

IRCAM — STMS :

Quelques approches pour la synthèse sonore par modélisation physique et l'analyse des Sons

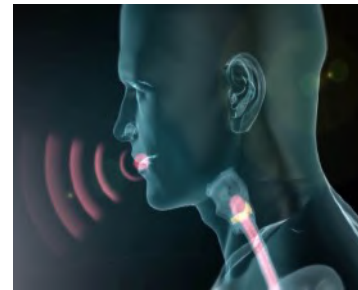
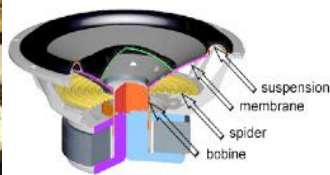
Thomas Hélie, CNRS

Equipe S3AM (*Systèmes et Signaux Sonores: Audio/acoustique, instruMents*)

Laboratoire des Sciences et Technologies de la Musique et du Son
IRCAM — CNRS — Sorbonne Université
Paris, France

GdR Géométrie Différentielle et Mécanique

Novembre, 2024 — Paris



IRCAM: Institute for Research and Coordination in Acoustics/Music



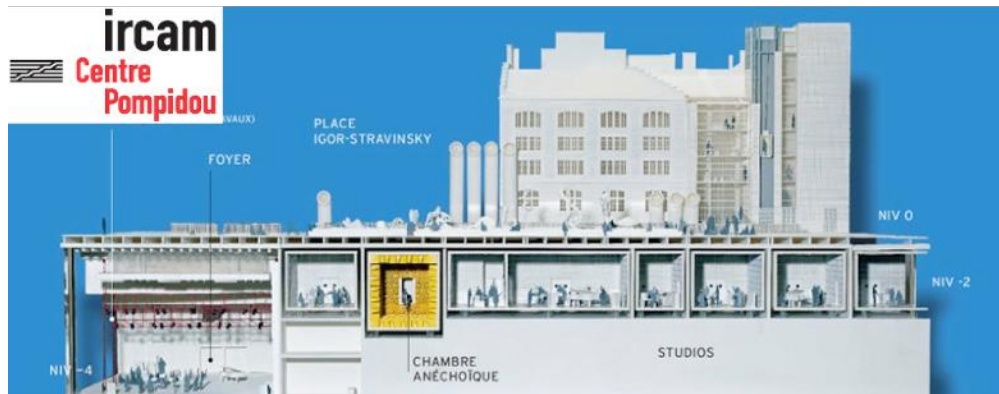
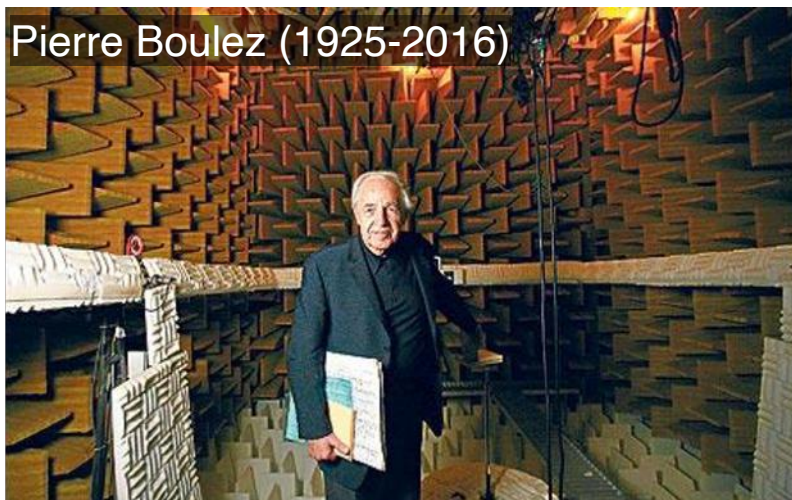
Creation in 1977 by Pierre Boulez,
associated with the Georges Pompidou Center

Vocation: interaction between

- Scientific research (sound/music)
- Technological development
- Contemporary music creation

Departments (~140 people):

- A. **STMS laboratory** (~100 people + 36 PhDs)
- B. Creation & Diffusion (concerts+20 creations/year)
- C. Education, Cultural Outreach and Resource Center
 - **Master ATIAM (sciences for music), Sorbonne Univ.**
 - Music curriculum (composers), Multimedia Library, ...
- D. Research/Creation Interface



IRCAM: Institute for Research and Coordination in Acoustics/Music



1981: 4X system by G. di Giugno



1994: voice sound processing
by P. Depalle, G. Garcia, X. Rodet

Reconstruction of a castrato voice

- 2 recorded singers:
 - coloratura soprano
 - countertenor
- analysis
- cross-synthesis

1995: creation of the **STMS laboratory**
(unit director, 2024 : N. Misdariis)



STMS laboratory: Sciences and Technologies for Music and Sound

Acoustic and Cognitive Spaces



ircam
Centre
Pompidou



SORBONNE
UNIVERSITÉ

7 teams

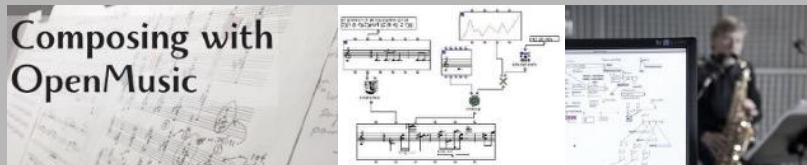
Analysis of Musical Practices



Sound Music Movement Interaction



Musical Representations



Sound Perception and Design



Sound Analysis-Synthesis

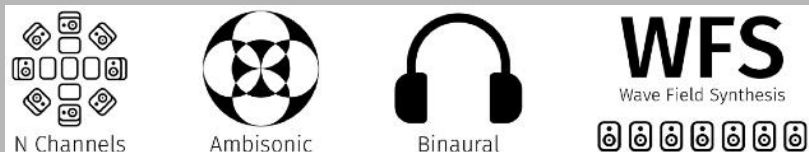


Sound Systems and Signals: Audio/Acoustics, Instruments



STMS laboratory: Sciences and Technologies for Music and Sound

Acoustic and Cognitive Spaces



ircam
Centre
Pompidou



SORBONNE
UNIVERSITÉ

7 teams

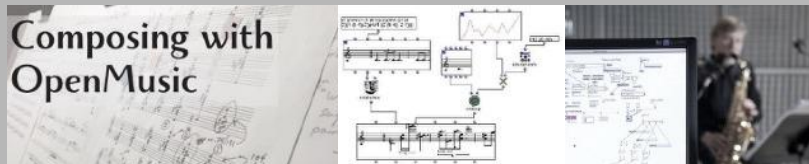
Analysis of Musical Practices



Sound Music Movement Interaction



Musical Representations



Sound Perception and Design



Sound Analysis-Synthesis

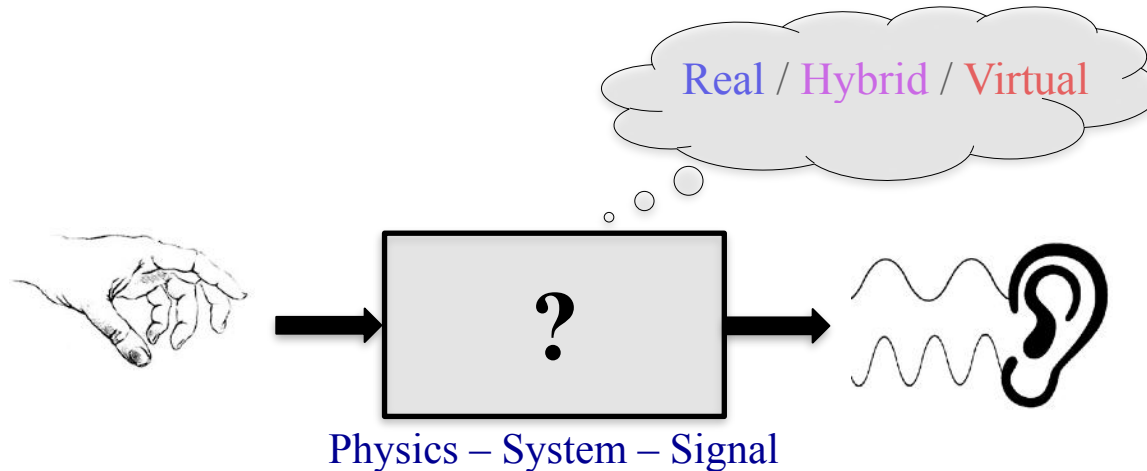


Sound Signals and Systems: Audio/Acoustics, Instruments



Sound Signals and Systems: Audio/Acoustics, Instruments

- Theoretical, technological, and experimental tools to address **multiphysics systems** and the sound signals they produce



- **Issues:** sound synthesis, controlled/augmented instruments, optimised design, tools for educational/health purposes
- **Emphasis on 3 points to reach realism**
 - **Nonlinearities:** (1) timbre & complex distortions, (2) self-oscillations & regimes
 - **Damping:** non trivial modelling (spectral colour)
 - **Passivity / power balance:** guarantee physical behaviours

Outline

A. IRCAM and the STMS lab

B. Modelling and simulation

B1. Volterra series

B2. Power-balanced multiphysics (Port-Hamiltonian systems)

C. Experimental work

D. Visualisation

B1. Volterra series

Motivation

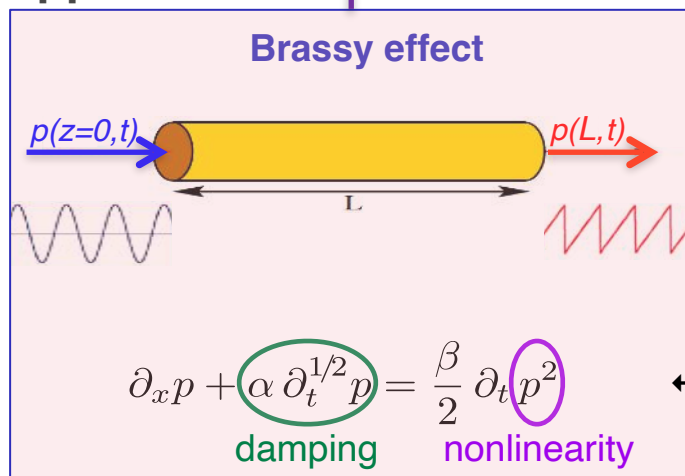
How to capture the timbre evolution with respect to the sound dynamics ?

→ Combine **weak nonlinearities** and **memory** (*generalised impulse responses*)

$$\begin{array}{c} \mathbf{u} \rightarrow \boxed{\{h_n\}} \rightarrow \mathbf{x}(t) = \underbrace{\sum_{n \in \mathbb{N}^*} \int_{\mathbb{R}^n} h_n(\tau_1, \dots, \tau_n) u(t - \tau_1) \dots u(t - \tau_n) d\tau_1 \dots d\tau_n}_{\text{Sum of multiple convolutions}} \end{array}$$

→ *distortions with memory for any input u*

Applications



← inspired from [Menguy, Gilbert, 2000]
“Weakly nonlinear gas oscillations in air-filled tubes (...)”

B1. Volterra series

Motivation

How to capture the timbre evolution with respect to the sound dynamics ?

→ Combine **weak nonlinearities** and **memory** (*generalised impulse responses*)

$$\begin{array}{c}
 \underline{u} \rightarrow \boxed{\{h_n\}} \rightarrow \underline{x}(t) = \underbrace{\sum_{n \in \mathbb{N}^*} \int_{\mathbb{R}^n} h_n(\tau_1, \dots, \tau_n) u(t - \tau_1) \dots u(t - \tau_n) d\tau_1 \dots d\tau_n}_{\text{Sum of multiple convolutions}} \rightarrow \text{distortions with memory for any input } u
 \end{array}$$

Applications

Brassy effect

$$\partial_x p + \underbrace{\alpha \partial_t^{1/2} p}_{\text{damping}} = \frac{\beta}{2} \partial_t \underbrace{p^2}_{\text{nonlinearity}}$$

Moog Ladder Filter

$$\partial_t x = \omega \begin{bmatrix} -\tanh x_1 + \tanh(u - 4rx_4) \\ -\tanh x_2 + \tanh x_1 \\ -\tanh x_3 + \tanh x_2 \\ -\tanh x_4 + \tanh x_3 \end{bmatrix}$$

String (large displacement)

$$\partial_t^2 u + \alpha \partial_t u - \beta \partial_t \partial_x^2 u - \left(1 + \varepsilon \left[\int_0^1 (\partial_x u(x,t))^2 dx \right] \right) \partial_x^2 u = \Phi f_{tot}$$

Sound examples - real-time plugins

1. Chet Baker : without / with **brassy effect**

2. **Moog Ladder Filter** without / with **distortion (x2)**

B1. Volterra series

Motivation

How to capture the timbre evolution with respect to the sound dynamics ?

→ Combine **weak nonlinearities** and **memory** (*generalised impulse responses*)

$$u \rightarrow \boxed{\{h_n\}} \rightarrow x(t) = \sum_{n \in \mathbb{N}^*} \underbrace{\int_{\mathbb{R}^n} h_n(\tau_1, \dots, \tau_n) u(t - \tau_1) \dots u(t - \tau_n) d\tau_1 \dots d\tau_n}_{\text{Sum of multiple convolutions}} \rightarrow \text{distortions with memory for any input } u$$

Other results:

1. **Space-time kernels**: *Green functions* → *Green-Volterra series*
2. **Convergence**: **computable convergence bounds** for classes of systems

Outline

A. IRCAM and the STMS lab

B. Modelling and simulation

B1. Volterra series

B2. Power-balanced multiphysics

B2.1. Introduction to Port-Hamiltonian Systems

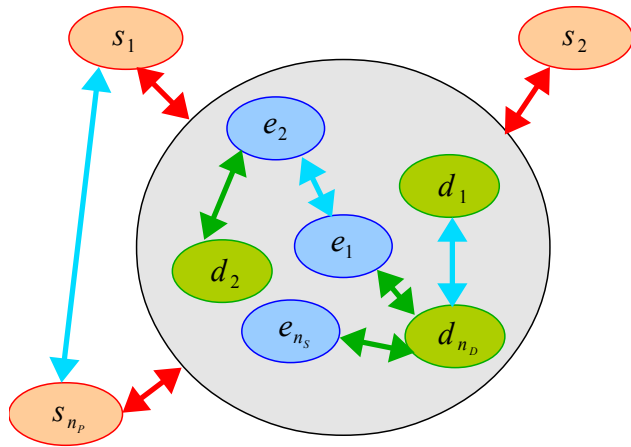
B2.2. Musical applications

C. Experimental work

D. Visualisation

B2.1 Power-balanced systems : Introduction to PHS

A physical system is made of...



(i) Energy-storing components

$$E = \sum_{n=1}^N e_n \geq 0$$

(ii) Memoryless passive components

$$P_{\text{diss}} = \sum_{m=1}^M d_m > 0 \text{ (dissipative) or } = 0 \text{ (conservative)}$$

(iii) External components

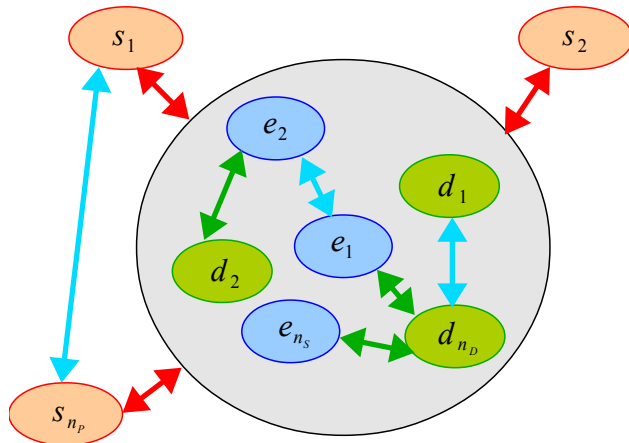
$$P_{\text{ext}} = \sum_{p=1}^P s_p$$

+ Conservative connections

B2.1 Introduction to Port Hamiltonian Systems (PHS)

A physical system is made of...

⚠ receiver convention



(i) Energy-storing components

→ store energy

$$E = \sum_{n=1}^N e_n \geq 0$$

(ii) Memoryless passive components

→ receive power

$$P_{\text{diss}} = \sum_{m=1}^M d_m > 0 \text{ (dissipative) or } = 0 \text{ (conservative)}$$

(iii) External components

→ receive power

$$P_{\text{ext}} = \sum_{p=1}^P s_p$$

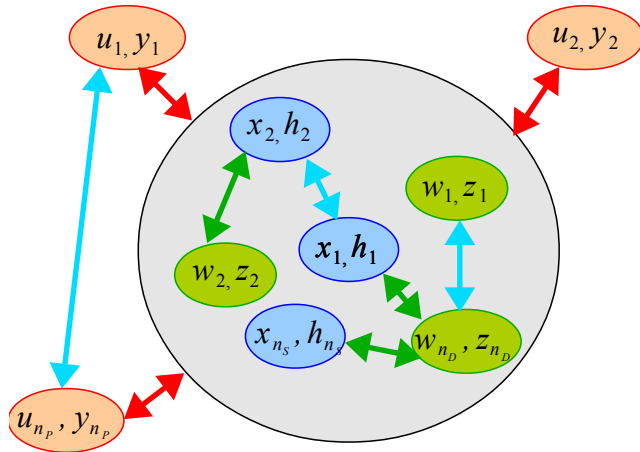
+ Conservative connections → sum of received powers is zero

$$P_{\text{stored}} + \underbrace{P_{\text{diss}}}_{\geq 0} + P_{\text{ext}} = 0 \text{ with } P_{\text{stored}} = \dot{E} \text{ (power balance)}$$

B2.1 Introduction to Port Hamiltonian Systems (PHS)

A physical system is made of...

⚠ receiver convention



(i) Energy-storing components

→ store energy

$$E = H(\mathbf{x}) = \sum_{n=1}^N H_n(x_n) \geq 0$$

(ii) Memoryless passive components

→ receive power

$$P_{\text{diss}} = \mathbf{z}(\mathbf{w})^T \mathbf{w} = \sum_{m=1}^M z_m(w_m) w_m \geq 0$$

(effort × flow : force × velocity, voltage × current, etc)

(iii) External components

→ receive power

$$P_{\text{ext}} = \mathbf{u}^T \mathbf{y} = \sum_{p=1}^P u_p y_p$$

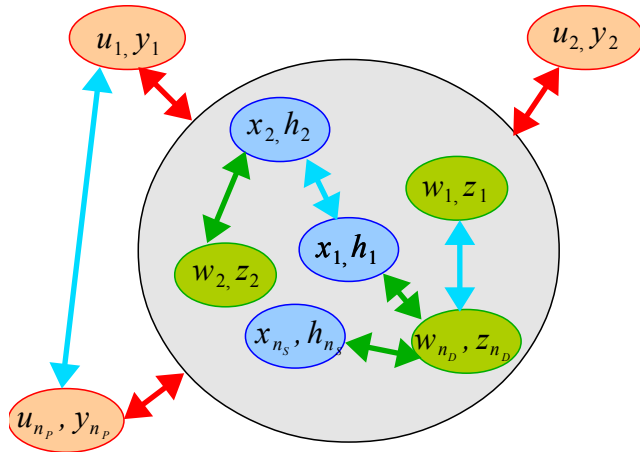
+ Conservative connections → sum of received powers is zero

$$\underbrace{\nabla H(\mathbf{x})^T \dot{\mathbf{x}}}_{P_{\text{stored}} = dE/dt} + \underbrace{\mathbf{z}(\mathbf{w})^T \mathbf{w}}_{\geq 0} + \mathbf{u}^T \mathbf{y} = 0 \quad \text{(power balance)}$$

B2.1 Introduction to Port Hamiltonian Systems (PHS)

A physical system is made of...

⚠ receiver convention



(i) Energy-storing components

→ store energy

$$E = H(\mathbf{x}) = \sum_{n=1}^N H_n(x_n) \geq 0$$

(ii) Memoryless passive components

→ receive power

$$P_{\text{diss}} = \mathbf{z}(\mathbf{w})^T \mathbf{w} = \sum_{m=1}^M z_m(w_m) w_m \geq 0$$

(effort × flow : force × velocity, voltage × current, etc)

(iii) External components

→ receive power

$$P_{\text{ext}} = \mathbf{u}^T \mathbf{y} = \sum_{p=1}^P u_p y_p$$

+ Conservative connections → sum of received powers is zero

$$\underbrace{\nabla H(\mathbf{x})^T \dot{\mathbf{x}}}_{P_{\text{stored}} = dE/dt} + \underbrace{\mathbf{z}(\mathbf{w})^T \mathbf{w}}_{\geq 0} + \mathbf{u}^T \mathbf{y} = 0 \quad \text{(power balance)}$$

PHS: Input-State-Output representation

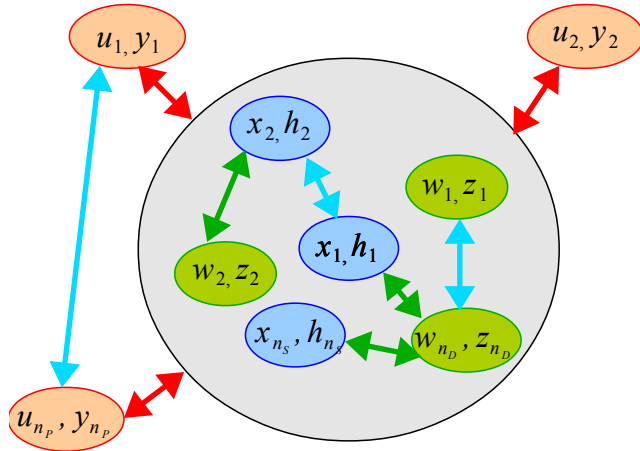
(**S**: interconnection matrix)

$$\underbrace{\begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{w} \\ \mathbf{y} \end{bmatrix}}_{\mathbf{f}} = \underbrace{\begin{bmatrix} \mathbf{S}_{xx} & \mathbf{S}_{xw} & \mathbf{S}_{xu} \\ * & \mathbf{S}_{ww} & \mathbf{S}_{wu} \\ * & * & \mathbf{S}_{yu} \end{bmatrix}}_{\text{with } \mathbf{S} = -\mathbf{S}^T} \underbrace{\begin{bmatrix} \nabla H(\mathbf{x}) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{bmatrix}}_{\mathbf{e}} \quad \begin{array}{l} (i) \text{ storage} \rightarrow \text{differential eq.} \\ (ii) \text{ memoryless} \rightarrow \text{algebraic eq.} \\ (iii) \text{ ports} \rightarrow \text{physical signals} \end{array} \quad (1)$$

B2.1 Introduction to Port Hamiltonian Systems (PHS)

A physical system is made of...

⚠ receiver convention



(i) Energy-storing components

→ store energy

$$E = H(\mathbf{x}) = \sum_{n=1}^N H_n(x_n) \geq 0$$

(ii) Memoryless passive components

→ receive power

$$P_{\text{diss}} = \mathbf{z}(\mathbf{w})^T \mathbf{w} = \sum_{m=1}^M z_m(w_m) w_m \geq 0$$

(effort × flow : force × velocity, voltage × current, etc)

(iii) External components

→ receive power

$$P_{\text{ext}} = \mathbf{u}^T \mathbf{y} = \sum_{p=1}^P u_p y_p$$

+ Conservative connections → sum of received powers is zero

$$\underbrace{\nabla H(\mathbf{x})^T \dot{\mathbf{x}}}_{P_{\text{stored}} = dE/dt} + \underbrace{\mathbf{z}(\mathbf{w})^T \mathbf{w}}_{\geq 0} + \mathbf{u}^T \mathbf{y} = 0 \quad (\text{power balance})$$

PHS: Input-State-Output representation

(**S**: interconnection matrix)

$$\underbrace{\begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{w} \\ \mathbf{y} \end{bmatrix}}_{\mathbf{f}} = \underbrace{\begin{bmatrix} \mathbf{S}_{xx} & \mathbf{S}_{xw} & \mathbf{S}_{xu} \\ * & \mathbf{S}_{ww} & \mathbf{S}_{wu} \\ * & * & \mathbf{S}_{yu} \end{bmatrix}}_{\text{with } \mathbf{S} = -\mathbf{S}^T} \underbrace{\begin{bmatrix} \nabla H(\mathbf{x}) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{bmatrix}}_{\mathbf{e}} \quad \left. \begin{array}{l} (i) \text{ storage} \rightarrow \text{differential eq.} \\ (ii) \text{ memoryless} \rightarrow \text{algebraic eq.} \\ (iii) \text{ ports} \rightarrow \text{physical signals} \end{array} \right| \quad (1)$$

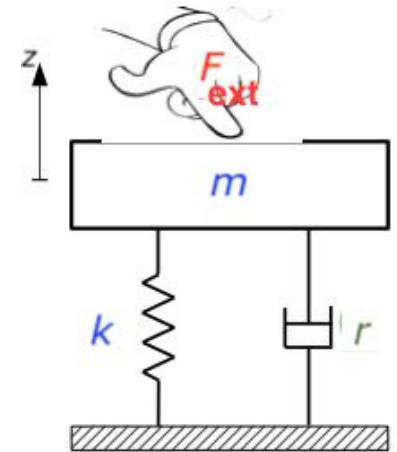
Power balance: $\mathbf{e}^T \mathbf{f} \stackrel{(1)}{=} \mathbf{e}^T \mathbf{S} \mathbf{e} = 0$ as $\mathbf{S} = -\mathbf{S}^T \Rightarrow \mathbf{e}^T \mathbf{S} \mathbf{e} = (\mathbf{e}^T \mathbf{S} \mathbf{e})^T = -(\mathbf{e}^T \mathbf{S} \mathbf{e})$

B2.1 Introduction to Port Hamiltonian Systems (PHS)

Example: damped mechanical oscillator excited by F_{ext} ($m\ddot{z} + r\dot{z} + kz = F_{\text{ext}}$)

(no gravity)

- 4 separate components

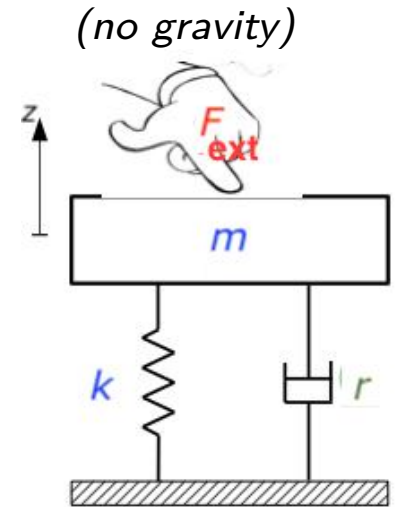


B2.1 Introduction to Port Hamiltonian Systems (PHS)

Example: damped mechanical oscillator excited by F_{ext} ($m\ddot{z} + r\dot{z} + kz = F_{\text{ext}}$)

- 4 separate components

(i₁) mass m of momentum $\pi = mv$ (energy: $\frac{1}{2}mv^2 = \frac{\pi^2}{2m}$),



	state	energy H_n	flow \mathbf{f}	effort \mathbf{e}
m	$x_1 := \pi$	$\pi^2/(2m)$	$\dot{x}_1 = \dot{\pi}$	$H'_1(x_1) = x_1/m$

blue : force
red : velocity

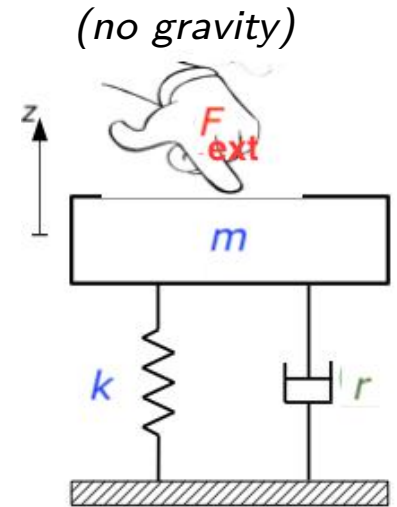
B2. Power-balanced systems

Example: damped mechanical oscillator excited by F_{ext} ($m\ddot{z} + r\dot{z} + kz = F_{\text{ext}}$)

- 4 separate components

(i₁) mass m of momentum $\pi = mv$ (energy: $\frac{1}{2}mv^2 = \frac{\pi^2}{2m}$),

(i₂) spring sp of elongation ξ



	state	energy H_n	flow \mathbf{f}	effort \mathbf{e}
m	$x_1 := \pi$	$\pi^2/(2m)$	$\dot{x}_1 = \dot{\pi}$	$H'_1(x_1) = x_1/m$
sp	$x_2 := \xi$	$k\xi^2/2$	$\dot{x}_2 = \dot{\xi}$	$H'_2(x_2) = kx_2$
		blue : force		
		red : velocity		

B2. Power-balanced systems

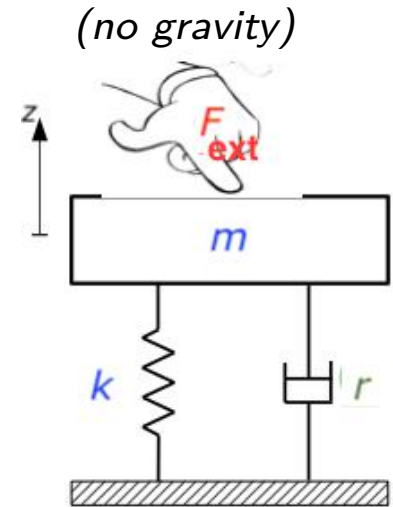
Example: damped mechanical oscillator excited by F_{ext} ($m\ddot{z} + r\dot{z} + kz = F_{\text{ext}}$)

- 4 separate components

(i₁) mass m of momentum $\pi = mv$ (energy: $\frac{1}{2}mv^2 = \frac{\pi^2}{2m}$),

(i₂) spring sp of elongation ξ

(ii) damper dp of velocity V_{dp}



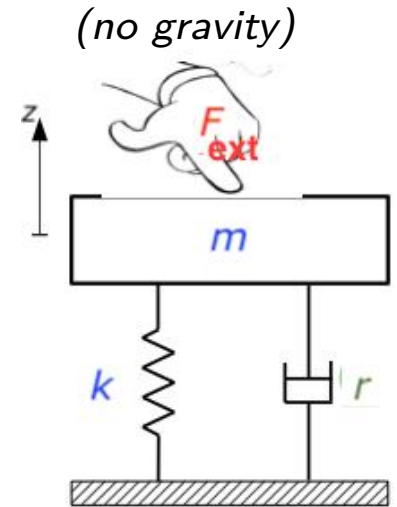
	state	energy H_n	flow \mathbf{f}	effort \mathbf{e}
m	$x_1 := \pi$	$\pi^2/(2m)$	$\dot{x}_1 = \dot{\pi}$	$H'_1(x_1) = x_1/m$
sp	$x_2 := \xi$	$k \xi^2/2$	$\dot{x}_2 = \dot{\xi}$	$H'_2(x_2) = k x_2$
dp	blue : force		$w := V_{\text{dp}}$	$z(w) := r w$
	red : velocity			

B2.1 Introduction to Port Hamiltonian Systems (PHS)

Example: damped mechanical oscillator excited by F_{ext} ($m\ddot{z} + r\dot{z} + kz = F_{\text{ext}}$)

• 4 separate components

- (i₁) mass m of momentum $\pi = mv$ (energy: $\frac{1}{2}mv^2 = \frac{\pi^2}{2m}$),
- (i₂) spring sp of elongation ξ
- (ii) damper dp of velocity V_{dp}
- (iii) actuator ext applying a force F_{ext}



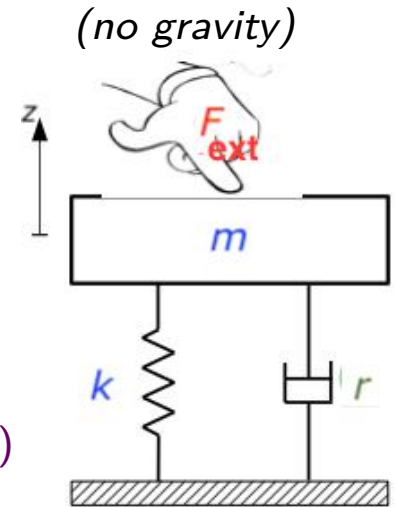
	state	energy H_n	flow \mathbf{f}	effort \mathbf{e}
m	$x_1 := \pi$	$\pi^2/(2m)$	$\dot{x}_1 = \dot{\pi}$	$H'_1(x_1) = x_1/m$
sp	$x_2 := \xi$	$k \xi^2/2$	$\dot{x}_2 = \dot{\xi}$	$H'_2(x_2) = k x_2$
dp	blue : force		$w := V_{\text{dp}}$	$z(w) := r w$
ext	red : velocity			

B2.1 Introduction to Port Hamiltonian Systems (PHS)

Example: damped mechanical oscillator excited by F_{ext} ($m\ddot{z} + r\dot{z} + kz = F_{\text{ext}}$)

• 4 separate components

- (i₁) mass m of momentum $\pi = mv$ (energy: $\frac{1}{2}mv^2 = \frac{\pi^2}{2m}$),
- (i₂) spring sp of elongation ξ
- (ii) damper dp of velocity V_{dp}
- (iii) actuator ext applying a force F_{ext} (\rightarrow your finger experiences $-F_{\text{ext}}$)



	state	energy H_n	flow \mathbf{f}	effort \mathbf{e}
m	$x_1 := \pi$	$\pi^2/(2m)$	$\dot{x}_1 = \dot{\pi}$	$H'_1(x_1) = x_1/m$
sp	$x_2 := \xi$	$k \xi^2/2$	$\dot{x}_2 = \dot{\xi}$	$H'_2(x_2) = k x_2$
dp	blue : force		$w := V_{\text{dp}}$	$z(w) := r w$
ext	red : velocity		$y := V_{\text{ext}}$	$u := -F_{\text{ext}}$

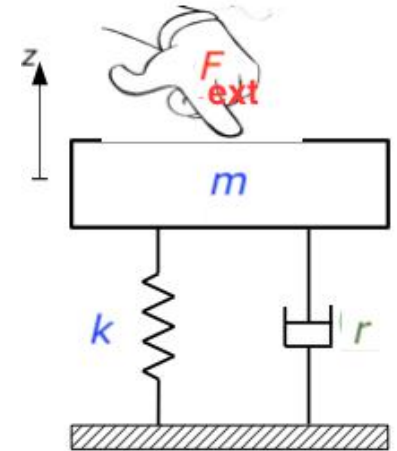
B2.1 Introduction to Port Hamiltonian Systems (PHS)

Example: damped mechanical oscillator excited by F_{ext} ($m\ddot{z} + r\dot{z} + kz = F_{\text{ext}}$)

(no gravity)

- 4 separate components

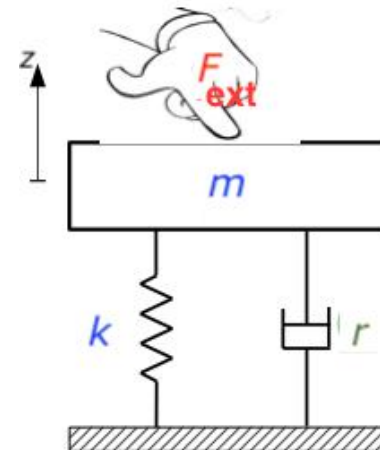
	state	energy H_n	flow \mathbf{f}	effort \mathbf{e}
m	$x_1 := \pi$	$\pi^2/(2m)$	$\dot{x}_1 = \dot{\pi}$	$H'_1(x_1) = x_1/m$
sp	$x_2 := \xi$	$k\xi^2/2$	$\dot{x}_2 = \dot{\xi}$	$H'_2(x_2) = kx_2$
dp		blue : force	$w := V_{\text{dp}}$	$z(w) := rw$
ext		red : velocity	$y := V_{\text{ext}}$	$u := -F_{\text{ext}}$



B2.1 Introduction to Port Hamiltonian Systems (PHS)

Example: damped mechanical oscillator excited by F_{ext} ($m\ddot{z} + r\dot{z} + kz = F_{\text{ext}}$)

(no gravity)



- 4 separate components

	state	energy H_n	flow f	effort e
m	$x_1 := \pi$	$\pi^2/(2m)$	$\dot{x}_1 = \dot{\pi}$	$H'_1(x_1) = x_1/m$
sp	$x_2 := \xi$	$k\xi^2/2$	$\dot{x}_2 = \dot{\xi}$	$H'_2(x_2) = kx_2$
dp		blue : force	$w := V_{\text{dp}}$	$z(w) := rw$
ext		red : velocity	$y := V_{\text{ext}}$	$u := -F_{\text{ext}}$

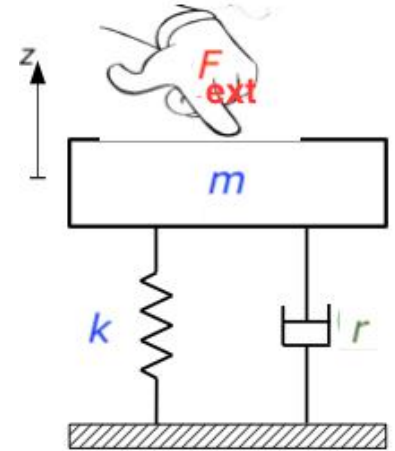
- assembled with rigid connections

$$\underbrace{\begin{array}{c} \dot{\pi} = F_m \\ \dot{\xi} = V_{\text{sp}} \\ V_{\text{dp}} \\ V_{\text{ext}} \end{array}}_f \underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ w \\ y \end{bmatrix}}_s = \underbrace{\begin{bmatrix} & & \\ & & \\ & & \\ & & \end{bmatrix}}_S \underbrace{\begin{bmatrix} H'_1(x_1) \\ H'_2(x_2) \\ z(w) \\ u \end{bmatrix}}_e \underbrace{\begin{array}{c} V_m = \pi/m \\ F_{\text{sp}} = k\xi \\ F_{\text{dp}} = rV_{\text{dp}} \\ -F_{\text{ext}} \end{array}}_e$$

B2.1 Introduction to Port Hamiltonian Systems (PHS)

Example: damped mechanical oscillator excited by F_{ext} ($m\ddot{z} + r\dot{z} + kz = F_{\text{ext}}$)

(no gravity)



- 4 separate components

	state	energy H_n	flow f	effort e
m	$x_1 := \pi$	$\pi^2/(2m)$	$\dot{x}_1 = \dot{\pi}$	$H'_1(x_1) = x_1/m$
sp	$x_2 := \xi$	$k\xi^2/2$	$\dot{x}_2 = \dot{\xi}$	$H'_2(x_2) = kx_2$
dp		blue : force	$w := V_{\text{dp}}$	$z(w) := rw$
ext		red : velocity	$y := V_{\text{ext}}$	$u := -F_{\text{ext}}$

- assembled with rigid connections

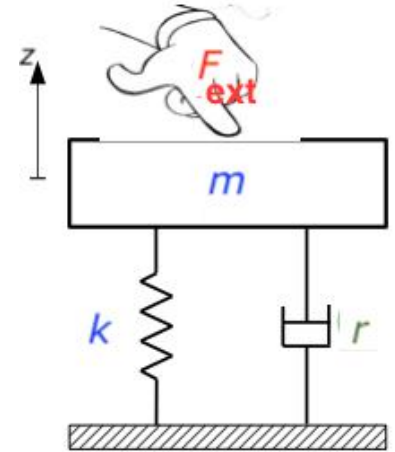
- internal forces are balanced $F_m + F_{\text{sp}} + F_{\text{dp}} + (-F_{\text{ext}}) = 0$

$$\underbrace{\begin{array}{c} \dot{\pi} = F_m \\ \dot{\xi} = V_{\text{sp}} \\ V_{\text{dp}} \\ V_{\text{ext}} \end{array}}_f \underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ w \\ y \end{bmatrix}}_s = \underbrace{\begin{bmatrix} 0 & -1 & -1 & -1 \\ \hline \hline \hline \hline \end{bmatrix}}_S \underbrace{\begin{bmatrix} H'_1(x_1) \\ H'_2(x_2) \\ z(w) \\ u \end{bmatrix}}_e \underbrace{\begin{array}{c} V_m = \pi/m \\ F_{\text{sp}} = k\xi \\ F_{\text{dp}} = rV_{\text{dp}} \\ -F_{\text{ext}} \end{array}}_e$$

B2.1 Introduction to Port Hamiltonian Systems (PHS)

Example: damped mechanical oscillator excited by F_{ext} ($m\ddot{z} + r\dot{z} + kz = F_{\text{ext}}$)

(no gravity)



- 4 separate components

	state	energy H_n	flow f	effort e
m	$x_1 := \pi$	$\pi^2/(2m)$	$\dot{x}_1 = \dot{\pi}$	$H'_1(x_1) = x_1/m$
sp	$x_2 := \xi$	$k\xi^2/2$	$\dot{x}_2 = \dot{\xi}$	$H'_2(x_2) = kx_2$
dp		blue : force	$w := V_{\text{dp}}$	$z(w) := rw$
ext		red : velocity	$y := V_{\text{ext}}$	$u := -F_{\text{ext}}$

- assembled with rigid connections

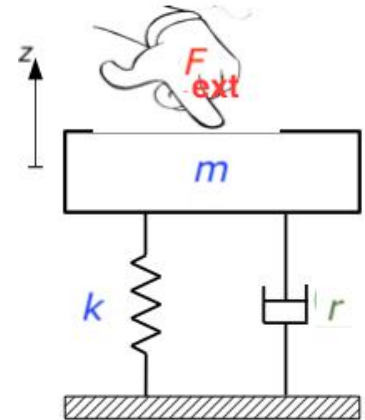
- internal forces are balanced $F_m + F_{\text{sp}} + F_{\text{dp}} + (-F_{\text{ext}}) = 0$
- all velocities are equal $V_m = V_{\text{sp}} = V_{\text{dp}} = V_{\text{ext}}$

$$\underbrace{\begin{array}{c} \dot{\pi} = F_m \\ \dot{\xi} = V_{\text{sp}} \\ V_{\text{dp}} \\ V_{\text{ext}} \end{array}}_f \underbrace{\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ w \\ y \end{array}} = \underbrace{\begin{bmatrix} 0 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{S = -S^T} \underbrace{\begin{array}{c} H'_1(x_1) \\ H'_2(x_2) \\ z(w) \\ u \end{array}}_e \underbrace{\begin{array}{c} V_m = \pi/m \\ F_{\text{sp}} = k\xi \\ F_{\text{dp}} = rV_{\text{dp}} \\ -F_{\text{ext}} \end{array}}$$

B2.1 Introduction to Port Hamiltonian Systems (PHS)

Example: damped mechanical oscillator excited by F_{ext} ($m\ddot{z} + r\dot{z} + kz = F_{\text{ext}}$)

(no gravity)



- 4 separate components

	state	energy H_n	flow f	effort e
m	$x_1 := \pi$	$\pi^2/(2m)$	$\dot{x}_1 = \dot{\pi}$	$H'_1(x_1) = x_1/m$
sp	$x_2 := \xi$	$k\xi^2/2$	$\dot{x}_2 = \dot{\xi}$	$H'_2(x_2) = kx_2$
dp		blue : force	$w := V_{\text{dp}}$	$z(w) := rw$
ext		red : velocity	$y := V_{\text{ext}}$	$u := -F_{\text{ext}}$

- assembled with rigid connections

- internal forces are balanced $F_m + F_{\text{sp}} + F_{\text{dp}} + (-F_{\text{ext}}) = 0$
- all velocities are equal $V_m = V_{\text{sp}} = V_{\text{dp}} = V_{\text{ext}}$

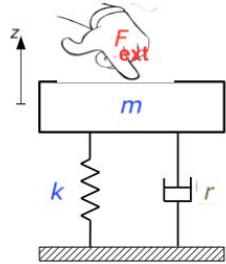
$$\underbrace{\begin{matrix} \dot{\pi} = F_m \\ \dot{\xi} = V_{\text{sp}} \\ V_{\text{dp}} \\ V_{\text{ext}} \end{matrix}}_{\mathbf{f}} \underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ w \\ y \end{bmatrix}}_{\mathbf{S} = -\mathbf{S}^T} = \underbrace{\begin{bmatrix} 0 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{S} = -\mathbf{S}^T} \underbrace{\begin{bmatrix} H'_1(x_1) \\ H'_2(x_2) \\ z(w) \\ u \end{bmatrix}}_{\mathbf{e}} \underbrace{\begin{matrix} V_m = \pi/m \\ F_{\text{sp}} = k\xi \\ F_{\text{dp}} = rV_{\text{dp}} \\ -F_{\text{ext}} \end{matrix}}_{\mathbf{e}}$$

→ Formulation (1) with $H(\mathbf{x}) = H_1(x_1) + H_2(x_2)$
 → $\mathbf{S} = -\mathbf{S}^T$ is canonical (no mechanical coefficients)

(ODE: with $z = \xi$)

B2.1 Introduction to Port Hamiltonian Systems (PHS)

Some variations: nonlinear components (modifying H or z) and also...



$$\begin{pmatrix} F_m \\ V_{sp} \\ V_{dp} \\ V_{ext} \end{pmatrix} = \left(\begin{array}{cc|cc} 0 & -1 & -1 & -1 \\ +1 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 \end{array} \right) \cdot \begin{pmatrix} V_m \\ F_{sp} \\ F_c \\ -F_{ext} \end{pmatrix}$$

Hamiltonian systems (conservative, autonomous)

$$\begin{pmatrix} F_m \\ V_{sp} \\ \cdot \\ \cdot \end{pmatrix} = \left(\begin{array}{cc|cc} 0 & -1 & \cdot & \cdot \\ +1 & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array} \right) \cdot \begin{pmatrix} V_M \\ F_{sp} \\ \cdot \\ \cdot \end{pmatrix}$$

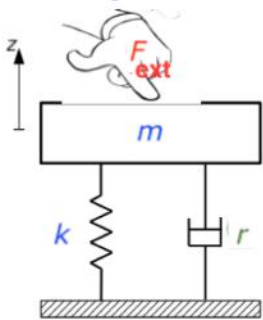
"Mass+Damper+Excitation" (spring removed)

$$\begin{pmatrix} F_m \\ \cdot \\ V_{dp} \\ V_{ext} \end{pmatrix} = \left(\begin{array}{cc|cc} 0 & \cdot & -1 & -1 \\ \cdot & \cdot & \cdot & \cdot \\ +1 & \cdot & 0 & 0 \\ +1 & \cdot & 0 & 0 \end{array} \right) \cdot \begin{pmatrix} V_m \\ \cdot \\ F_c \\ -F_{ext} \end{pmatrix}$$

"Mass+Excitation"

$$\begin{pmatrix} F_m \\ \cdot \\ \cdot \\ V_{ext} \end{pmatrix} = \left(\begin{array}{cc|cc} 0 & \cdot & \cdot & -1 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ +1 & \cdot & \cdot & 0 \end{array} \right) \cdot \begin{pmatrix} V_m \\ \cdot \\ \cdot \\ -F_{ext} \end{pmatrix}$$

B2.1 Introduction to Port Hamiltonian Systems (PHS)



$$\begin{array}{l} F_m \\ V_{sp} \\ V_{dp} \\ V_{ext} \end{array} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ w \\ y \end{pmatrix} = \left(\begin{array}{cc|cc} 0 & -1 & -1 & -1 \\ +1 & 0 & 0 & 0 \\ \hline +1 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 \end{array} \right) \cdot \begin{pmatrix} \frac{\partial_{x_1} H(\mathbf{x})}{\partial_{x_2} H(\mathbf{x})} \\ \frac{z(w) = r w}{u} \end{pmatrix} \begin{array}{l} V_m \\ F_{sp} \\ F_C \\ -F_{ext} \end{array}$$

$$\downarrow F_C = -r \partial_{x_1} H(\mathbf{x})$$

$$\begin{array}{l} F_m \\ V_{sp} \\ V_{ext} \end{array} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ y \end{pmatrix} = \left(\begin{pmatrix} 0 & -1 & -1 \\ +1 & 0 & 0 \\ +1 & 0 & 0 \end{pmatrix} - \begin{pmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right) \cdot \begin{pmatrix} \frac{\partial_{x_1} H(\mathbf{x})}{\partial_{x_2} H(\mathbf{x})} \\ u \end{pmatrix} \begin{array}{l} V_m \\ F_{sp} \\ -F_{ext} \end{array}$$

Differential formulation of PHS in “J-R”

$$\underbrace{\begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{y} \end{bmatrix}}_{\mathbf{f}} = \left(\underbrace{\begin{bmatrix} \mathbf{J}_{xx} & \mathbf{J}_{xu} \\ * & \mathbf{J}_{yu} \end{bmatrix}}_{\mathbf{J} = -\mathbf{J}^T} - \underbrace{\begin{bmatrix} \mathbf{R}_{xx} & \mathbf{R}_{xu} \\ * & \mathbf{R}_{yu} \end{bmatrix}}_{\mathbf{R} = \mathbf{R}^T \succeq 0} \right) \underbrace{\begin{bmatrix} \nabla H(\mathbf{x}) \\ \mathbf{u} \end{bmatrix}}_{\mathbf{e}} \rightarrow \left| \begin{array}{l} \text{power balance with} \\ P_{diss} = \mathbf{e}^T \mathbf{R} \mathbf{e} \geq 0 \end{array} \right.$$

+ Power-balanced numerical methods

B2.1 Introduction to Port Hamiltonian Systems (PHS)

Power-balanced numerical method : discrete gradient

Classical numerical schemes for $\frac{dx}{dt} = f(x)$:

- efficiently approximate $\frac{d}{dt}$ and exploit f
- *a posteriori* analysis of stability

B2.1 Introduction to Port Hamiltonian Systems (PHS)

Power-balanced numerical method : discrete gradient

Classical numerical schemes for $\frac{dx}{dt} = f(x)$:

- efficiently approximate $\frac{d}{dt}$ and exploit f
- *a posteriori* analysis of stability

A discrete power-balanced method (PHS)

Exploit differentiation chain rule

$$\frac{dE}{dt} = \sum_n \frac{\partial H}{\partial x_n} \frac{dx_n}{dt} \simeq \sum_n \underbrace{\frac{H_n(x_n[k+1]) - H_n(x_n[k])}{x_n[k+1] - x_n[k]}}_{\left[\nabla_D H(x[k], \delta x[k]) \right]_n} \underbrace{\frac{x_n[k+1] - x_n[k]}{\delta t}}_{[\delta x[k]/\delta t]_n} = \frac{E[k+1] - E[k]}{\delta t}$$

Jointly substitute $\dot{x} \rightarrow \delta x/\delta t$ and $\nabla H(x) \rightarrow \nabla_D H(x, \delta x)$:

$$\underbrace{\begin{pmatrix} \frac{\delta x}{\delta t} \\ w \\ -y \end{pmatrix}}_{f[k]} = S \underbrace{\begin{pmatrix} \nabla_D H(x, \delta x) \\ z(w) \\ u \end{pmatrix}}_{e[k]}$$

Simulation : solve $(\delta x, w)$ at each time step k (e.g. Newton-Raphson algo.)

B2.1 Introduction to Port Hamiltonian Systems (PHS)

Power-balanced numerical method : discrete gradient

Classical numerical schemes for $\frac{dx}{dt} = f(x)$:

- efficiently approximate $\frac{d}{dt}$ and exploit f
- *a posteriori* analysis of stability

A discrete power-balanced method (PHS)

Exploit differentiation chain rule

$$\frac{dE}{dt} = \sum_n \frac{\partial H}{\partial x_n} \frac{dx_n}{dt} \approx \sum_n \underbrace{\frac{H_n(x_n[k+1]) - H_n(x_n[k])}{x_n[k+1] - x_n[k]}}_{[\nabla_D H(x[k], \delta x[k])]_n} \underbrace{\frac{x_n[k+1] - x_n[k]}{\delta t}}_{[\delta x[k]/\delta t]_n} = \frac{E[k+1] - E[k]}{\delta t}$$

Jointly substitute $\dot{x} \rightarrow \delta x/\delta t$ and $\nabla H(x) \rightarrow \nabla_D H(x, \delta x)$:

$$\underbrace{\begin{pmatrix} \frac{\delta x}{\delta t} \\ \mathbf{w} \\ -\mathbf{y} \end{pmatrix}}_{\mathbf{f}[k]} = \mathbf{S} \underbrace{\begin{pmatrix} \nabla_D H(x, \delta x) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{pmatrix}}_{\mathbf{e}[k]}$$

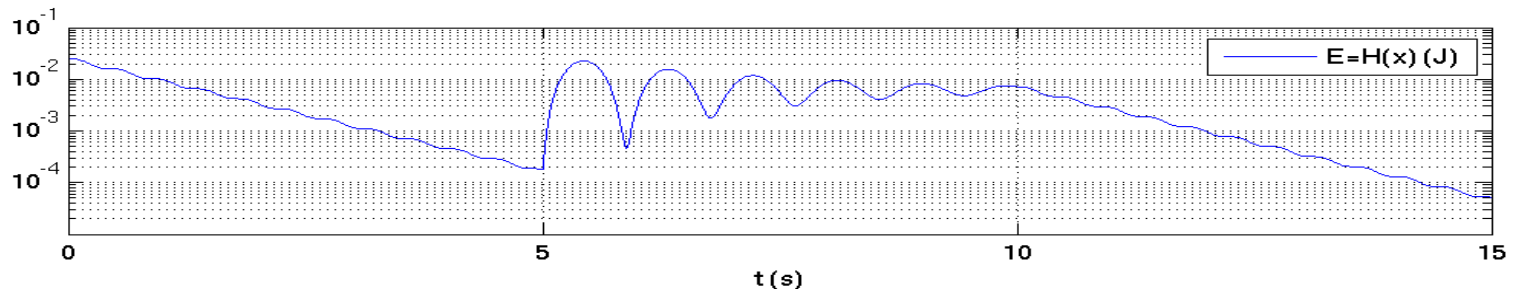
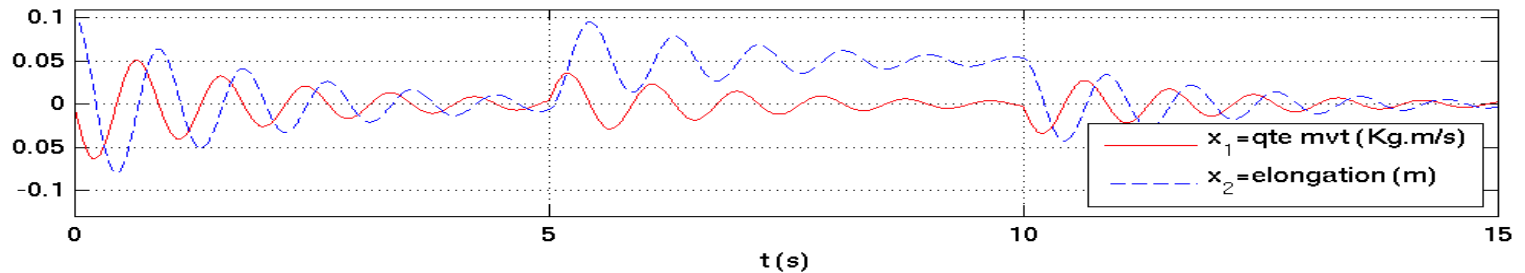
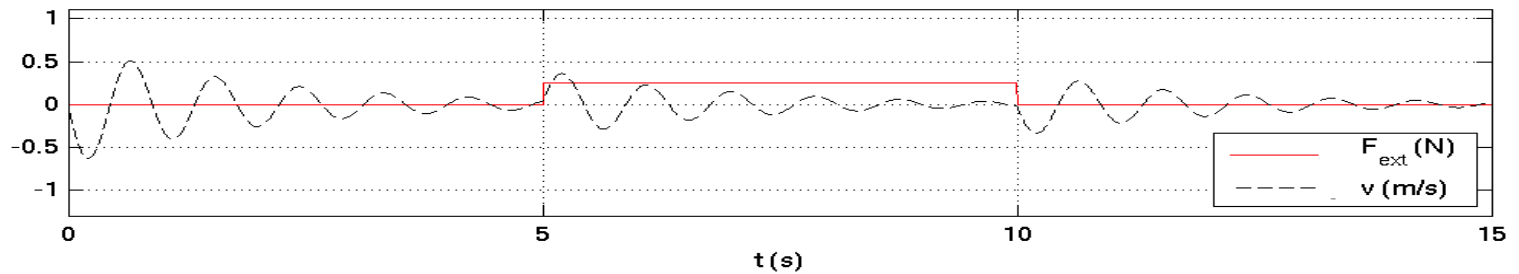
Simulation : solve $(\delta x, \mathbf{w})$ at each time step k (e.g. Newton-Raphson algo.)

- Skew-symmetry of S preserved $\Rightarrow 0 = \mathbf{e}^T \mathbf{S} \mathbf{e} = \mathbf{e}^T \mathbf{f} = \delta E/\delta t + \mathbf{z}(\mathbf{w})^T \mathbf{w} + \mathbf{u}^T \mathbf{y}$
- For **linear systems**, $\nabla_D H(x, \delta x) = \nabla H(x + \delta x/2)$ restores the **mid-point scheme**.
- Method also applies to nonlinear components and non separate Hamiltonian
- Power-balanced Runge-Kutta scheme (non iterative) [Lopes et al., LHMNC'2015]

B2.1 Introduction to Port Hamiltonian Systems (PHS)

Simulation 1: mass-spring-damper

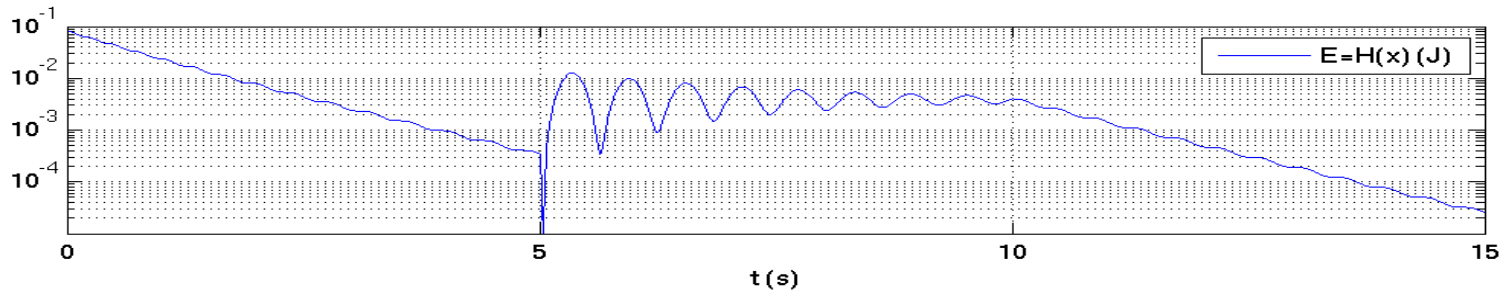
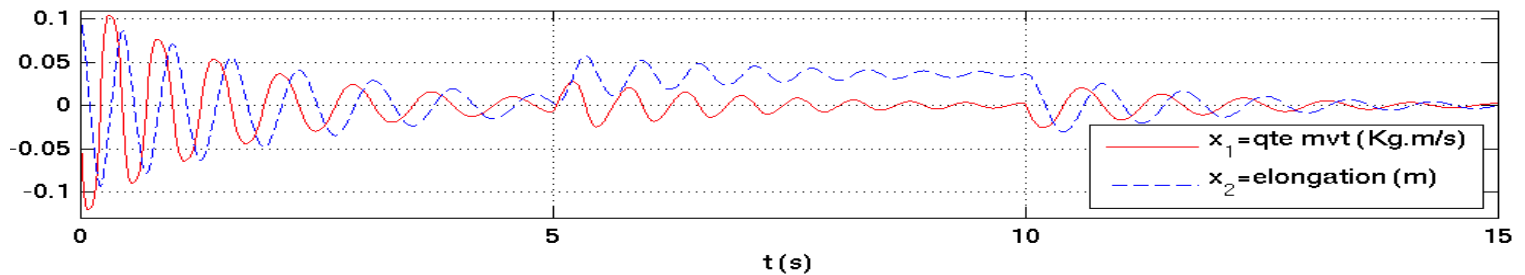
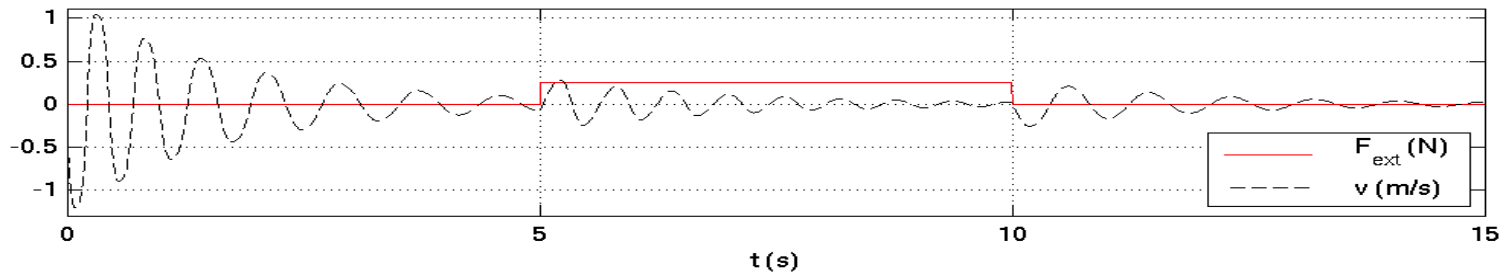
- **Parameters:** $M=100$ g, $K=5$ N/m, $C=0.1$ N.s/m et $\delta t=5$ ms
- **Initial conditions:** $x_0 = [mv_0=0, \ell_0=10 \text{ cm}]^T$
- **Excitation:** $F_{\text{ext}}(t) = F_{\text{max}} \mathbf{1}_{[5\text{s},10\text{s}]}(t)$ with $F_{\text{max}} = K\ell_0/2 = 0.25$ N



B2.1 Introduction to Port Hamiltonian Systems (PHS)

Simulation 2: idem with a hardening spring

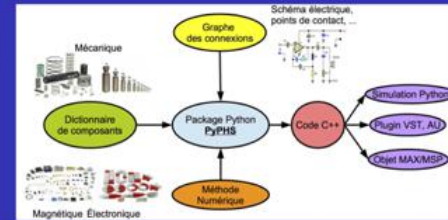
- **Potential energy:** $H_2^{\text{NL}}(x_2) = K L^2 [\cosh(x_2/L) - 1]$ ($\sim k x_2^2/2$)
- **Physical law:** $F_2 = (H_2^{\text{NL}})'(x_2) = K L \sinh(x_2/L)$ ($\sim K x_2$)
- **Reference elongation:** $L = \ell_0/4 = 25 \text{ mm}$



B2.2a Musical Applications (PHS)

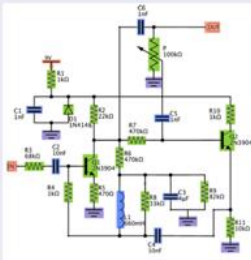
PhD, July 2016: Antoine Falaize

Passive modelling, simulation, code generation and correction of audio multi-physical systems



→ **librairie open-source** : <https://pyphs.github.io/pyphs>

Wah pedal (CryBaby): netlist → **PyPHS** → LateX eq. & Simulation C code



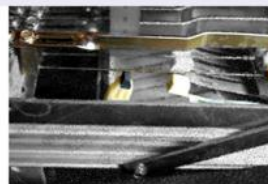
Components	Number
Storage	7 linear
Dissipative	18 (5 NL, 2 modulated)
Ports	3 (IN, OUT, battery)

Audio PlugIn:

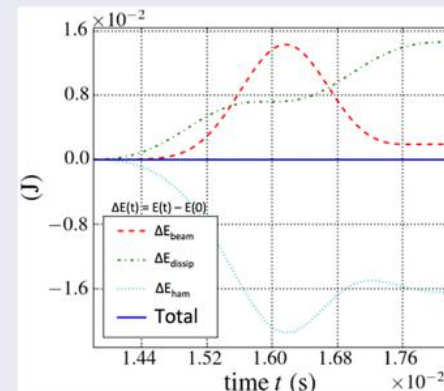
Sound 1 : dry

Sound 2 : wah

A simplified Fender-Rhodes Piano



Components	Hammer	1 beam	Pickup/RC-circuit
Storage	2 NL	2M lin.	2 lin. (+ NL connection)
Dissipative	1 NL	M lin.	1 lin.
Ports	2	1	1



B2.2b Musical Applications (PHS)

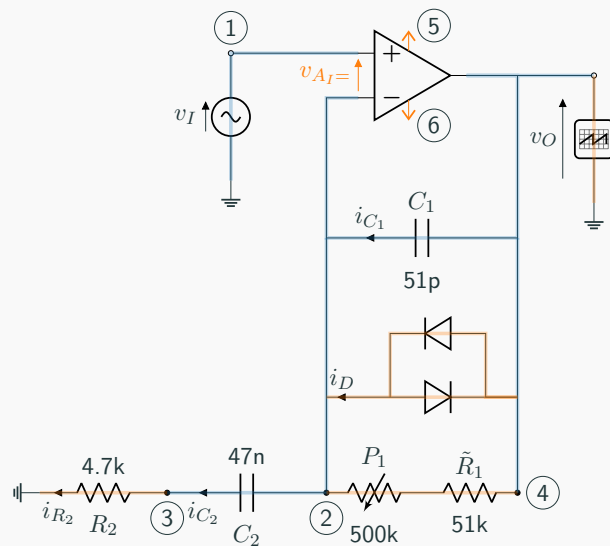
PhD, July 2021: Rémy Müller

Time-continuous power-balanced simulation of nonlinear audio circuits:
realtime processing framework and aliasing rejection



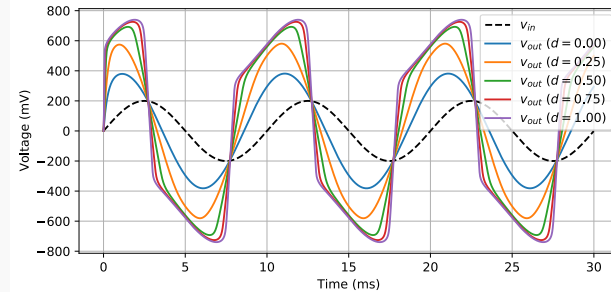
Tube screamer effect

Circuit



- 13 branches
- PH-ODE simulation

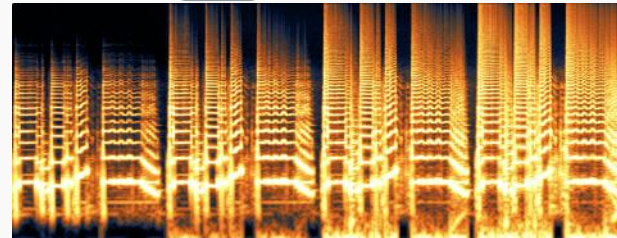
Simulation



projection \equiv RPM(1,0)

Newton method: avg 1.52 iteration

Sounds [▶ play](#)



Dry \rightarrow Drive = 0 \rightarrow 50 \rightarrow 100 %

(constant loudness: -14dBLKFS)

B2.2e Musical Applications (PHS)

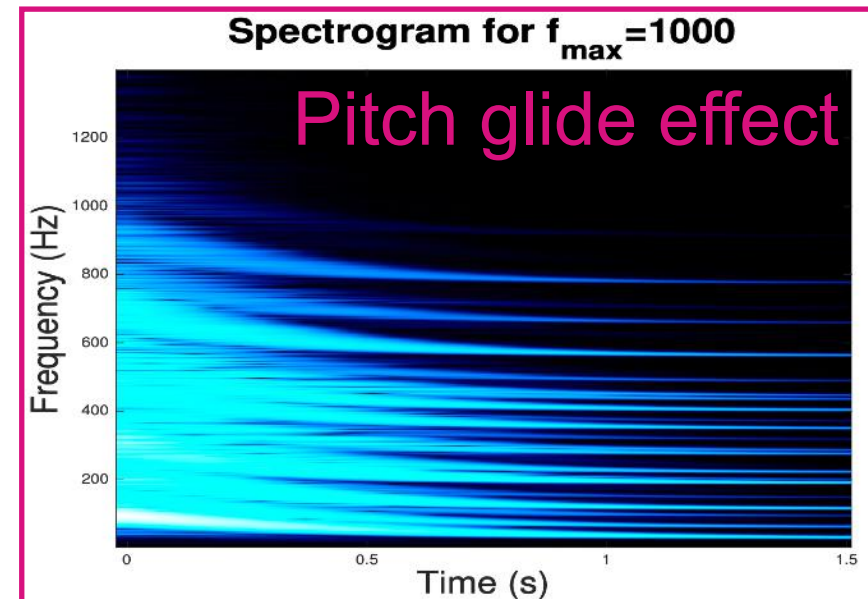
2D Berger plate + fluid damping + nonlinearity [CFA'18 with D. Roze, STMS]

$$\partial_t^2 w + \alpha \partial_t w + \Delta \Delta w - \epsilon \iint |\nabla w|^2 dS \Delta w = f \quad + \text{Boundary Conditions (simply supported)}$$

$$\begin{cases} \partial_t x &= (\mathcal{J} - \mathcal{R}) \delta_x \mathcal{H}(x) + \mathcal{G} u \\ y &= \mathcal{G}^* \delta_x \mathcal{H}(x) \end{cases}$$

Exact modal reduction

$$\partial_t X = (J - R) \nabla H(X) + G U$$



B2.2f Musical Applications (PHS)

$$M(z)\ddot{w}(z, t) + \mathcal{K}(z)w(z, t) = f(z, t) \text{ for all } (z, t) \in \Omega \times \mathbb{R}_+$$

Linear conservative mechanical boundary problems

+ linear damping models preserving eigenmodes (generalise Caughey series)

= musical instruments for various materials [with D. Matignon, Sup'Aéro]

Linear
Beam

$$\underbrace{\partial_t^2 q}_{\mathcal{M} \equiv Id} + \underbrace{(c_0 + c_1 \partial_\ell^4)}_c \partial_t q + \underbrace{\partial_\ell^4 q}_{\mathcal{K}} = f_{\text{ext}}$$

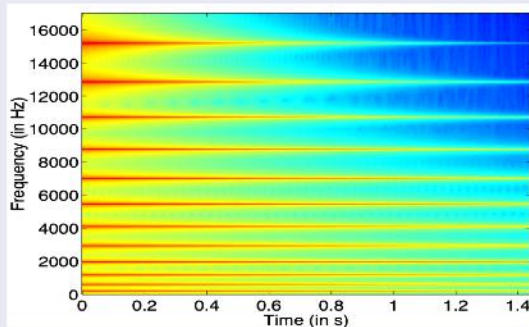
+ Boundary Conditions
(simply supported)

PHS

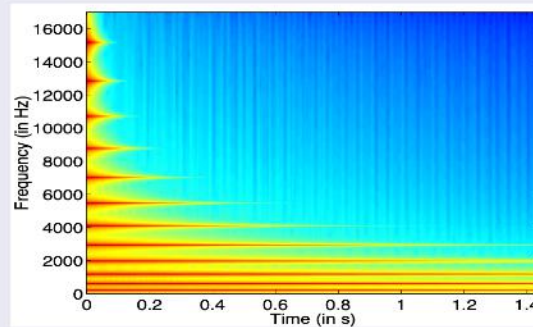
+ modal decomposition

+ power-balanced simulation

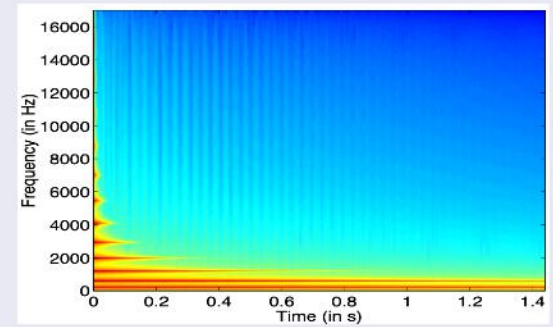
metal ($c_1 \sim 10^{-6}$)



glass ($c_1 \sim 10^{-5}$)



wood ($c_1 \sim 10^{-4}$)

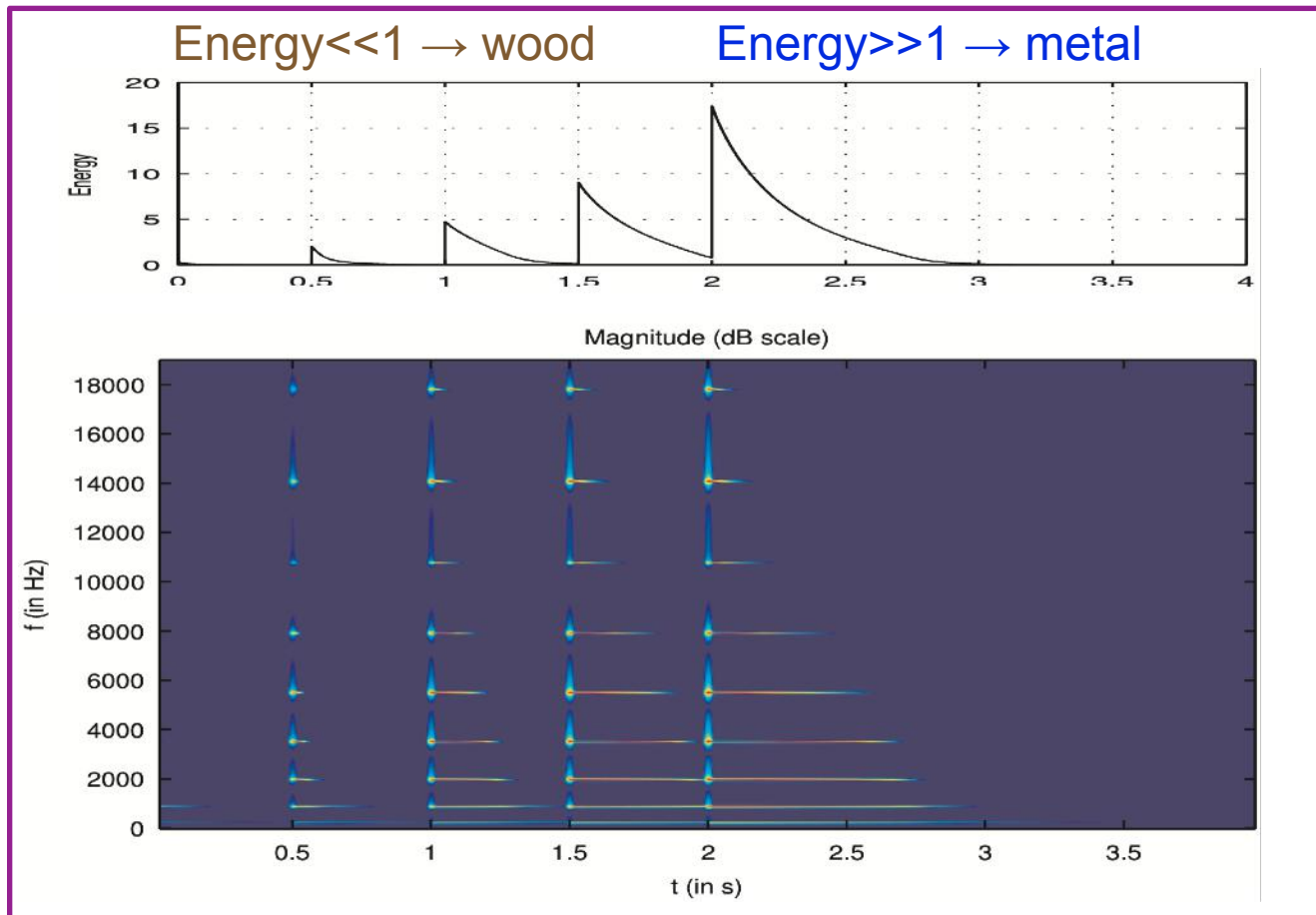


B2.2f Musical Applications (PHS)

Linear conservative mechanical boundary problems

+ nonlinear damping models preserving eigenmodes (generalises Caughey series)

= musical instruments with material morphing [with D. Matignon, Sup'Aéro]



B2.2f Musical Applications (PHS)

Idem for a 2D plate

	PDE	Hamiltonian
Function	w	$X := \begin{bmatrix} \vec{q} := \text{grad } w \\ p := \rho_0 \partial_t w \end{bmatrix}$
Energy	E	$= H(X) = \frac{1}{2} \int_{\Omega} (T_0 \vec{q}^2 + p^2 / \rho_0) dx dy$
Formal dyn. eq	$M \ddot{w} + \mathcal{K} w = f$	$\partial_t X = \begin{bmatrix} 0 & \mathcal{J}_{qp} \\ -\mathcal{J}_{qp}^* & 0 \end{bmatrix} \underbrace{\delta_X H(X)}_{\mathcal{L} X} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f$
Membrane equation	$\rho_0 \ddot{w} - T_0 \Delta w = f$	$\partial_t X = \begin{bmatrix} 0 & \text{grad} \\ \text{div} & 0 \end{bmatrix} \begin{bmatrix} T_0 l_2 & 0 \\ 0 & \frac{1}{\rho_0} \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f$

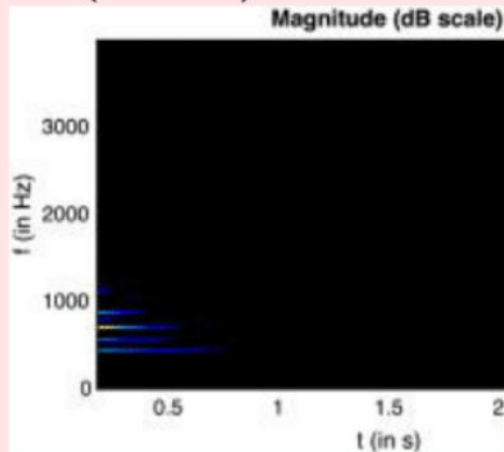
PHS +NL damping R

$$\partial_t X = (\mathcal{J} - \mathcal{R}) \mathcal{L} X + [0, 1]^T f$$

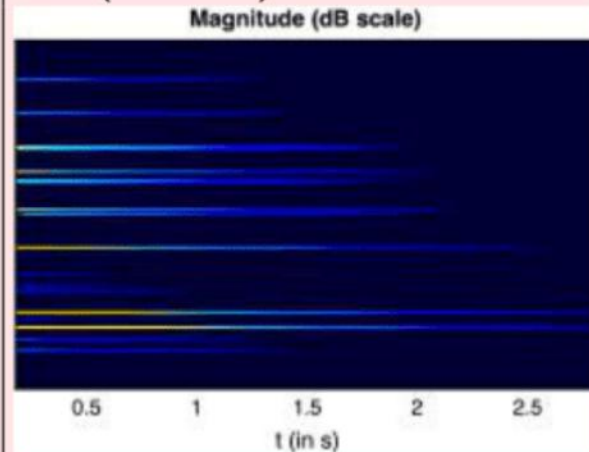
$$\mathcal{L} \mathcal{R} = \underbrace{\begin{bmatrix} 0_{qq} & 0_{qp} \\ 0_{pq} & I_{pp} \end{bmatrix}}_{\text{momentum selection}} \mathbb{P}[(\mathcal{L} \mathcal{J})^* (\mathcal{L} \mathcal{J})]$$

Simulation & Spectrograms

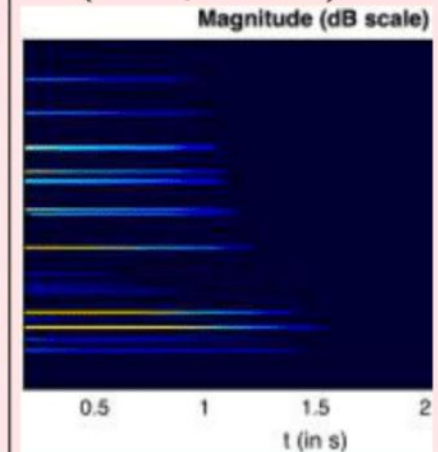
0 (~wood)



∞ (~metal)



c (interpolated)



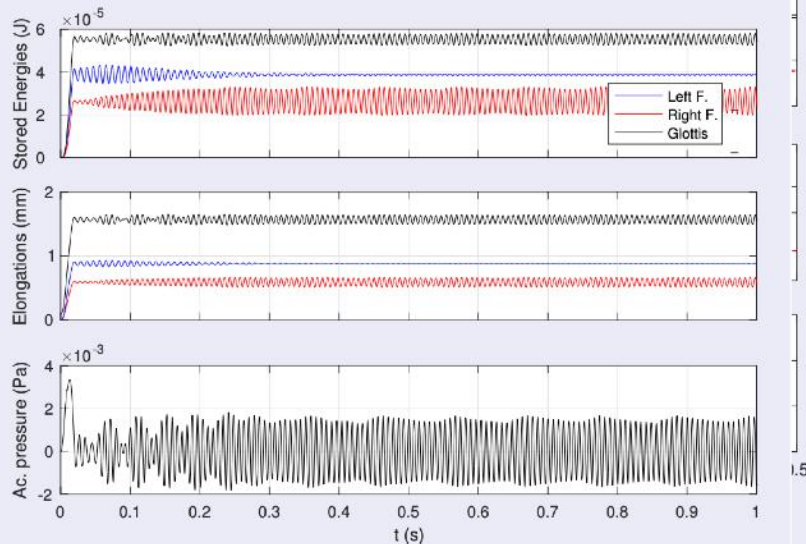
B2.2g Minimal power-balanced Vocal Apparatus

Strong asymmetry

$$k_l = 100 \text{ N m}^{-1} \text{ (112 Hz)}$$

$$k_r = 149 \text{ N m}^{-1} \text{ (137 Hz)}$$

with adduction ($h_r = 0.1 \text{ mm}$)



Quasi-periodic oscillations:

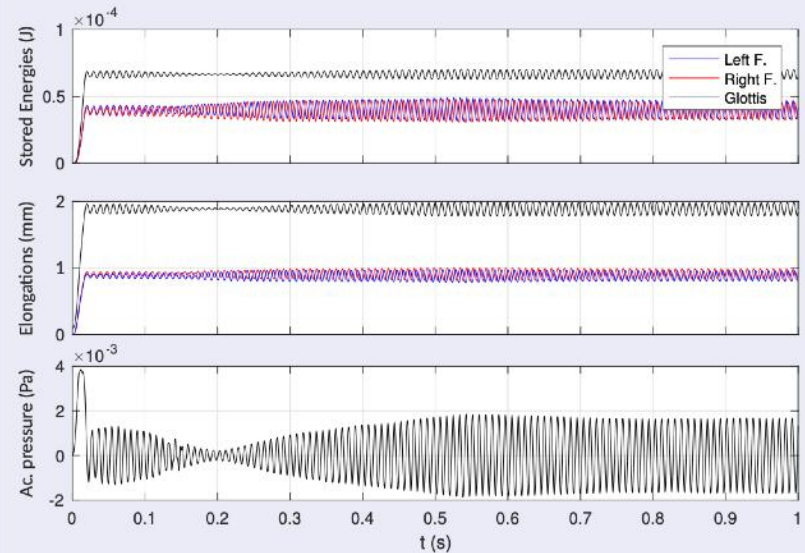
- starting on the left (lax) vocal fold,
- transferred to the right (stiffer) fold for the steady state regime.

Adduction (nearly closed, $h_r = 0.1 \text{ mm}$)

Slightly detuned vocal folds

$$k_l = 100 \text{ N m}^{-1} \text{ (112 Hz)}$$

$$k_r = 97 \text{ N m}^{-1} \text{ (110 Hz)}$$



Periodic oscillations:

- Oscillation stabilized after some transient
- Synchronized folds vibrations even without contact between folds

👉 *Self-oscillation threshold & Phonation & Dysphonia*

+ [V. Wetzel, PhD'21] + [T. Risse, PhD'25] + [ANR AVATARS 2023-2027]

Outline

A. IRCAM and the STMS lab

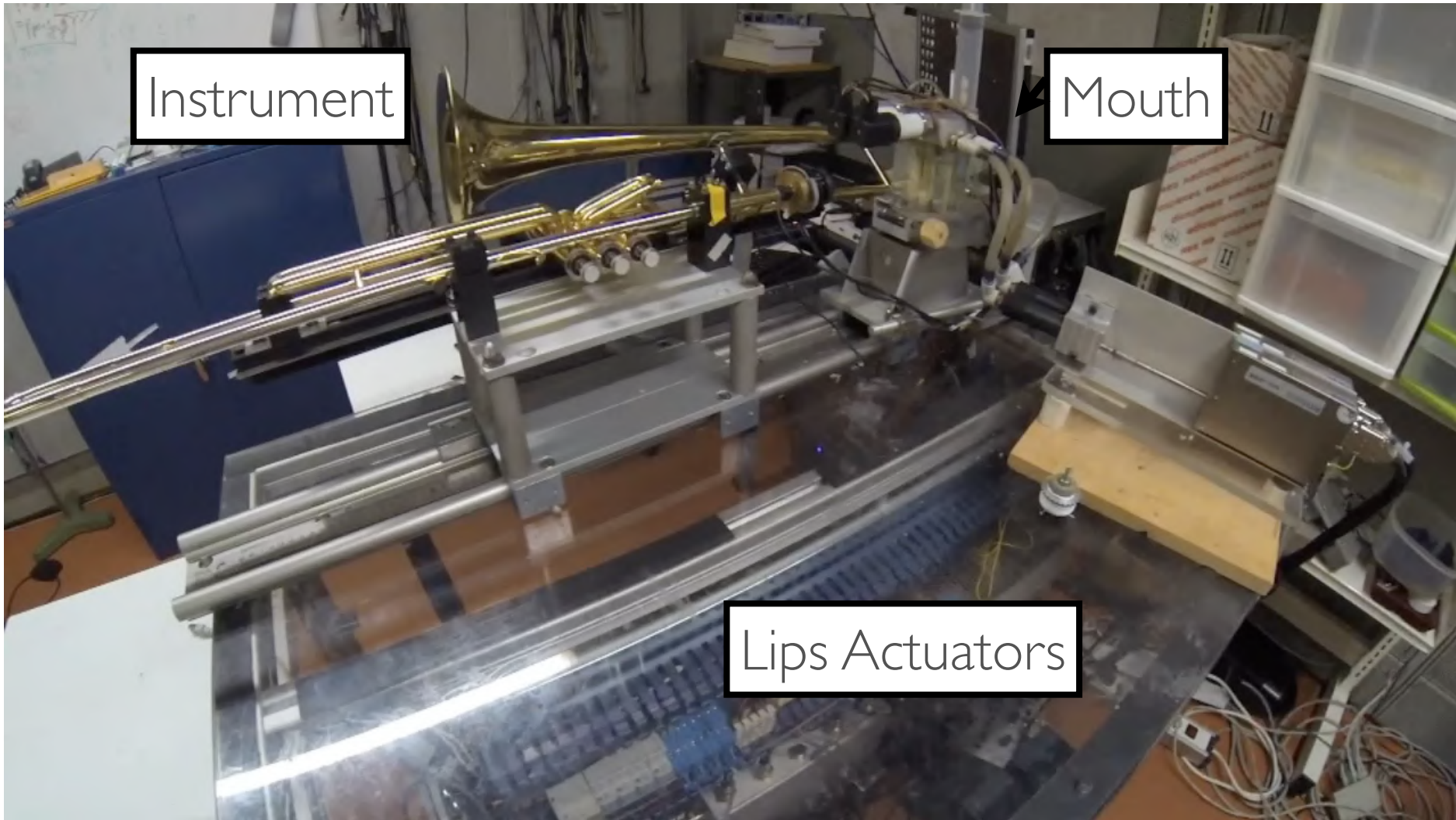
B. Modelling and simulation

C. Experimental work

Robotised artificial mouth for brass instruments

D. Visualisation

Robotised Artificial Mouth for brass instruments



Robotised Artificial Mouth for brass instruments

Valve trombone Bb

Mouthpiece

Valves

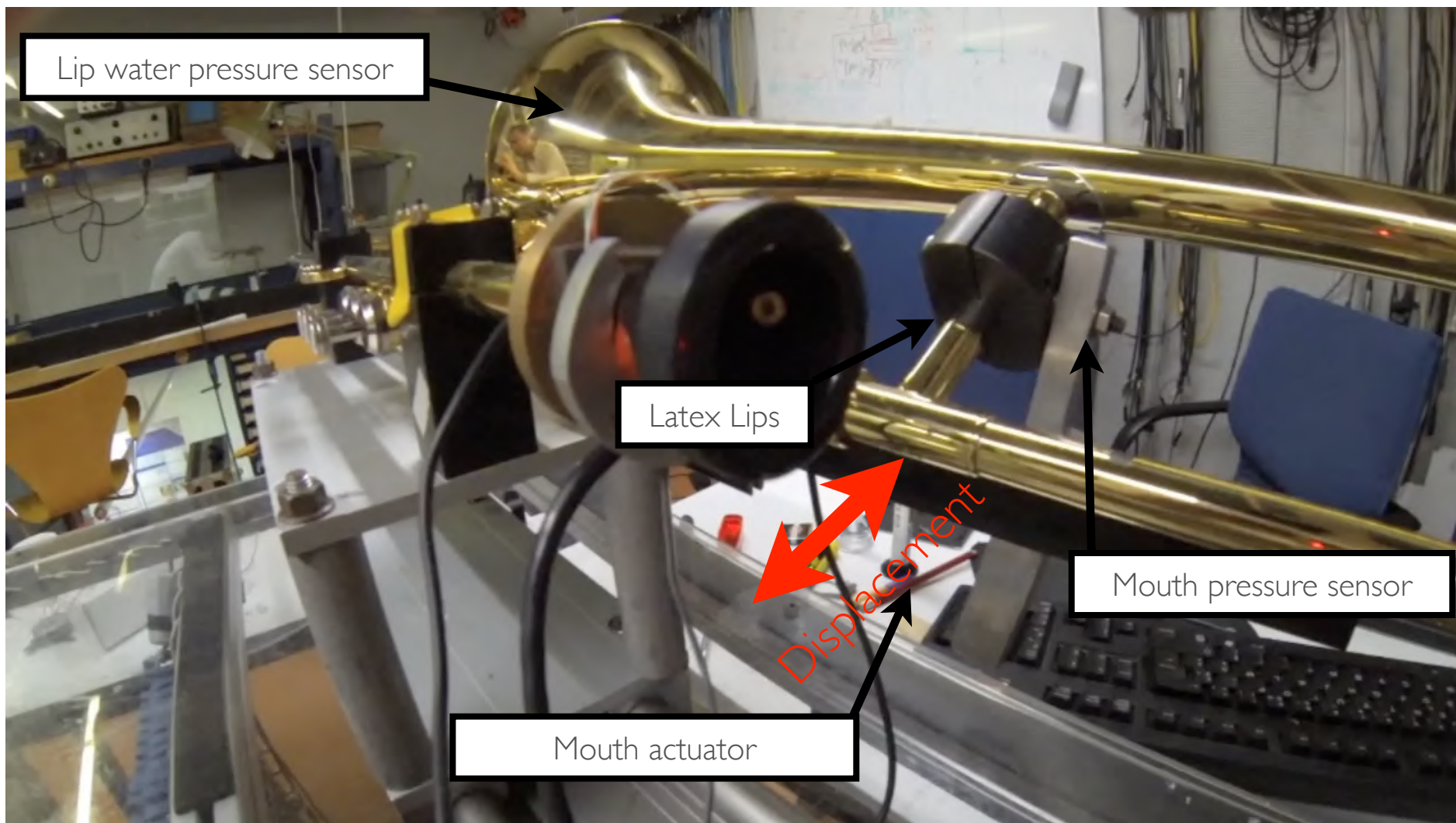
Robotised Artificial Mouth for brass instruments

Modified mouthpiece

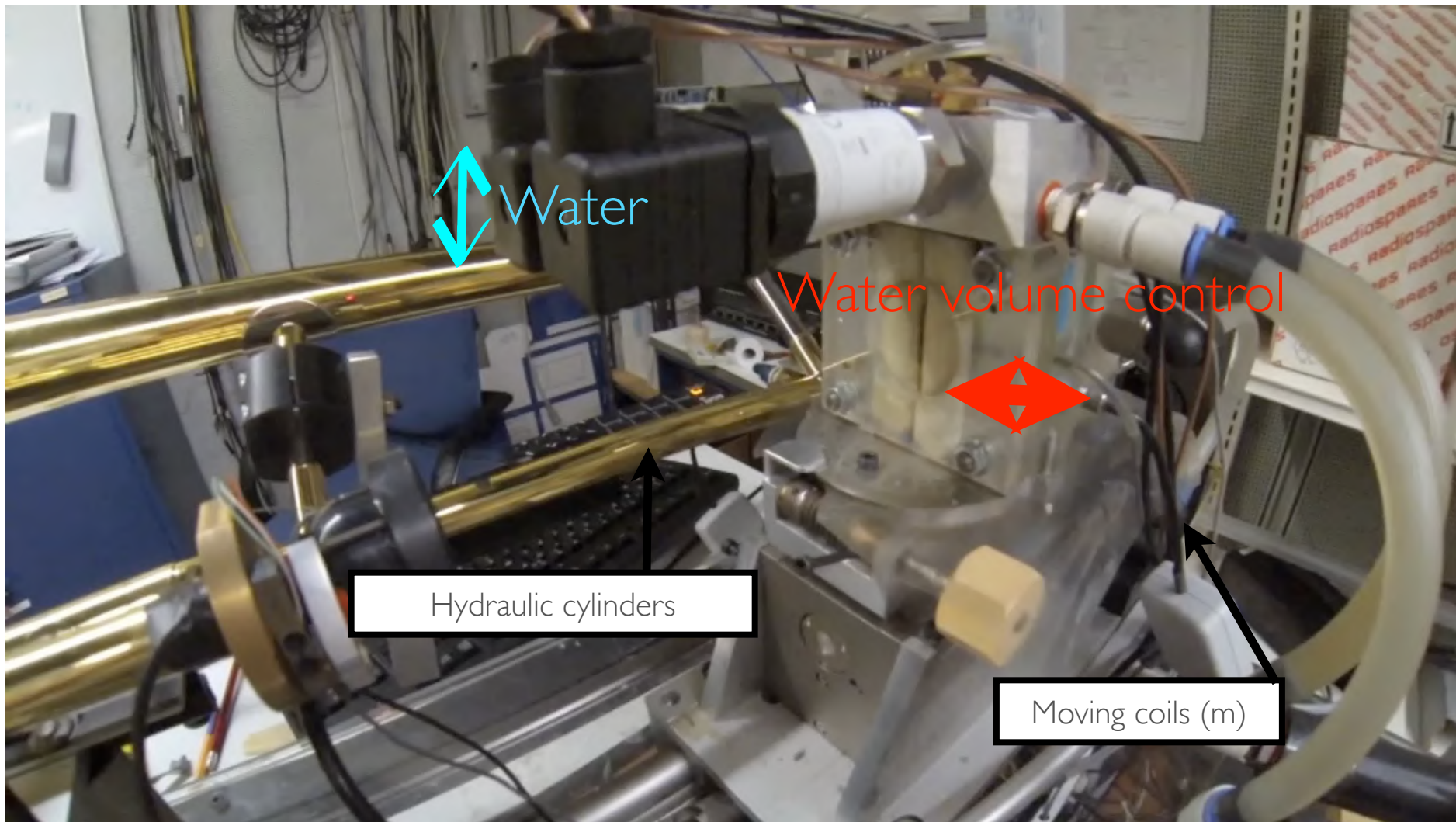
Pressure sensor

Force sensor

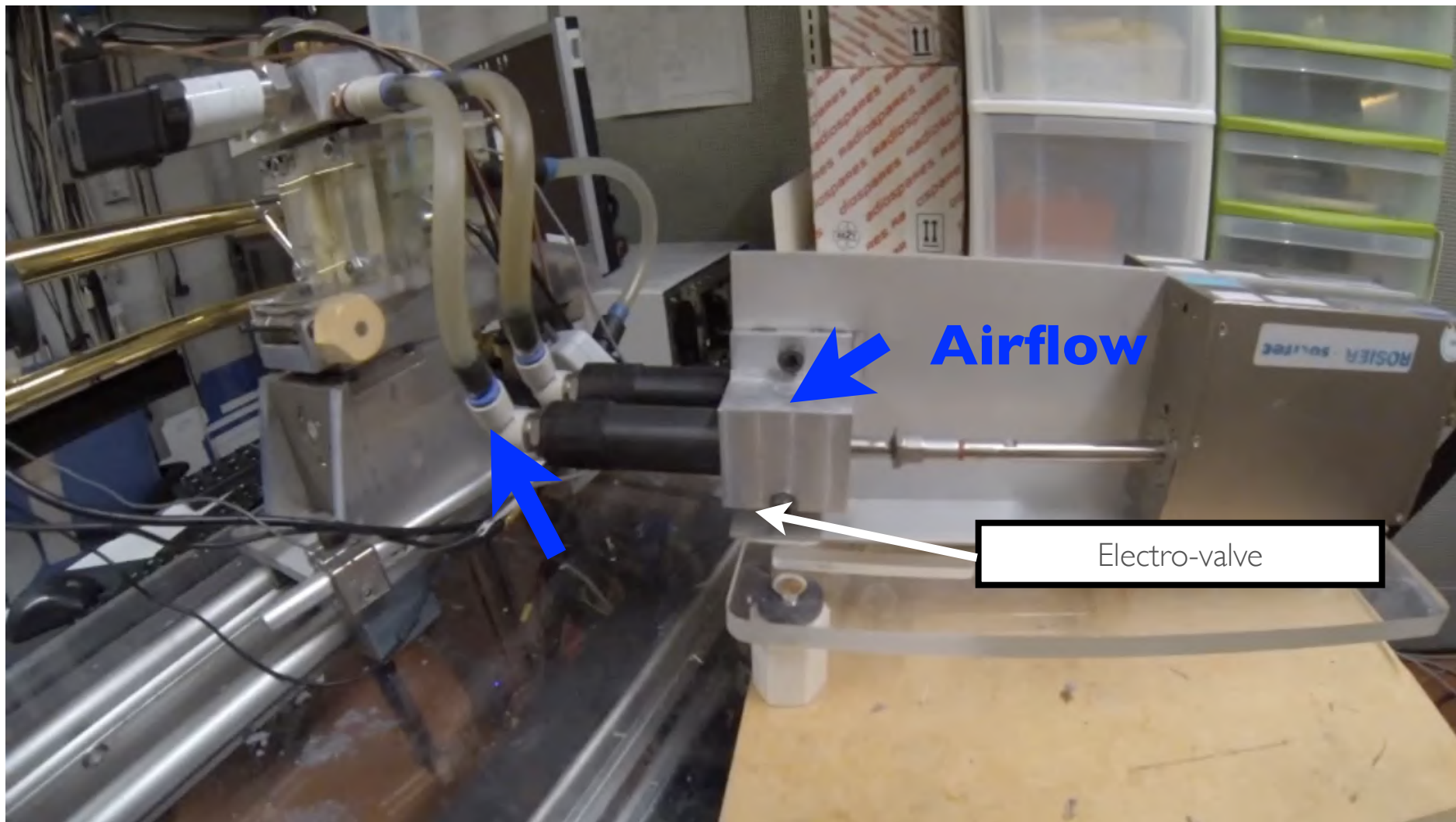
Robotised Artificial Mouth for brass instruments



Robotised Artificial Mouth for brass instruments



Robotised Artificial Mouth for brass instruments

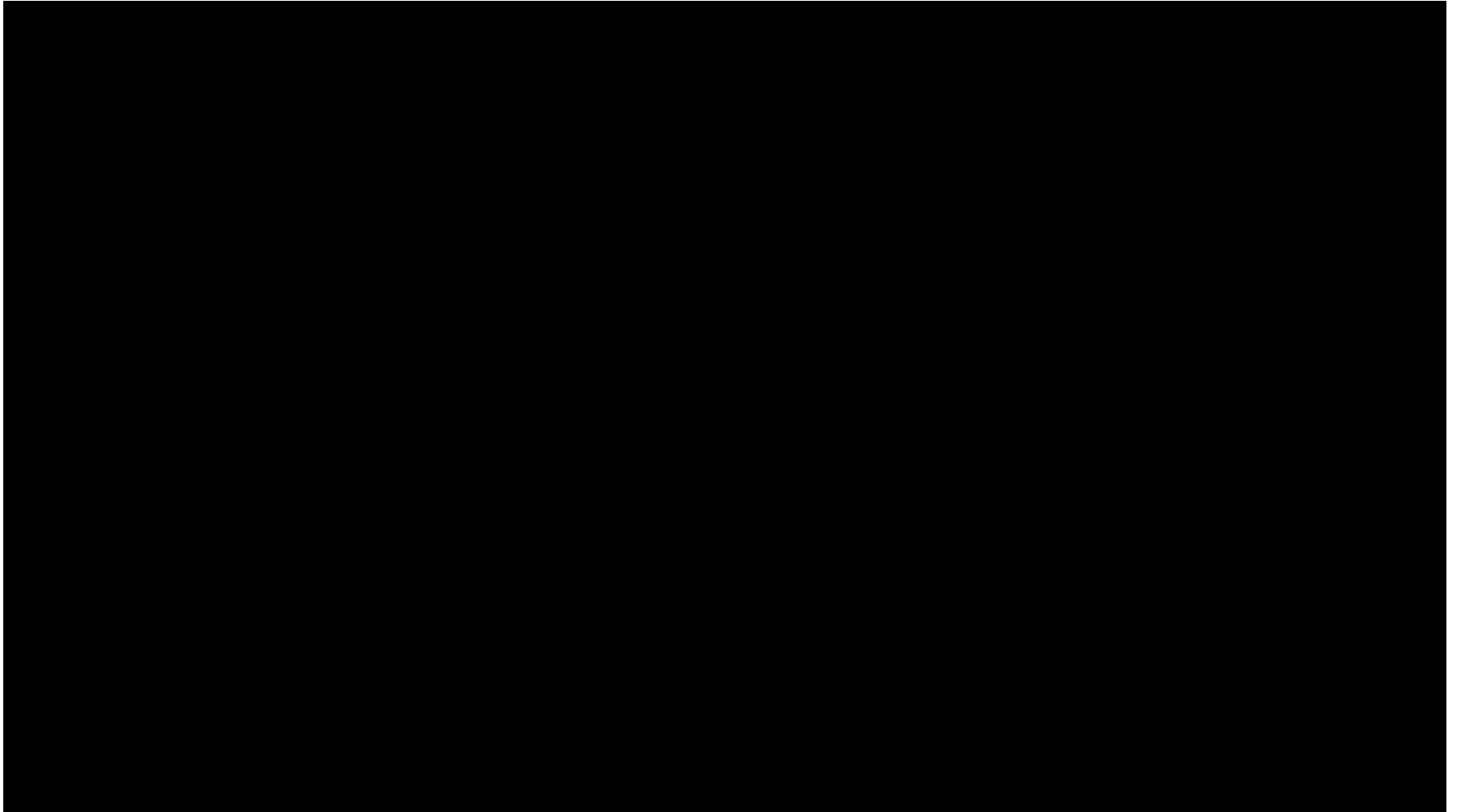


Robotised Artificial Mouth for brass instruments

First experiment on a trumpet (2009)

Fixed: all the lip parameters

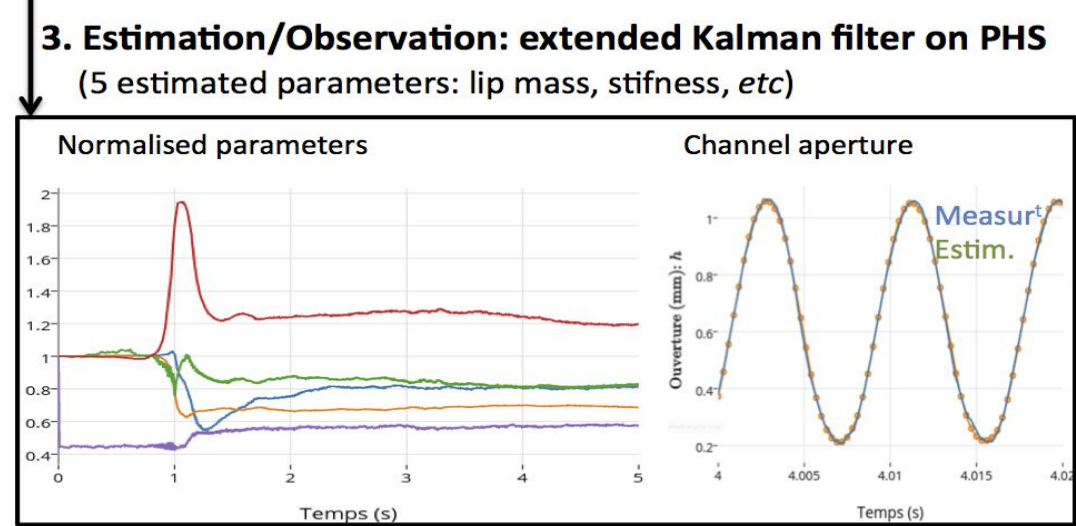
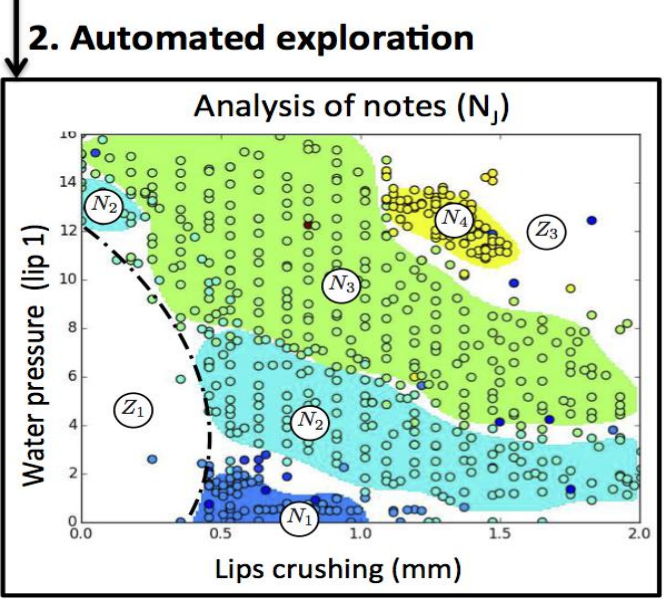
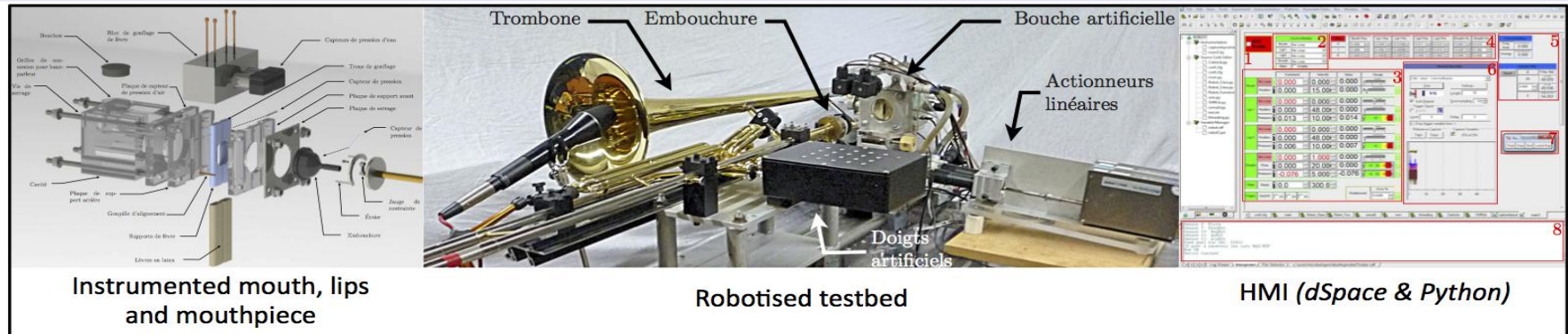
Open control: **airflow** (*servo-valve*)



Robotised Artificial Mouth for brass instruments

PhD, June 2016: Nicolas Lopes

Passive modelling, simulation and experimental study of a robotised artificial mouth playing brass instruments



Outline

A. IRCAM and the STMS lab

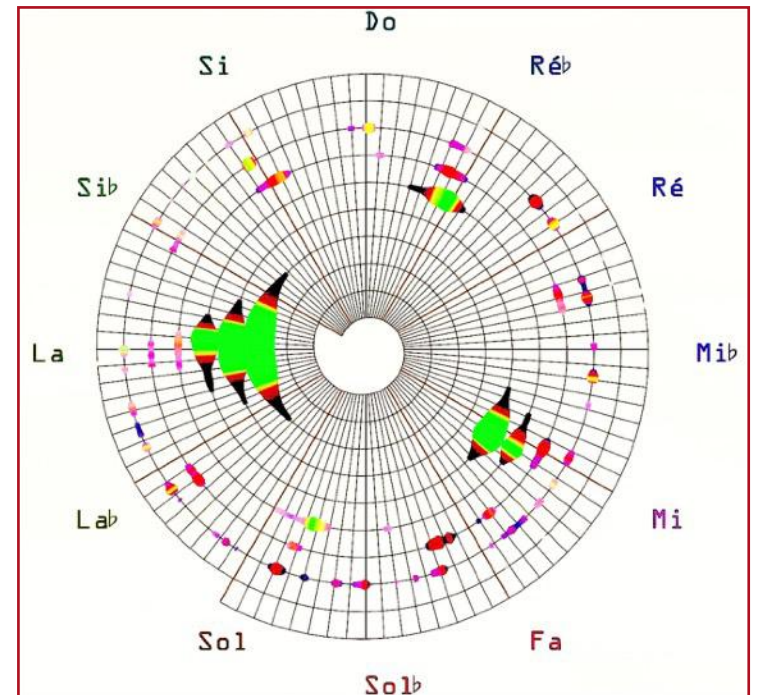
B. Modelling and simulation

C. Experimental work

D. Visualisation

D1. Principle (snail-analyser)

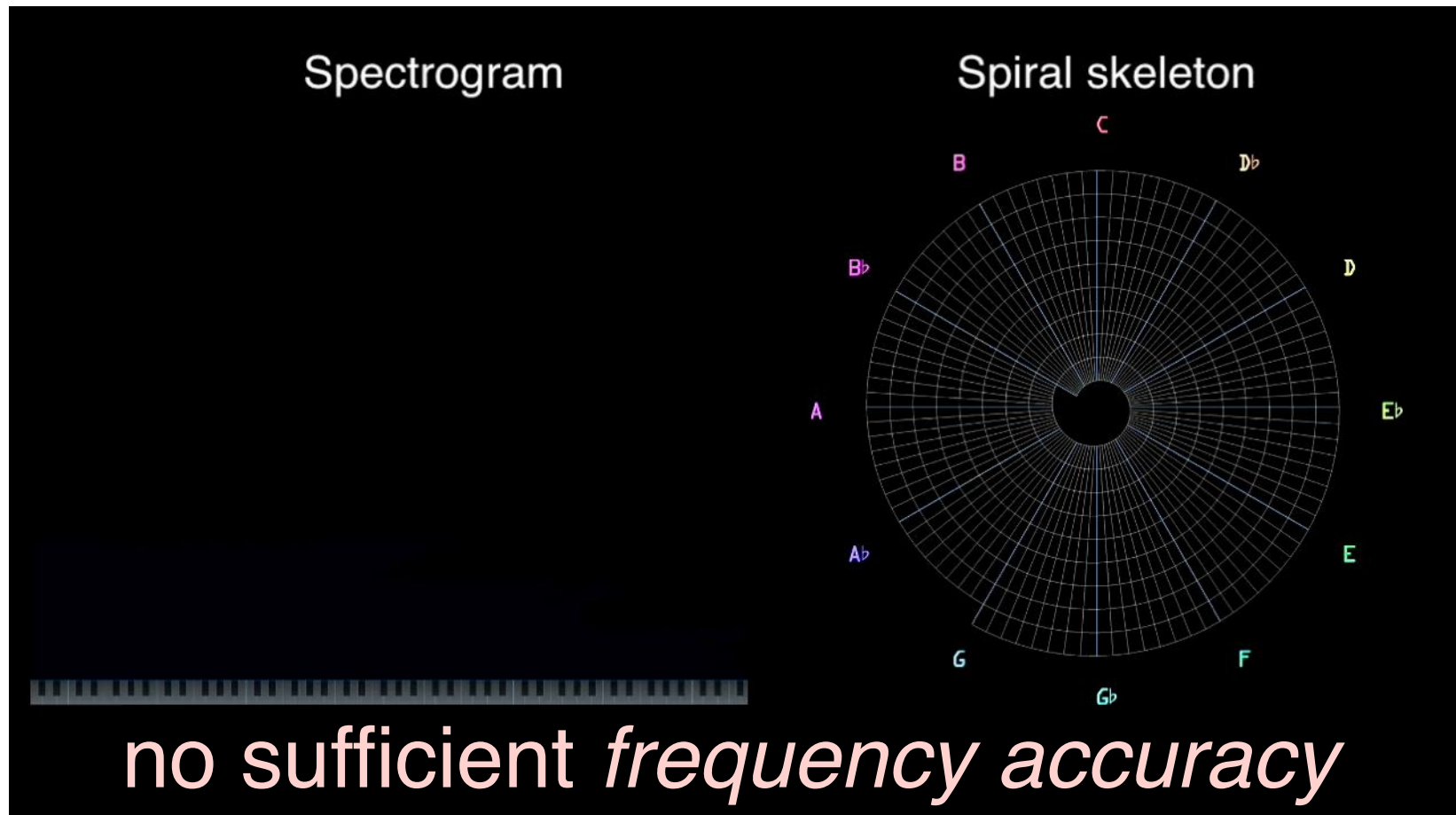
D2. Views



D1. Principle (1/2)

1. Musical representation of the spectrum on a spiral skeleton

- +1 round = +1 octave
- angle = chroma (*note name*)

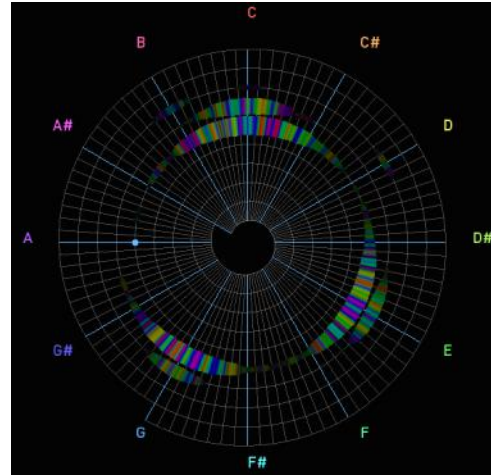


D1. Principle (2/2)

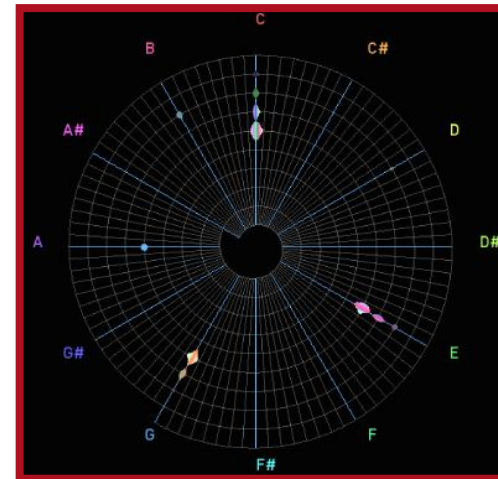
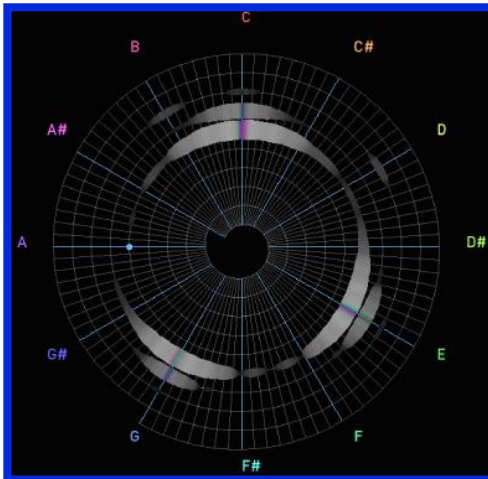
2. Algorithm to improve the frequency accuracy

➔ build the **demodulated phase**

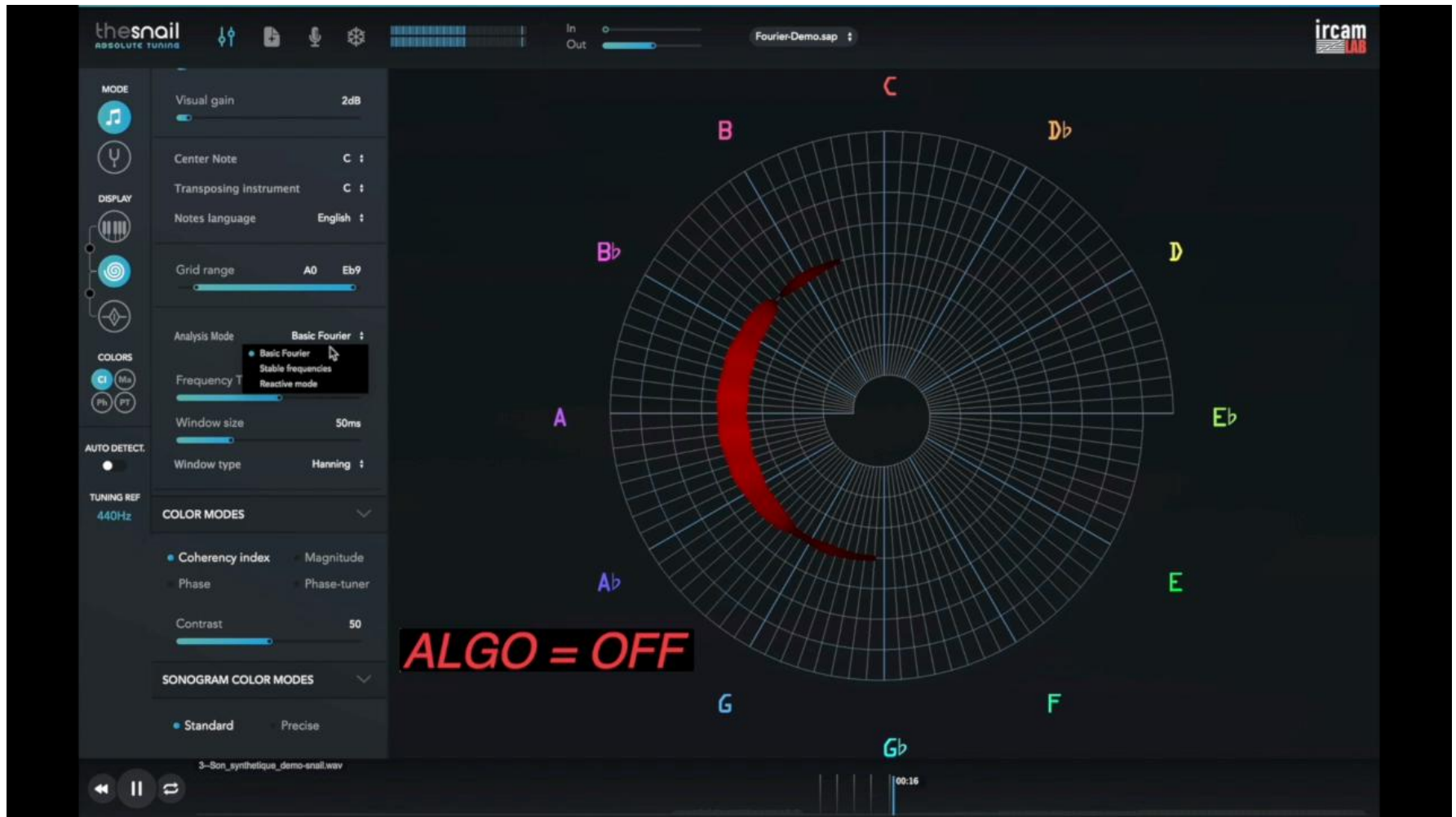
$$\phi_d(t, f) = \phi_{Fourier}(t, f) - 2\pi ft$$



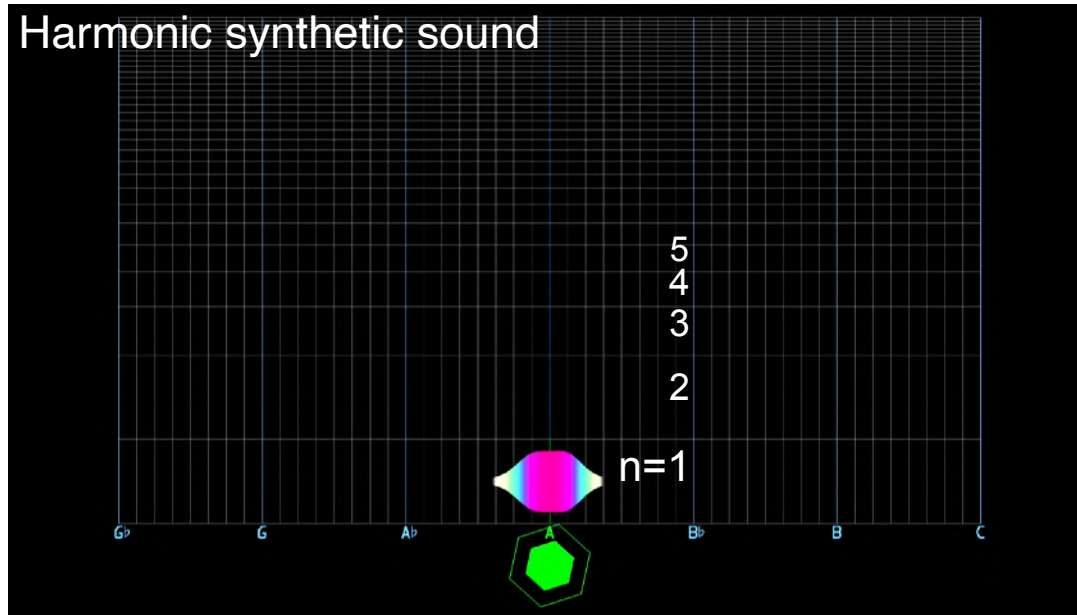
➔ only the **slowly oscillating** phases $\phi_d(t, f)$ are **selected**



D2. Views: demonstration



D2. Views: harmonic representation



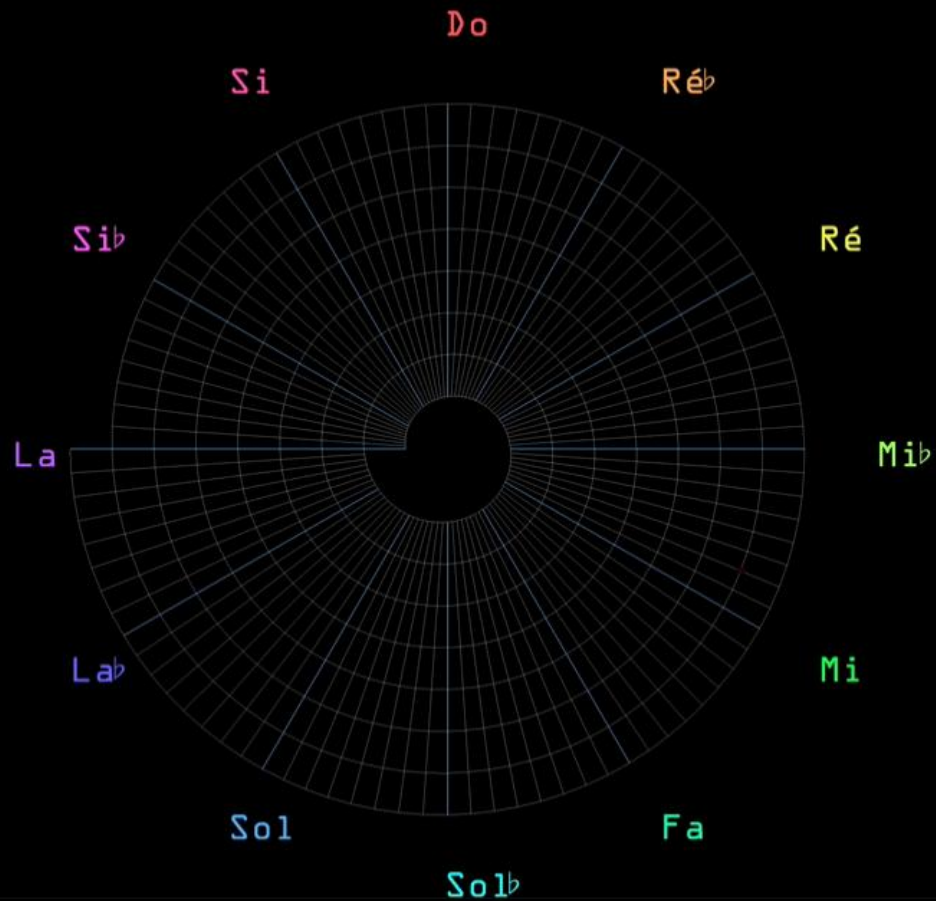
Stage $n = \text{Harmonic } n$

Harmonicity
= vertical alignment

Inharmonicity
= deviation with n

Illustration of a sound illusion

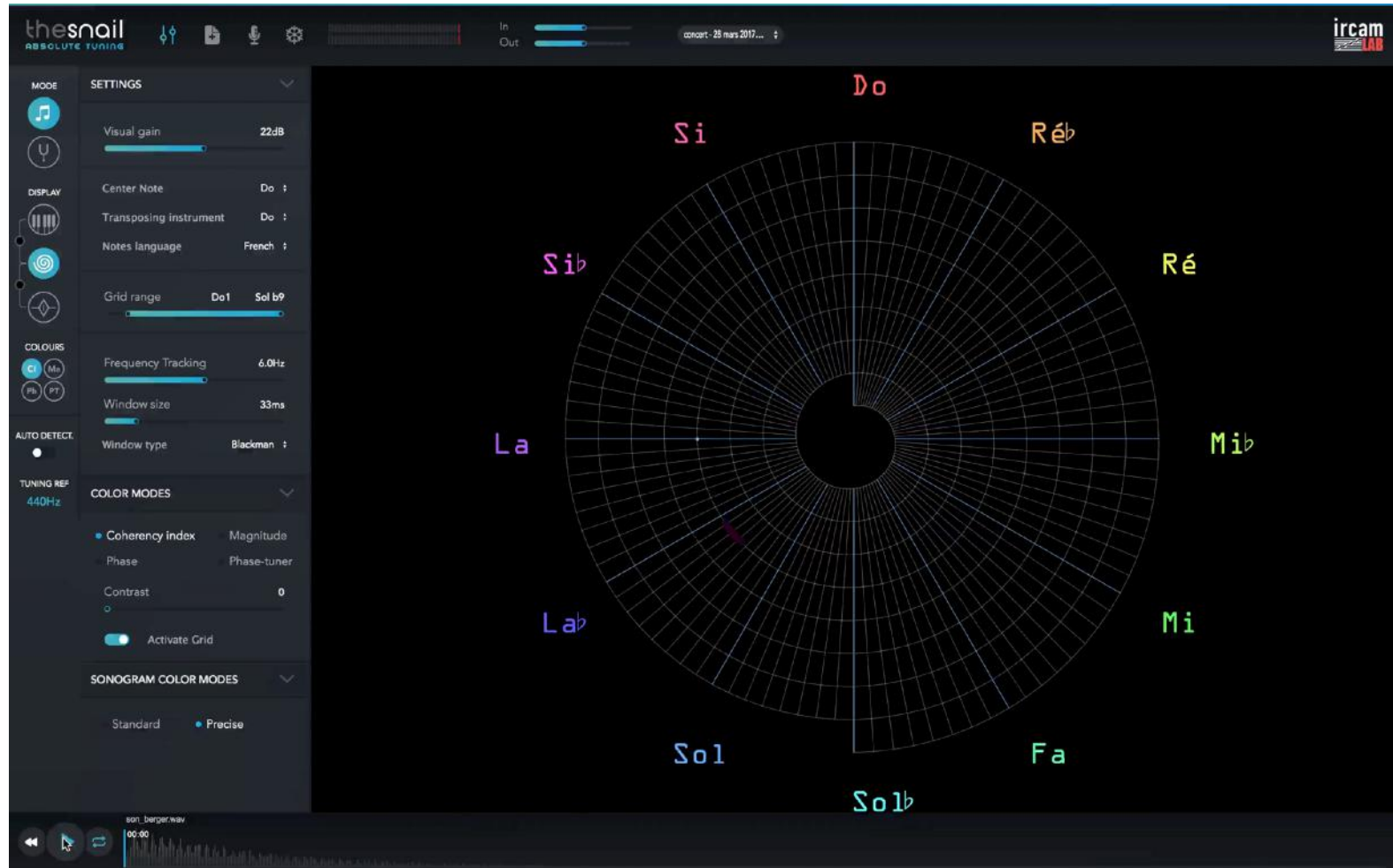
Infinite continually ascending glissando (continuous Risset scale)



D2. Views

Simulation of the nonlinear Berger membrane

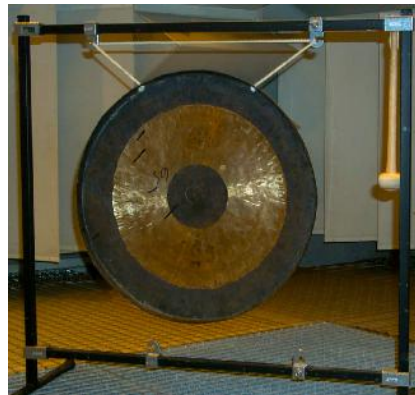
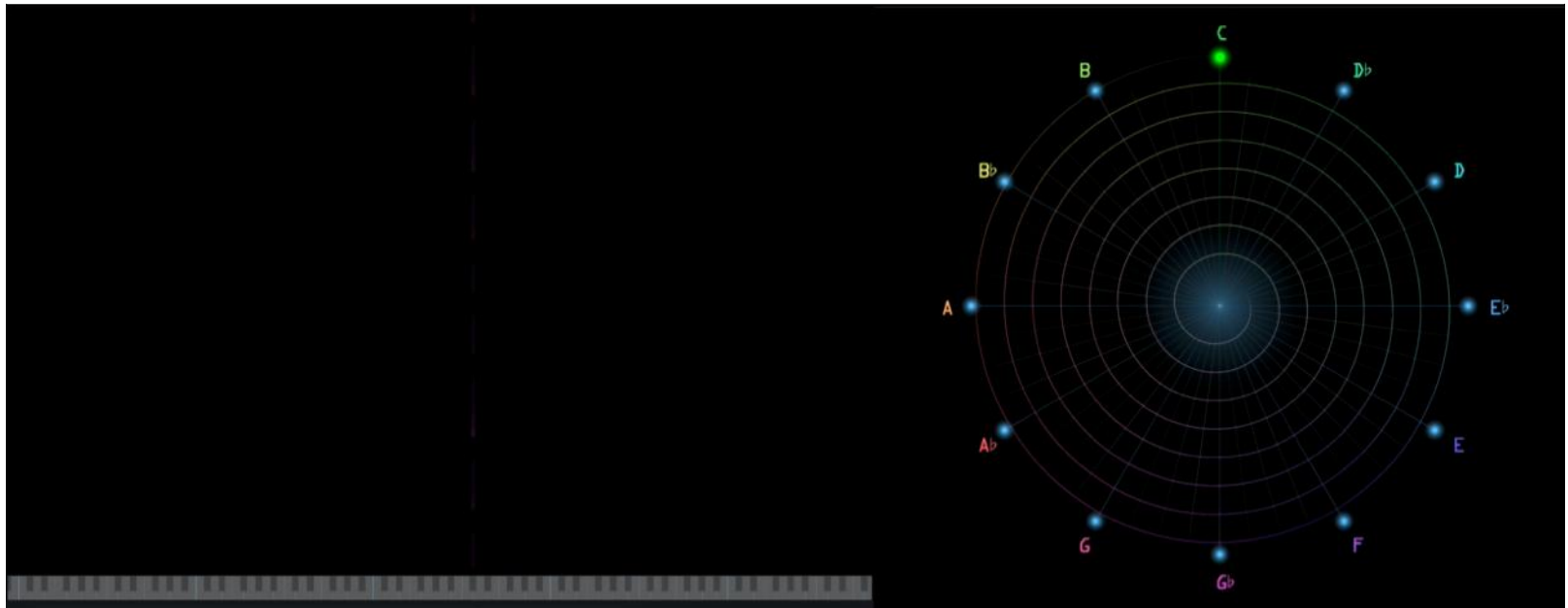
[CFA'2018 with David Roze]



D2. Views

Sound of a gong excited by a sinusoid with increasing amplitude

Courtesy of A. Chaigne, O. Thomas, C. Touzé



Thank you for your attention

Acknowledgements:

- **Volterra series:** M. Hasler (EPFL, Switzerland), B. Laroche (INRAE), D. Roze (STMS), V. Smet
- **Port-Hamiltonian systems:** A. Falaize, N. Lopes, D. Matignon (ISAE-Sup'aéro), J. Najnudel, R. Müller, D. Roze (STMS), F. Silva (LMA) & *ANR Hamecmopsys* & *ANR Infidhem*
- **Artificial mouth:** R. Caussé, N. Lopes, B. Véricel, C. Vergez (LMA, Marseille) & *Mechatronics group-Mines-ParisTech* & *ANR Consonnes*
- **Snail Analyser:** Q. Lamerand, C. Picasso, R. Piéchaud, F. Rousseau & *Plugivery* & *Buffet*
- **Gong sound:** courtesy of A. Chaigne, O. Thomas, C. Touzé
- **& Ongoing work on 4D-PHS :** E. Rouhaud (UTT) et coll.

