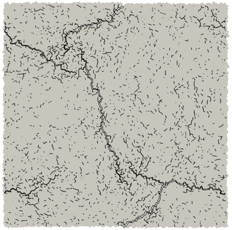

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Orthotropy $\Delta_{\mathcal{O}rt} = 0.013$

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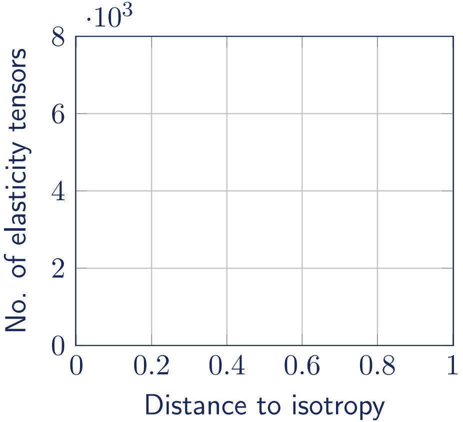
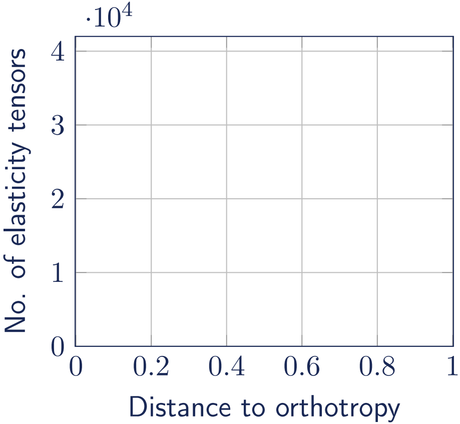
Damage variable – Anisotropy of measured effective elasticity tensors





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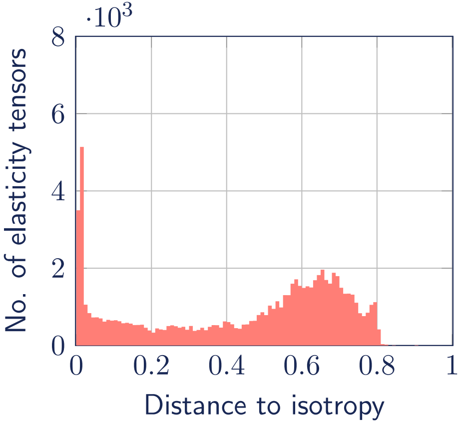

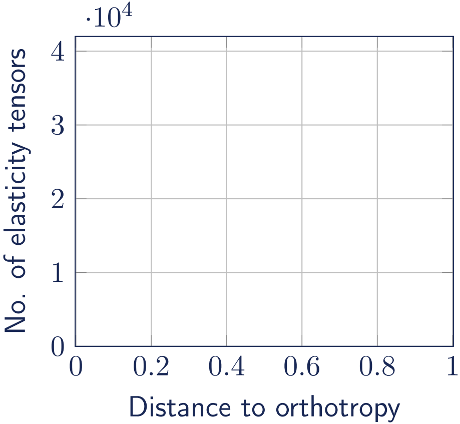

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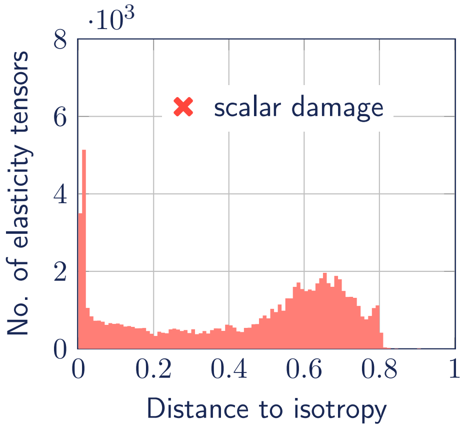

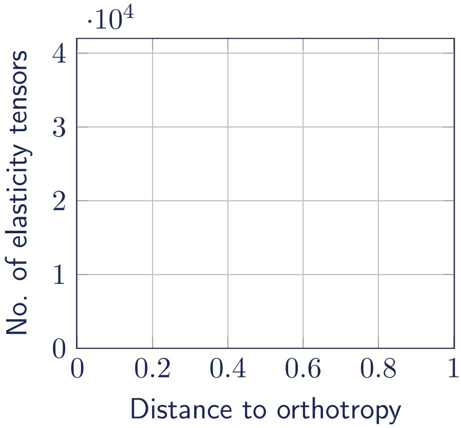

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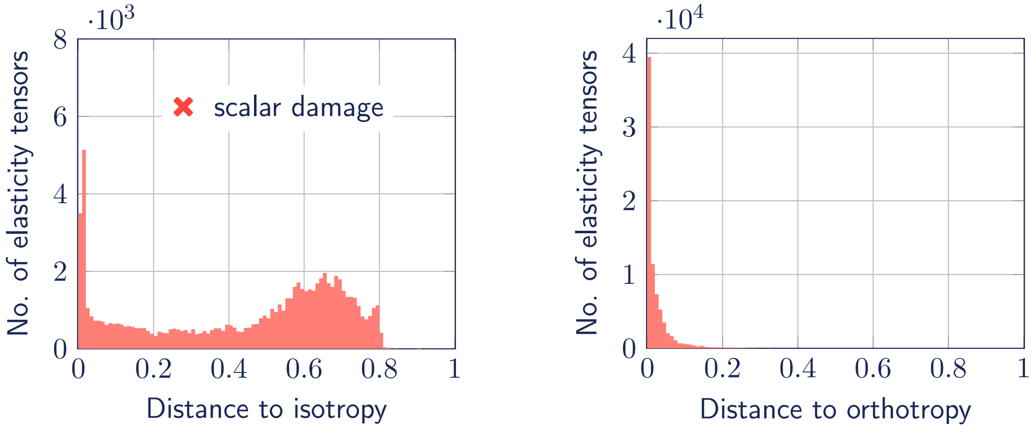

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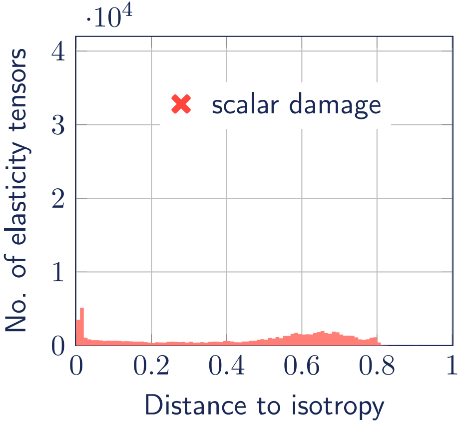

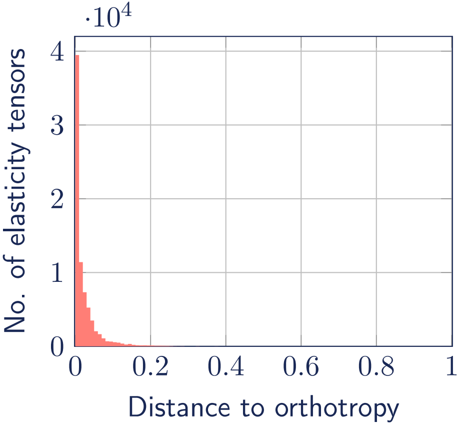

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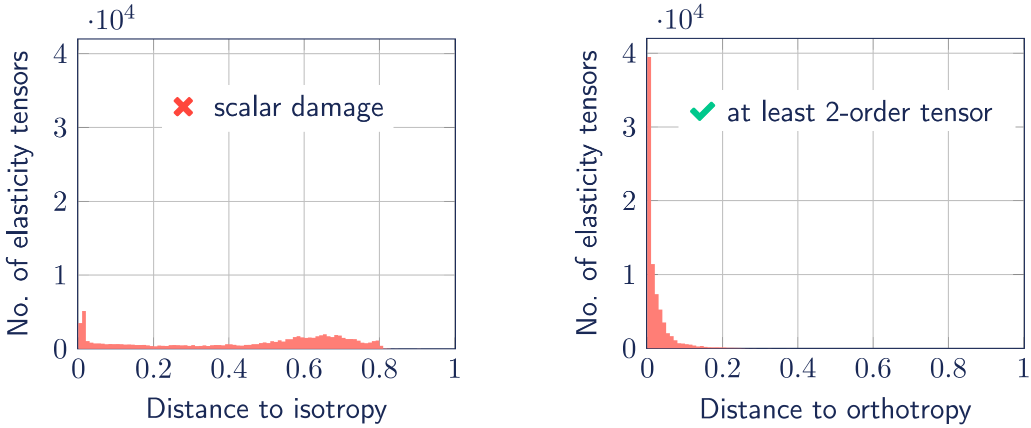

Damage variable – 2D Harmonic decomposition

Applications to elasticity tensor: 3D (Backus, 1970), 2D (Blinowski et al., 1996)

$$\mathbf{E} \cong (\mu, \kappa, \mathbf{d}', \mathbf{H}) \in \mathbb{H}^1(\mathbb{R}^2) \oplus \mathbb{H}^1(\mathbb{R}^2) \oplus \mathbb{H}^2(\mathbb{R}^2) \oplus \mathbb{H}^4(\mathbb{R}^2)$$

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Damage variable – State model basis and damage definition

Knowing isotropic $\mathbf{E}_0 \cong (\mu_0, \kappa_0, \mathbf{0}_2, \mathbf{0}_4)$ and $\mathbf{D}$, we want to model

$$\mathbf{E}(\mathbf{D}) = 2\mu(\mathbf{D})\mathbf{J} + \kappa(\mathbf{D})\mathbf{1}_2 \otimes \mathbf{1}_2 + \frac{1}{2}\left(\mathbf{d}'(\mathbf{D}) \otimes \mathbf{1}_2 + \mathbf{1}_2 \otimes \mathbf{d}'(\mathbf{D})\right) + \mathbf{H}(\mathbf{D})$$

How to define damage?

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Damage variable

$$\mathbf{D} = (\mathbf{d}_0 - \mathbf{d}) \cdot \mathbf{d}_0^{-1} = \mathbf{1}_2 - \frac{1}{2\kappa_0}\mathbf{d}$$

Damage variable – State model basis and damage definition

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$$\mathbf{E}(\mathbf{D}) = 2\mu(\mathbf{D})\mathbf{J} + \kappa(\mathbf{D})\mathbf{1}_2 \otimes \mathbf{1}_2 + \frac{1}{2}\left(\mathbf{d}'(\mathbf{D}) \otimes \mathbf{1}_2 + \mathbf{1}_2 \otimes \mathbf{d}'(\mathbf{D})\right) + \mathbf{H}(\mathbf{D})$$

How to define damage? Using the harmonic decomposition

$$\mu = \frac{1}{8}\left(2 \operatorname{tr} \mathbf{v} - \operatorname{tr} \mathbf{d}\right) \qquad \mathbf{d}' = \mathbf{d} - \frac{1}{2} \operatorname{tr} \mathbf{d}\ \mathbf{1}_2$$

$$\kappa = \frac{1}{4} \operatorname{tr} \mathbf{d} \qquad \mathbf{H} = \mathbf{E} - \mathbf{Iso} - \mathbf{Dil}$$

Damage variable

$$\mathbf{D} = (\mathbf{d}_0 - \mathbf{d}) \cdot \mathbf{d}_0^{-1} = \mathbf{1}_2 - \frac{1}{2\kappa_0}\mathbf{d} \iff \mathbf{d} = 2\kappa_0(\mathbf{1}_2 - \mathbf{D})$$

State model – Advantages of the damage definition

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State model – Partial model formulation

Objectives of the presentation

- ✓ Generate a dataset of effective elasticity tensors
- ✓ Define damage variable (quantify micro-cracking)
- ○ Formulate a state model $\mathbf{E}(\mathbf{D})$

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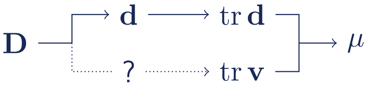

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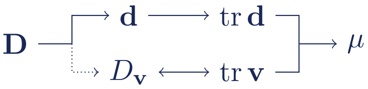

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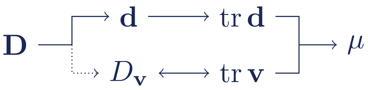

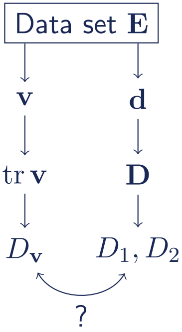

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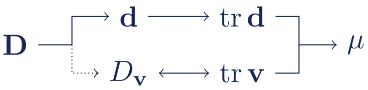

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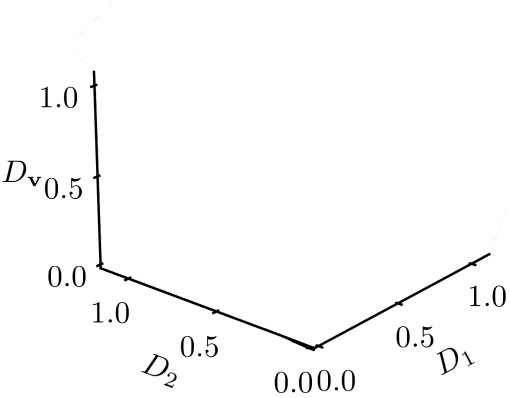

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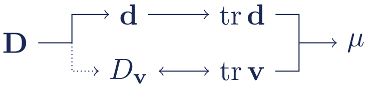

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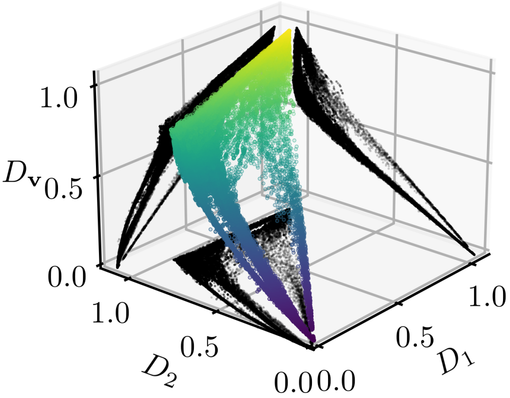

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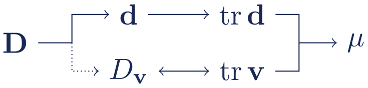

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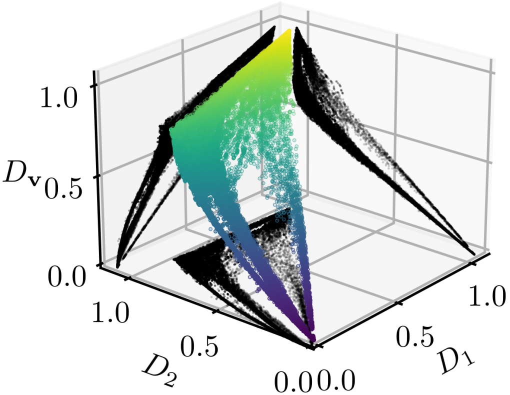

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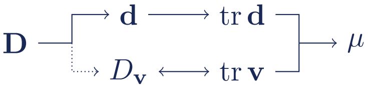

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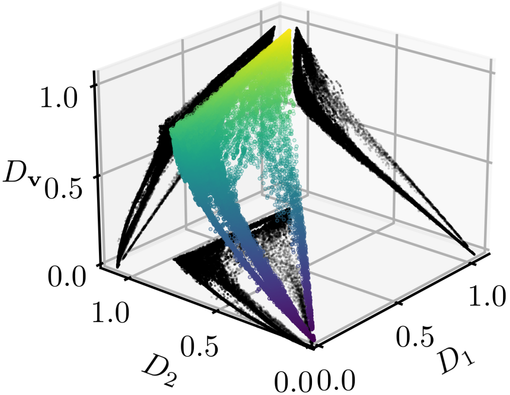

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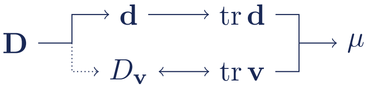

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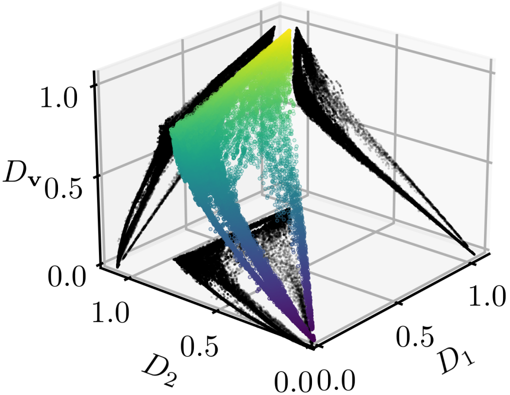

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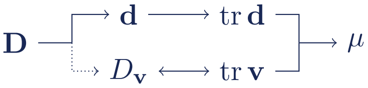

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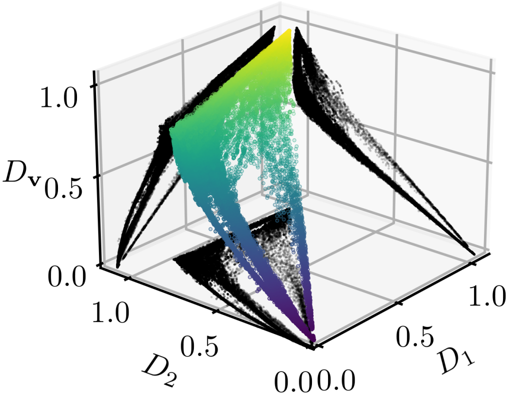

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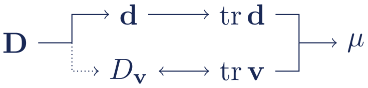

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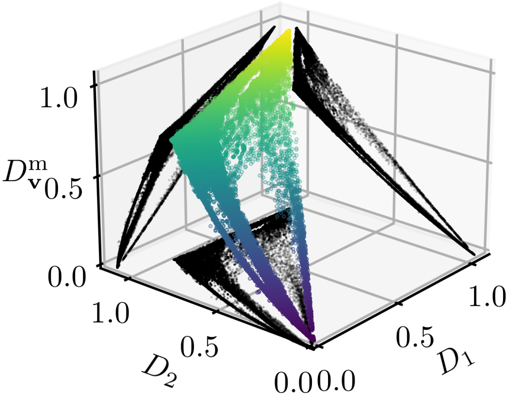

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How to parametrize the harmonic part?
(Vannucci, 2005) (Desmorat & Desmorat, 2015)

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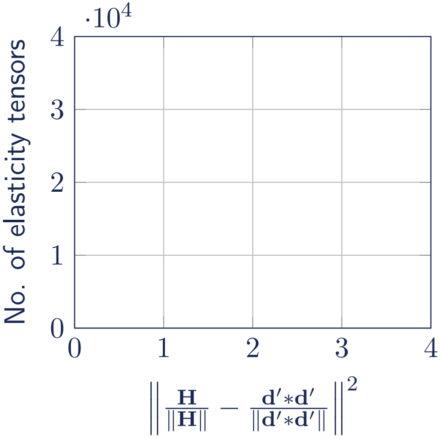

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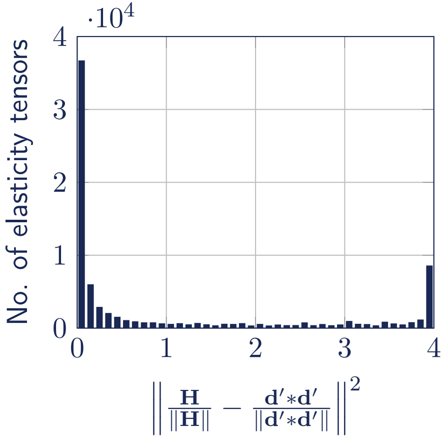

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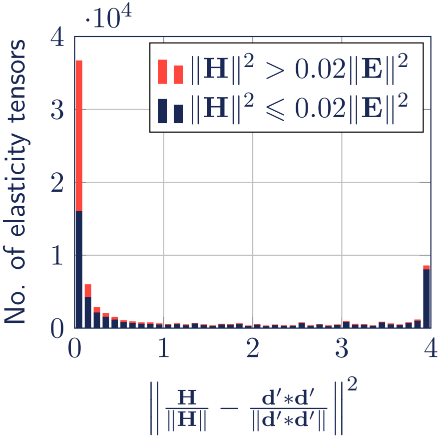

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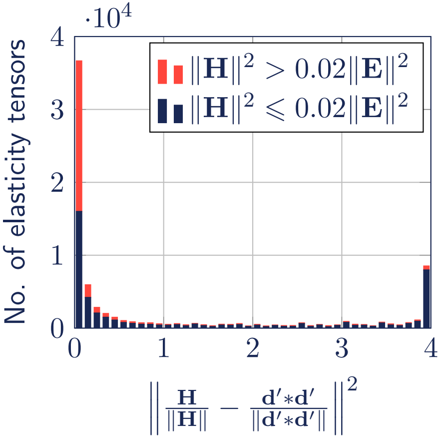
$$\text{Orthotropy} \implies \mathbf{H} = \|\mathbf{H}\| \left(+ \frac{\mathbf{d}' * \mathbf{d}'}{\|\mathbf{d}' * \mathbf{d}'\|} \right)$$

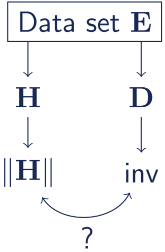
where $\mathbf{d}' * \mathbf{d}' = \mathbf{d}' \otimes \mathbf{d}' - \frac{1}{2}(\mathbf{d}':\mathbf{d}')\mathbf{J}$.



Questions

- ✓ Model orientation ($\pm$)?
- ○ Model norm $H(\mathbf{D}) = \|\mathbf{H}\|$?

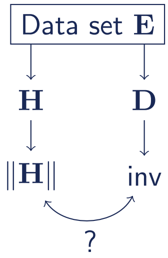
State model – Harmonic part – Norm modelling $H^{\mathrm{m}} : \mathbf{D} \mapsto H^{\mathrm{m}}(\mathbf{D}) \approx \|\mathbf{H}\|$?



State model – Harmonic part – Norm modelling $H^{\mathrm{m}} : \mathbf{D} \mapsto H^{\mathrm{m}}(\mathbf{D}) \approx \|\mathbf{H}\|$?

Invariants

$$I_1\left(\mathbf{D}\right) = \operatorname{tr}(\mathbf{D}) \quad I_2\left(\mathbf{D}'\right) = \mathbf{D}' : \mathbf{D}'$$



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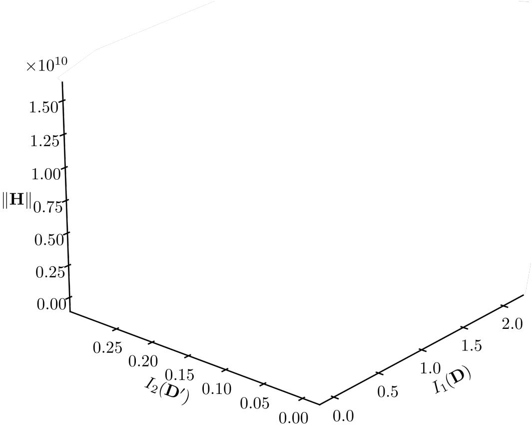

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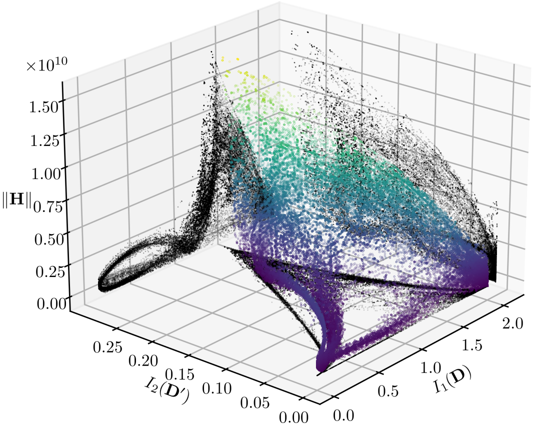


State model – Harmonic part – Norm modelling $H^{\mathrm{m}} : \mathbf{D} \mapsto H^{\mathrm{m}}(\mathbf{D}) \approx \|\mathbf{H}\|$?

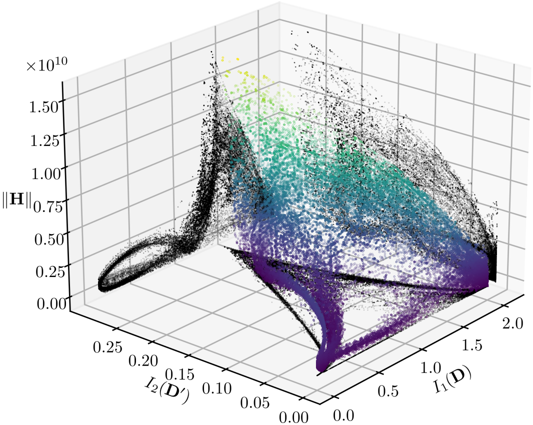
Invariants

$$I_1(\mathbf{D}) = \mathrm{tr}(\mathbf{D}) \quad I_2(\mathbf{D}') = \mathbf{D}':\mathbf{D}'$$

Assumptions

$$H^{\mathrm{m}}(\mathbf{D} = \mathbf{0}_2) = \mathbf{0}_4 \qquad \text{(Initial isotropy)}$$

$$H^{\mathrm{m}}(\mathbf{D} = \mathbf{1}_2) = \mathbf{0}_4 \qquad \text{(Fully damaged)}$$



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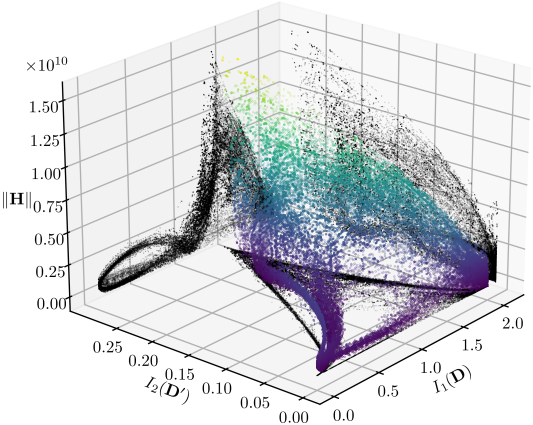
Model – Polynomial of invariants

$$H^{\mathrm{m}}(\mathbf{D}) = \sum_{n,m} c_{n,m} \, I_1(\mathbf{D})^n \cdot I_2(\mathbf{D}')^m$$



State model – Harmonic part – Norm modelling $H^{\mathrm{m}} : \mathbf{D} \mapsto H^{\mathrm{m}}(\mathbf{D}) \approx \|\mathbf{H}\|$?

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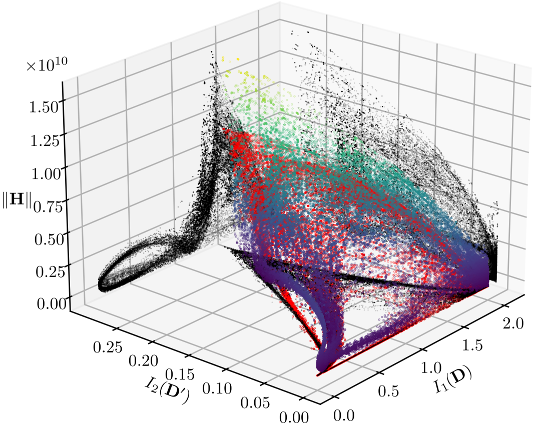


Sparse regression $\implies H^{\mathrm{m}}(\mathbf{D}) = 12 \cdot 10^9 \cdot I_1(\mathbf{D})^4 \cdot I_2(\mathbf{D}')$

State model – Conclusion

Knowing κ_0, μ_0 and $\mathbf{D}$, the elasticity tensor can be modelled as

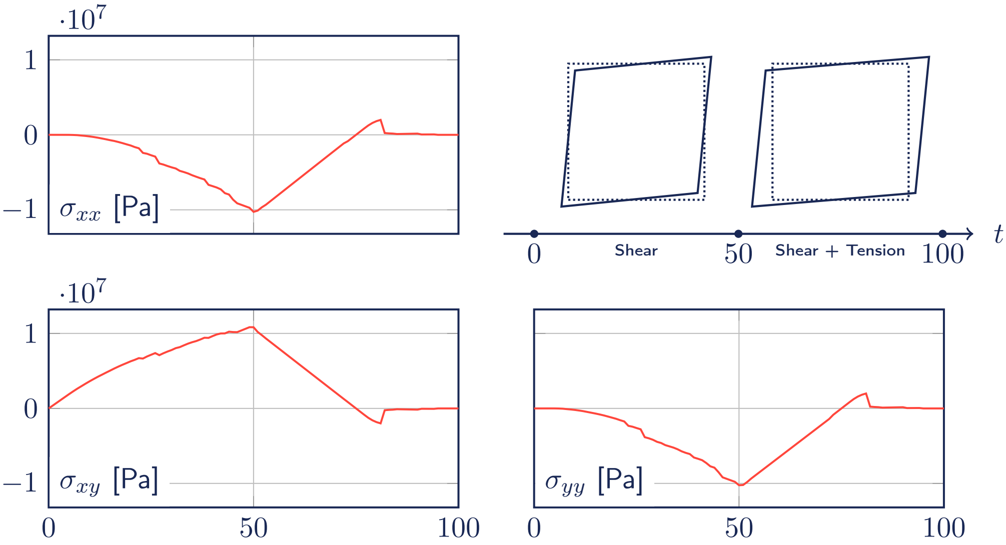
$$\mathbf{E}(\mathbf{D}) = 2\mu(\mathbf{D})\mathbf{J} + \kappa(\mathbf{D})\mathbf{1}_2 \otimes \mathbf{1}_2 + \frac{1}{2}\left(\mathbf{d}'(\mathbf{D}) \otimes \mathbf{1}_2 + \mathbf{1}_2 \otimes \mathbf{d}'(\mathbf{D})\right) + \mathbf{H}(\mathbf{D})$$

where the invariants and covariants models are

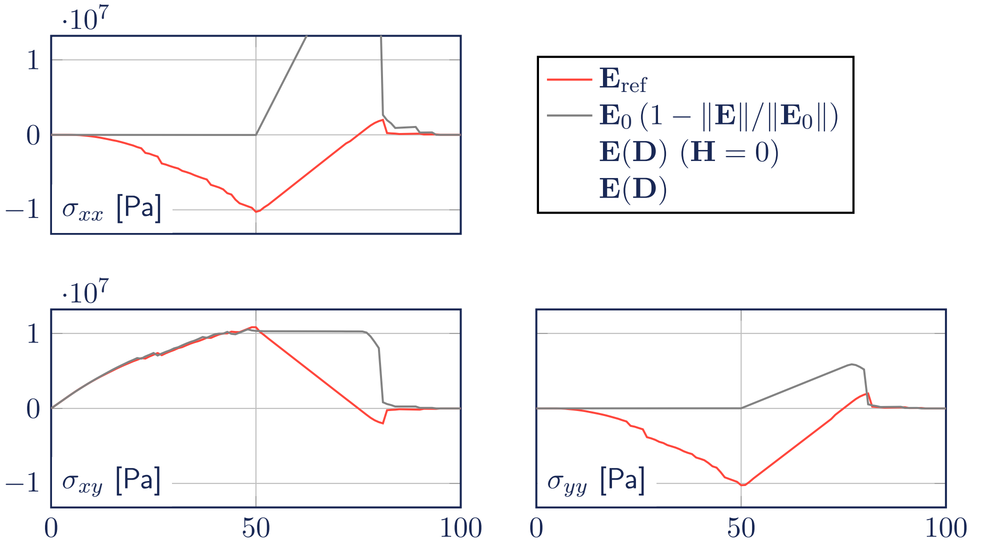
$$\mu(\mathbf{D}) = \mu_0 - \frac{\kappa_0}{4}(\operatorname{tr}\mathbf{D}) + \frac{\kappa_0 - 2\mu_0}{4}(\mathbf{D}:\mathbf{D}) \qquad \mathbf{d}'(\mathbf{D}) = -2\kappa_0\mathbf{D}'$$

$$\kappa(\mathbf{D}) = \kappa_0\left(1 - \frac{1}{2}\operatorname{tr}\mathbf{D}\right) \qquad \mathbf{H}^{\mathrm{m}}(\mathbf{D}) = h(\operatorname{tr}\mathbf{D})^4\mathbf{D}'*\mathbf{D}'$$

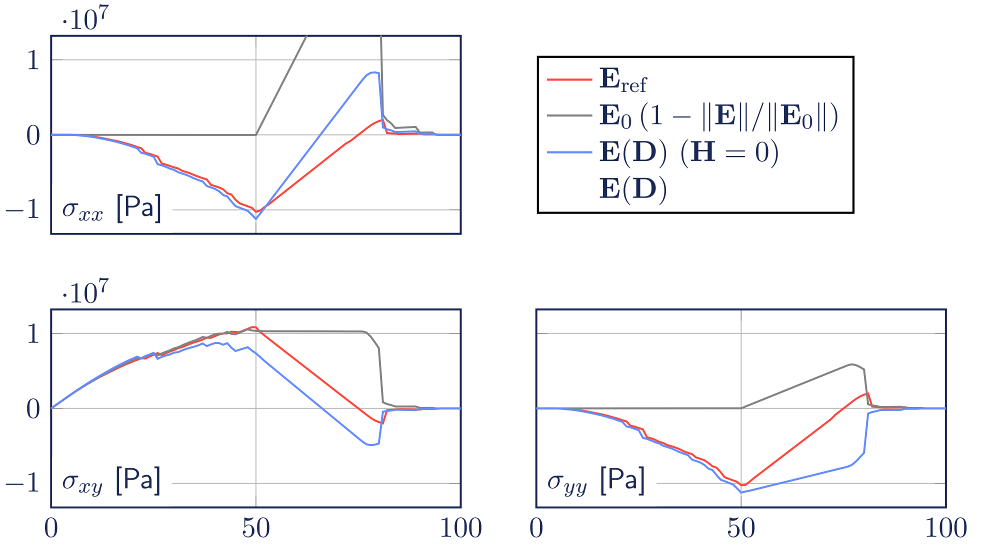
Verifications – Reconstruction of stress σ from (exact) damage



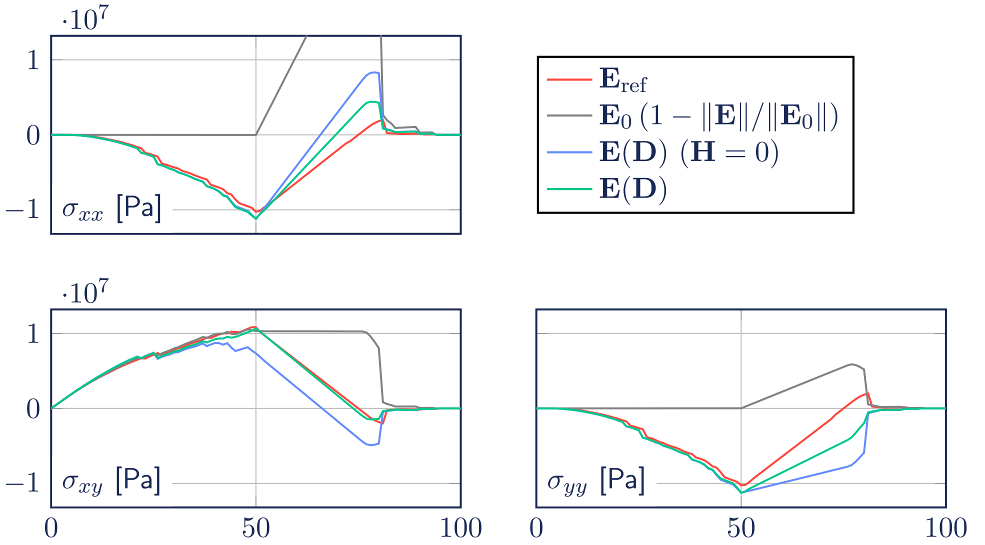
Verifications – Reconstruction of stress σ from (exact) damage



Verifications – Reconstruction of stress σ from (exact) damage

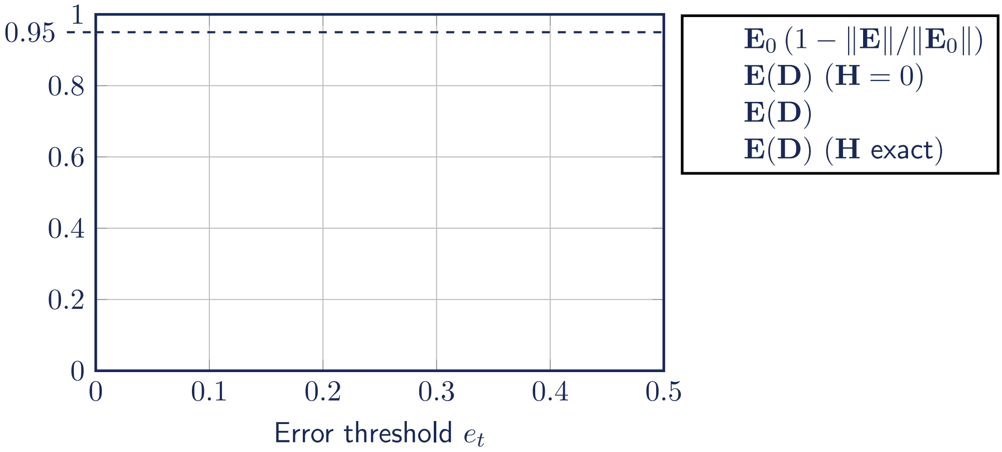


Verifications – Reconstruction of stress σ from (exact) damage

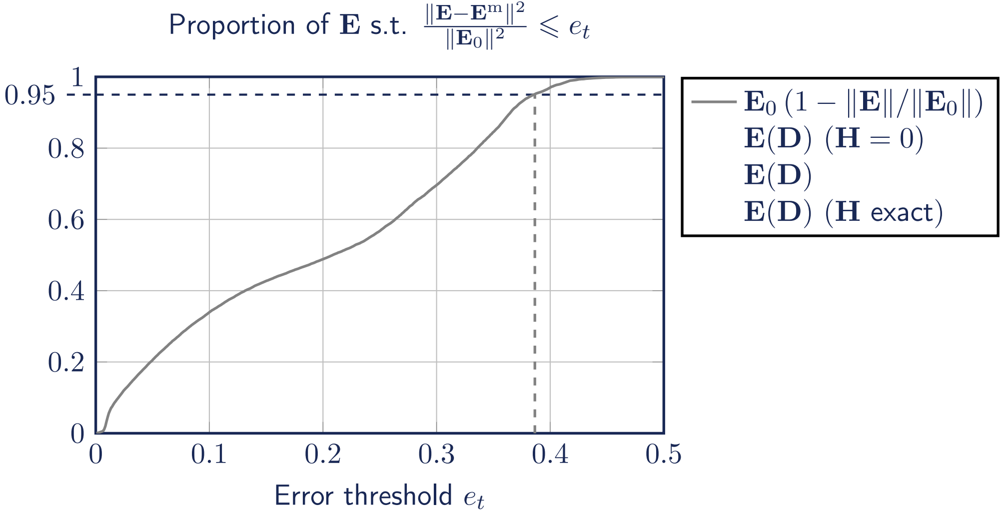


Verifications – Over the whole dataset

Proportion of $\mathbf{E}$ s.t. $\frac{\|\mathbf{E}-\mathbf{E}^{\mathrm{m}}\|^2}{\|\mathbf{E}_0\|^2} \leqslant e_t$



Verifications – Over the whole dataset



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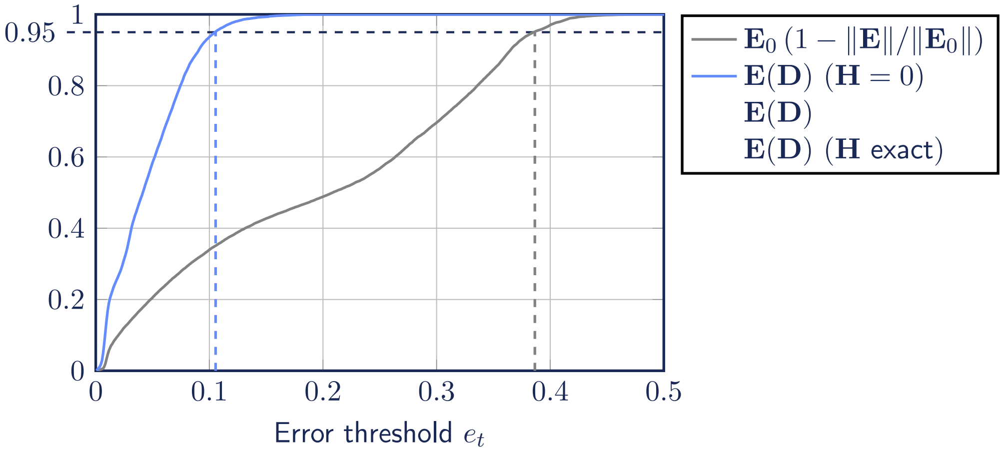
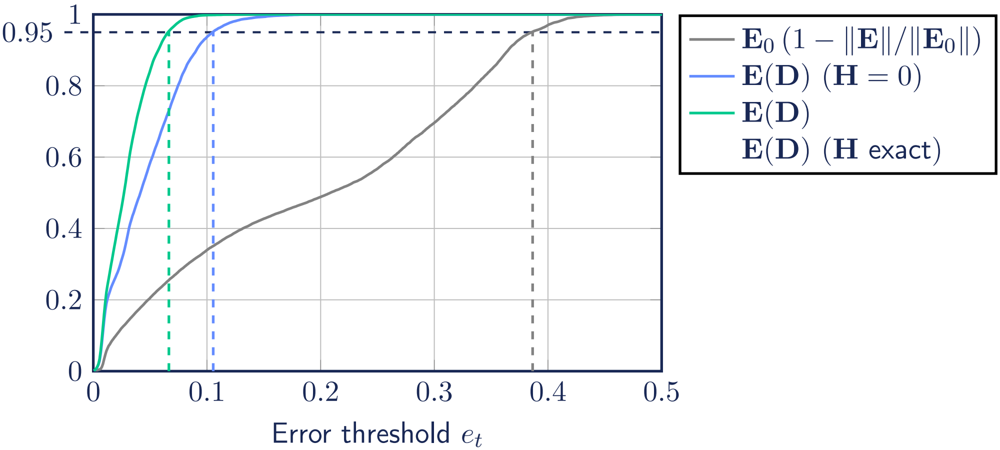


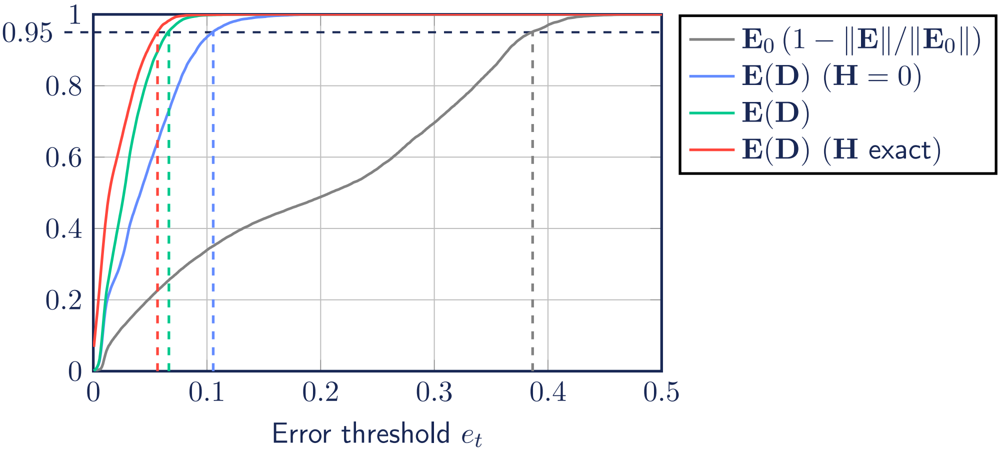
Verifications – Over the whole dataset

Proportion of $\mathbf{E}$ s.t. $\frac{\|\mathbf{E}-\mathbf{E}^{\mathrm{m}}\|^2}{\|\mathbf{E}_0\|^2} \leqslant e_t$



Verifications – Over the whole dataset

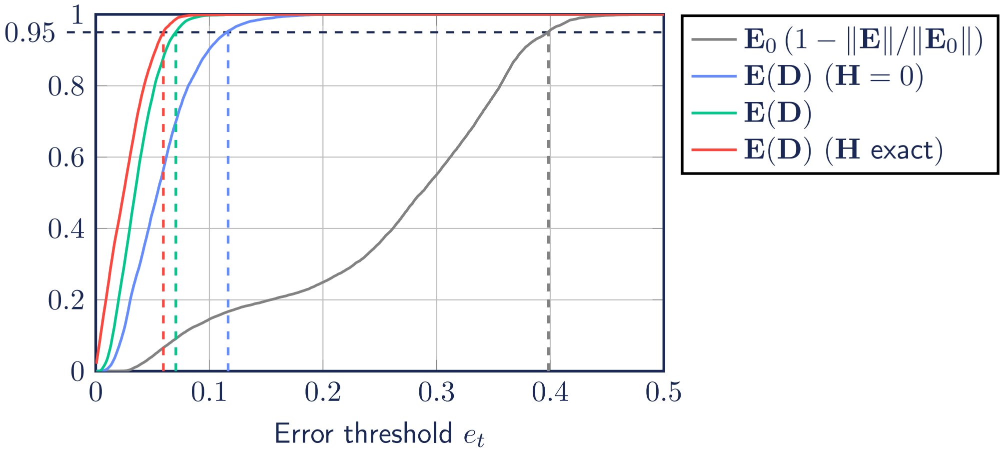
Proportion of $\mathbf{E}$ s.t. $\frac{\|\mathbf{E}-\mathbf{E}^{m}\|^2}{\|\mathbf{E}_0\|^2} \leqslant e_t$



Verifications – Highly damaged tensors

$\max(D_1, D_2) > 0.5$ and $D_1 < 0.95$ and $D_2 < 0.95$

Proportion of $\mathbf{E}$ s.t. $\frac{\|\mathbf{E}-\mathbf{E}^m\|^2}{\|\mathbf{E}_0\|^2} \leqslant e_t$



Next steps – Evolution model: Basis

Defintion by case

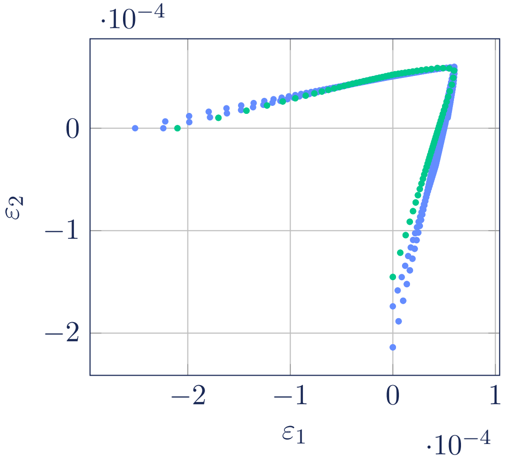
Criterion function

$$f(\boldsymbol{\varepsilon}, \mathbf{D}) = \varepsilon_{\text{eq}} - C(\mathbf{D})$$

Damage evolution

$$\dot{\mathbf{D}} = \begin{cases} 0 & \text{if } f(\boldsymbol{\varepsilon}, \mathbf{D}) < 0 \\ \dot{\lambda}\langle \boldsymbol{\Delta} \rangle_+ & \text{if } f(\boldsymbol{\varepsilon}, \mathbf{D}) = 0 \end{cases}$$

Remark $C(\mathbf{D})$ and $\dot{\lambda}$ are by $\dot{f}(\boldsymbol{\varepsilon}, \mathbf{D}) = 0$.



Next steps – Evolution model: Preliminary results

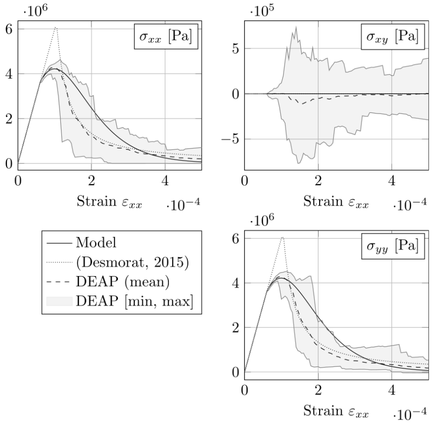


Figure: Bitension

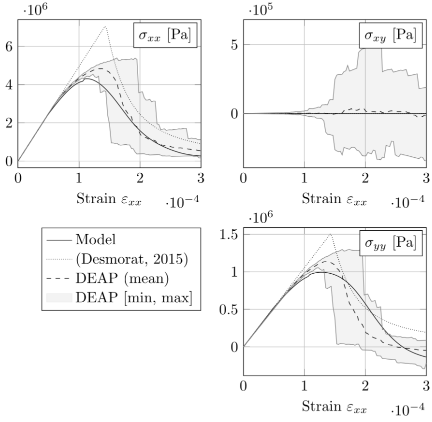


Figure: Tension

Thank you for your attention!

Rencontre annuelle du GDR-GDM
La Rochelle, 28–30 Juin 2023.

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Next steps – References I

Backus, G. (1970). A geometrical picture of anisotropic elastic tensors. *Reviews of Geophysics*, *8*(3), 633–671. https://doi.org/10.1029/RG008i003p00633

Terrien, M. (1980). Emission acoustique et comportement mécanique post-critique d'un béton sollicité en traction. *Bulletin de liaison des laboratoires des ponts et chaussees*, *1980*(105), 65–71.

Berthaud, Y. (1991). Damage measurements in concrete via an ultrasonic technique. part i experiment. *Cement and Concrete Research*, *21*(1), 73–82. https://doi.org/10.1016/0008-8846(91)90033-E

Kachanov, M. (1992). Effective elastic properties of cracked solids: Critical review of some basic concepts [Publisher: American Society of Mechanical Engineers Digital Collection]. *Applied Mechanics Reviews*, *45*(8), 304–335. https://doi.org/10.1115/1.3119761

Blinowski, A., Ostrowska-Maciejewska, J., & Rychlewski, J. (1996). Two-dimensional hooke's tensors - isotropic decomposition, effective symmetry criteria [Number: 2]. *Archives of Mechanics*, *48*(2), 325–345. https://doi.org/10.24423/aom.1345

Vannucci, P. (2005). Plane Anisotropy by the Polar Method*. *Meccanica*, *40*(4), 437–454. https://doi.org/10.1007/s11012-005-2132-z

Next steps – References II

Desmorat, B., & Desmorat, R. (2015). Tensorial Polar Decomposition of 2D fourth-order tensors. *Comptes Rendus Mécanique*, *343*(9), 471–475. https://doi.org/10.1016/j.crme.2015.07.002

Vassaux, M., Oliver-Leblond, C., Richard, B., & Ragueneau, F. (2016). Beam-particle approach to model cracking and energy dissipation in concrete: Identification strategy and validation. *Cement and Concrete Composites*, *70*, 1–14. https://doi.org/10.1016/j.cemconcomp.2016.03.011

Antonelli, A., Desmorat, B., Kolev, B., & Desmorat, R. (2022). Distance to plane elasticity orthotropy by Euler–Lagrange method. *Comptes Rendus. Mécanique*, *350*(G2), 413–430. https://doi.org/10.5802/crmeca.122