# Cosmological Constant : Physical or Geometrical Necessity ?

L Rakotomanana

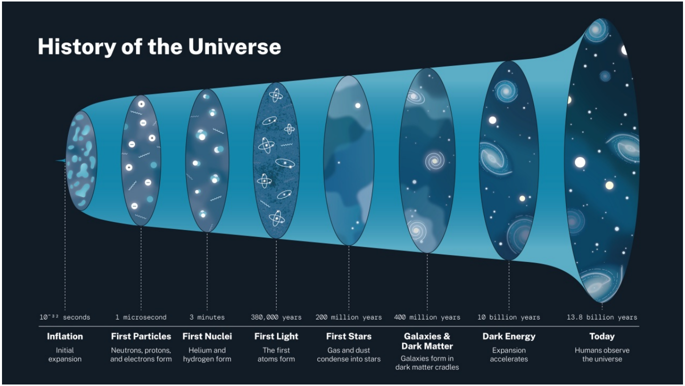
IRMAR, UMR 6625, Université de RENNES, France

GDR-GDM, Paris, November $20^{th} - 22^{nd}$, 2024

# Outine

1. Introduction and Motivation
2. Mathematical Model of Vacuum Spacetime
3. Cosmological Constant : Geometrical and/or Physical ?
4. Final Remarks and Outlook

# I. Introduction and Motivation




We focus on the era of **Galaxies & Dark Matter → Dark Energy → Today** , the early period from Bing-Bang until First Light being better approached with **Quantum Physics** (Astrophysics)!

# INTR-1 : Cosmological Constant $\Lambda$ is a necessary evil

- **Einstein 1917** (Concept RG) : The Universe is **static** and a Cosmological Constant $\Lambda$ is necessary to satisfy this belief:

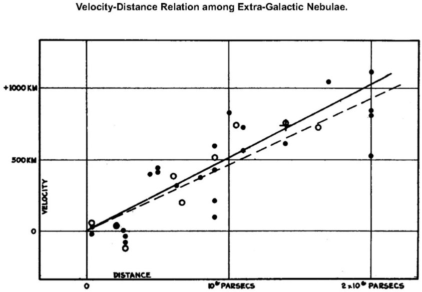
$$\frac{1}{2\chi}\left(\overline{G}_{\mu\nu} + \Lambda\, g_{\mu\nu}\right) = 0 \qquad \text{with} \qquad \overline{G}_{\mu\nu} := \overline{\Re}_{\mu\nu} - \frac{\overline{\mathcal{R}}}{2}\, g_{\mu\nu}$$

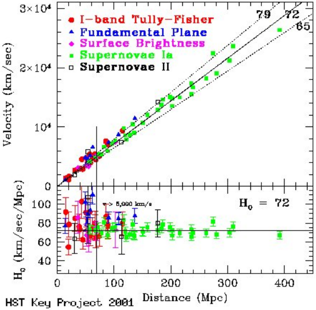
- **Friedman equations 1922** (Theory) models **expansion** of Universe, homogeneous and isotropic spacetime (FLRW metric), filled of **perfect fluid** with $T^{\text{fl}}_{\mu\nu} := (\rho + p)u_\mu u_\nu - p\, g_{\mu\nu}$

$$\begin{cases} \left(\dfrac{\dot{R}}{R}\right)^2 = \dfrac{8\pi G}{3}\rho - \dfrac{\Lambda}{3} - \dfrac{k}{R^2} \\ \dfrac{\ddot{R}}{R} = -\dfrac{4\pi G}{3}(\rho + 3p) - \dfrac{\Lambda}{3} \end{cases}$$

- **Hubble 1929** (Experimental) : The Universe is **expanding uniformly** governed by matter and radiation ($\Lambda$ **not needed?** - Eddington -).

# INTR-2 : Uniform Expansion of the Universe

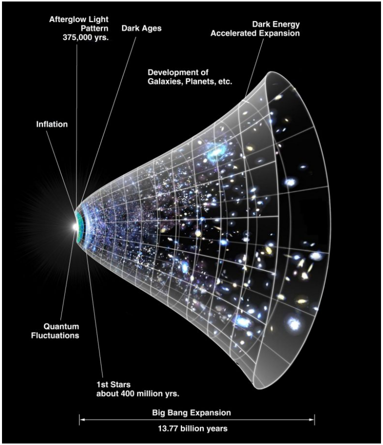



- (Left) Original curve from **Hubble** (PNAS 1929) : $v = H_0 \; D$.
- (Right) Results from **Hubble Space Telescope** projects (2001)
- There are more recent values (Hubble Tension Problem).
- Key Review on Cosmological Constant : **Weinberg 1989** !

# INTR-3 : : Acceleration of the Universe Expansion

**Perlmutter & al, and Riess & al 1998** (Observation) : Two teams found independently that the Universe expansion is **accelerating**.



# INTR-4 : Cosmological Constant Λ is a necessary evil

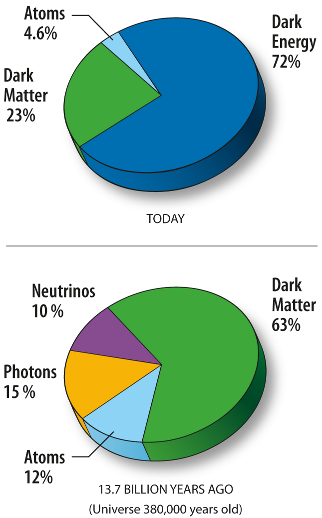
1. **Change of Paradigm** : To fit measurements a **Cosmological Constant** Λ is thus **re-introduced** to mimic source of acceleration.

$$\begin{cases} \left(\dfrac{\dot{R}}{R}\right)^2 & = \dfrac{8\pi G}{3}\rho - \dfrac{\Lambda}{3} - \dfrac{k}{R^2} \\ \dfrac{\ddot{R}}{R} & = -\dfrac{4\pi G}{3}(\rho + 3p) - \dfrac{\Lambda}{3} \end{cases}$$

2. **Standard Model** is kept: $\Lambda < 0 \implies$ **Dark Energy** is a repulsive energy

3. **Dark Energy Survey** (https://www.darkenergysurvey.org): DES aims to map hundreds of **millions** of galaxies, detect **thousands** of supernovae to explain why the Universe expansion is accelerating:
   - Start : 2013,
   - Scientifics number $> 400$,
   - 25 institutes, in **7 countries**
   - Most recent 5-years results **Abbott et al. 2024**.

# INTR-5 : Observational Findings & Data fitting



# INTR-6 : Some remarks on Λ

- **Phenomenological approach** : Data fitting of constant Λ from cosmological observations gives very **small values**:

$$\Lambda_{\rm obs} \simeq 1.09 \times 10^{-52} m^{-2}$$

- **Quantum Physics approach** : Model of quantum fluctuations in a vacuum (e.g. Zel'dovich 1967, Rugh & Zinkernagel 2002, ... ) gives:

$$\frac{\Lambda_{\rm qua}}{\Lambda_{\rm obs}} \geq 10^{120} \qquad \textbf{too big !}$$

- **Other approaches** : Electromagnetic field (e.g. Jimenez et al. 2009), Spacetime Defects (e.g. Ivanov & Wellenzohn 2016, Milton 2022), numerous ongoing works searching for **new physics** ?

- **Our strategy**: Instead of new physics, **go back to basics** !

**Challenge** : Propose a quite simple model of **Physics** and/or **Geometry** potentially explaining an origin of **Dark Energy**.

- **Physics aspects** : Consider "only" classical forces as **Gravitation** and **Electromagnetism** for cosmology models.
- **Geometry aspects** : Assume **Riemann-Cartan** manifold endowed with:
  - **Metric** $g_{\alpha\beta}$,
  - **Connection** $\Gamma^{\gamma}_{\alpha\beta}$, metric-compatible with torsion $\aleph^{\gamma}_{\alpha\beta}$,
  - **Volume-form** $\omega_n = \Omega \; dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3, \quad \Omega > 0$.
- **Lagrangian** (Model of vacuum spacetime):

$$\mathscr{S}_{GEM} = \int_{\mathscr{M}} \mathscr{L} \; (\text{geometry}, \text{physics}) \; \omega_n$$

# II. Mathematical Model of Vacuum Spacetime[1]

[1] We focus on the particular case of torsion-vector model.

**Einstein RG equations** are derived from:

- the **Hilbert-Einstein** action (with Riemann volume-form):

$$\mathscr{S}_{HE} := \frac{1}{2\chi} \int_{\mathscr{B}} g^{\beta\lambda}\, \overline{\Re}_{\beta\lambda}\, \overline{\omega}_n, \qquad \overline{\omega}_n := \sqrt{|\mathrm{Det}\mathbf{g}|}\; dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

- the **variation** equation:

$$\delta\mathscr{S}_{HE} := \frac{1}{2\chi} \int_{\mathscr{B}} \left( \delta g^{\beta\lambda}\, \overline{\Re}_{\beta\lambda}\, \overline{\omega}_n + g^{\beta\lambda}\, \delta\overline{\Re}_{\beta\lambda}\, \overline{\omega}_n + g^{\beta\lambda}\, \overline{\Re}_{\beta\lambda}\, \delta\overline{\omega}_n \right) = 0$$

  - for curvature: $\delta\overline{\Re}_{\beta\lambda} = \delta^{\alpha}_{\gamma} \left[ \overline{\nabla}_{\alpha}(\delta\overline{\Gamma}^{\gamma}_{\ \beta\lambda}) - \overline{\nabla}_{\beta}(\delta\overline{\Gamma}^{\gamma}_{\ \alpha\lambda}) \right]$;
  - for volume-form: $\delta\overline{\omega}_n = -\frac{1}{2} g_{\beta\lambda}\, \delta g^{\beta\lambda}\, \overline{\omega}_n$.

**Compatibility** (implicitly):

1. connection $\overline{\nabla}$ with metric $\mathbf{g}$ ($\rightarrow$ Levi-Civita connection)
2. connection $\overline{\nabla}$ with the volume-form $\overline{\omega}_n$ ($\rightarrow$ Riemann volume-form)

**FLRW metric** is the "gold standard of Cosmology":

$$ds^2 = (dx^0)^2 - R^2(x^0)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right)$$

obtained from:

- **Isotropy** and **Homogeneity** of spacetime (Cosmological Principle)
- **Birkhoff theorem** (symmetry) and **Killing constraint** $\mathcal{L}_\xi g_{\mu\nu} = 0$ !.

**Friedmann equations** are based on:

- Variation $\delta \mathscr{S}_{HEdS} := \frac{1}{2\chi}\int_{\mathscr{B}} \left(\overline{\mathcal{R}} - 2\Lambda\right)\overline{\omega}_n = 0$ (with **ad hoc** $\Lambda$)

$$\Longrightarrow \frac{1}{2\chi}\left(\overline{G}_{\beta\lambda} + \Lambda\, g_{\beta\lambda}\right) = 0, \quad \text{with} \quad \overline{G}_{\beta\lambda} := \Re_{\beta\lambda} - \frac{\overline{\mathcal{R}}}{2}\, g_{\beta\lambda}$$

- Perfect fluid source gives road to **Friedmann equations**:

$$\frac{1}{2\chi}\left(\overline{G}_{\beta\lambda} + \Lambda\, g_{\beta\lambda}\right) = (\rho + p)\, u_\beta u_\lambda - p\, g_{\beta\lambda} \qquad \square$$

- Electromagnetic radiation source gives (not isotropic) GR-EM equations:

$$\frac{1}{2\chi}\left(\overline{G}_{\beta\lambda} + \Lambda\, g_{\beta\lambda}\right) + T^{\mathrm{em}}_{\beta\lambda} = 0$$

# ECST-1 : Defects and evolution of the Universe

**Generation of defects** in the spacetime continuum is increasingly **considered** (e.g. Ruggieri 2003, Ivanov & Wellenzohn 2016, Katanaev 2021)

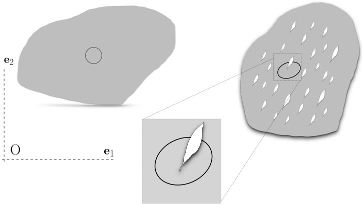
- In early phase of inflation, drastic **Universe cooling** (light release) creates potential mismatches leading to **defect formation** at higher density.
- Seeded defects are mainly point, **line**, and **surface** defects corresponding to non-metricity, **torsion**, and **curvature** respectively.
- Initial moments of Universe formation are essentially governed by particles behavior at the quantum level $\simeq$ Astrophysics.

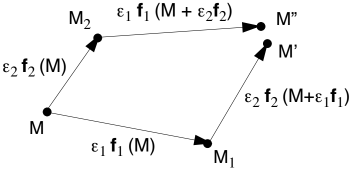
  **Cosmology approach** is more convenient for **astronomical** & **galactic dimensions**, and **long-term evolution**.

**Geometry Framework** : We thus consider a **Generalized Continuum** for the vacuum spacetime, embodied by the Einstein-Cartan spacetime (e.g. R 2018).

# ECST-2 : Einstein-Cartan Spacetime

**Generalized Continuum $\mathscr{B}$** : Smooth with loops and defects (e.g. R 1996)




**Einstein-Cartan Spacetime** $(\mathscr{B}, \mathbf{g}, \nabla, \omega_n)$ with:

- **Torsion-vector** $\aleph^{\lambda}_{\mu\nu} = \aleph_{\mu}\ \delta^{\lambda}_{\nu} - \aleph_{\nu}\ \delta^{\lambda}_{\mu}$
- **Curvature** $\Re^{\gamma}_{\alpha\beta\lambda} = (\partial_\alpha \Gamma^{\gamma}_{\beta\lambda} + \Gamma^{\gamma}_{\alpha\mu}\ \Gamma^{\mu}_{\beta\lambda}) - (\partial_\beta \Gamma^{\gamma}_{\alpha\lambda} + \Gamma^{\gamma}_{\beta\mu}\ \Gamma^{\mu}_{\alpha\lambda})$
- **Volume-form** $\omega_n = \Omega\ dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$

**Requirement** : **g**, $\nabla$, and $\omega_n$ should be **compatible** *i.e.* $\mathcal{L}_\xi \omega_n = (\nabla_\mu \xi^\mu)\, \omega_n$ !

# ECST-3 : Compatible volume-form $\omega_n$

**Theorem**

*For an Riemann-Cartan manifold, the following* **volume-form** $\omega_n$ *is compatible with connection* $\nabla$ *(Mosna & Saa, 1995-2005):*

$$\omega_n = e^{\vartheta(\tau)}\, \overline{\omega}_n, \qquad \vartheta = \vartheta(\tau), \qquad \tau := g^{\mu\nu}\, \aleph_\mu \aleph_\nu$$

**Proof**: Compatibility of $\nabla$ with $\omega_n$ is expressed with the Lie derivative along $\xi$:

$$\mathcal{L}_\xi \omega_n = \left(\nabla_\mu \xi^\nu\right) \omega_n$$

The Lie derivative of the volume-form components are (e.g. R 2018):

$$\begin{aligned}
\mathcal{L}_\xi \omega_{\alpha_1,\cdots\alpha_n} &= \xi^\mu \nabla_\mu \omega_{\alpha_1,\cdots\alpha_n} + \left(\nabla_{\alpha_1}\xi^\mu\right)\omega_{\mu,\alpha_2,\cdots\alpha_n} \\
&+ \left(\nabla_{\alpha_2}\xi^\mu\right)\omega_{\alpha_1,\mu,\alpha_3,\cdots\alpha_n} + \cdots + \left(\nabla_{\alpha_n}\xi^\mu\right)\omega_{\alpha_1,\cdots,\alpha_{n-1},\mu} \\
&+ \xi^\mu \aleph^\nu_{\mu\alpha_1}\, \omega_{\nu,\alpha_2,\cdots\alpha_n} + \cdots + \xi^\mu \aleph^\nu_{\mu\alpha_n}\, \omega_{\alpha_1,\cdots\alpha_{n-1},\nu}
\end{aligned}$$

Since $\dim\mathscr{B} = n$ there is only one component $\Omega := \omega_{\alpha_1<\alpha_2\cdots<\alpha_n}$, it gives:

$$\mathcal{L}_\xi \Omega = \xi^\mu \nabla_\mu \Omega + \left(\nabla_\mu \xi^\mu\right)\Omega + \tau_\mu\, \xi^\mu\, \Omega, \quad \text{with} \quad \tau_\mu := \aleph^\nu_{\mu\nu} = \sum_{\nu=0}^{\nu=3} \aleph^\nu_{\mu\nu}$$

Compatibility of $\nabla$ with $\omega_n$ implies: $\xi^\mu \nabla_\mu \Omega + \tau_\mu \, \xi^\mu = 0.$

By definition $\Omega > 0$, say a change of variable $\Omega := e^\vartheta$, we obtain:[2]

$$\boxed{\nabla_\mu \vartheta = -\tau_\mu}$$

Projecting $\omega_n$ onto $dx^1 \wedge \cdots dx^n$, we may write:

$$\omega_n = e^\vartheta \; \omega_{12\cdots n} \; dx^1 \wedge \cdots \wedge dx^n = e^\vartheta \; \underbrace{\mathrm{Det}(\partial_1, \partial_2, \cdots, \partial_n) \; dx^1 \wedge \cdots \wedge dx^n}_{Riemann \;\; volume-form}$$

A compatible volume-form with $\nabla$ thus takes the general form:

$$\omega_n = e^\vartheta \; \overline{\omega}_n, \qquad \vartheta = \vartheta(\mathbf{g}, \aleph)$$

Focusing only on torsion-vector case: $\aleph^\gamma_{\mu\nu} = \aleph_\mu \delta^\gamma_\nu - \aleph_\nu \delta^\gamma_\mu$, we write $\vartheta(\aleph_\mu)$ and thanks to Cauchy's theorem (e.g. R 2003) we get $\vartheta\,(\tau)$ $\square$.

[2]Function $\vartheta$ **corresponds to Dilaton** of **String Theory of Gravitation** (e.g. Brans & Dicke 1961, Lemoine & Lemoine 1995).

For **Gravitation**, we consider a (slightly extended) **Einstein-Palatini** action:

$$\boxed{\mathscr{S}_{EP} := \frac{1}{2\chi}\int_{\mathscr{B}} \mathcal{R}\left(\mathbf{g},\nabla\right)\ \omega_n}, \qquad \mathcal{R} := g^{\beta\lambda}\ \Re_{\beta\lambda}\left(\nabla\right)$$

inspired from:

- **Hilbert-Einstein** action (1915) ($\overline{\nabla}$: Levi-Civita connection)

$$\mathscr{S}_{HE} := \frac{1}{2\chi}\int_{\mathscr{B}} \overline{\mathcal{R}}\left(\mathbf{g},\overline{\nabla}\right)\overline{\omega}_n \qquad \text{Einstein RG}$$

- **Reichenbach** action (1929) to forecast electromagnetic field:

$$\mathscr{S}_{RB} = \int_{\mathscr{B}} \mathscr{L}\left(g_{\alpha\beta},\Gamma^{\gamma}_{\alpha\beta},\partial_\lambda\Gamma^{\gamma}_{\alpha\beta}\right)\overline{\omega}_n$$

It was covariantized in (Antonio & R 2011)

The **variation of the gravitation action** induces:

$$\delta \mathscr{S}_{EP} = \frac{1}{2\chi} \int_{\mathscr{M}} \left[ G_{\mu\nu} + \Lambda_{\aleph}\ g_{\mu\nu} \right] \delta g^{\mu\nu} \omega_n + \frac{1}{2\chi} \int_{\mathscr{M}} \left[ 3 + 2\ \mathcal{R}\ \vartheta'(\tau) \right] g^{\mu\nu} \aleph_\nu\ \delta \aleph_\mu\ \omega_n$$

in which we defined (to keep the same shape as Einstein original form):

- an extended **Einstein tensor**:

$$G_{\mu\nu} = \overline{G}_{\mu\nu} + (\overline{\nabla}_\mu \aleph_\nu + \overline{\nabla}_\nu \aleph_\mu) + \left[ 2 + \mathcal{R}\vartheta'(\tau) \right] \aleph_\mu \aleph_\nu$$

- a (local) **"Cosmological Constant"** (factor of the metric $g_{\mu\nu}$):

$$\boxed{\Lambda_{\aleph} := g^{\alpha\beta} \left[ \aleph_\alpha \aleph_\beta - (\overline{\nabla}_\alpha \aleph_\beta + \overline{\nabla}_\beta \aleph_\alpha) \right]} \quad \in \mathbb{R}$$

**Remark**

*Locally, the scalar field* $\Lambda_{\aleph}$ *acts as a* **energy source**.

# ECEM-1 : Magnetic fields are pervading ...

**REVIEWS on Magnetic Fields** research point out some features:

- Magnetic fields existed in the **early Universe**: **Seeds of Primordial** fields amplified by dynamo (e.g. Grasso & Rubinstein 2001, Zweibel 2013)
- **Magnetized Universe** has strong influences on **spiral filaments & clusters of galaxies** (e.g. Giovannini 2004, Vernstrom et al. 2019)
- **Primordial** magnetic fields is pervading in the **intergalactic vacuum spacetime** and strongly influence the large-scale structure formation (e.g. Durrer & Neronov 2013, Mtchedlidze et al 2022)

- **Magnetic Monopoles** in Universe (Dirac 1931) are **not excluded** (theoretically or experimentally) from current physics and play important role in gauge invariance (e.g. Preskill 1984, Mavromatos & Mitsou 2020)
- **Magnetic Monopoles** are emergence from **spacetime torsion defect** and induce an apparent magnetic charge (e.g. Gera & Segupta 2021)
- **Magnetogenesis**: Emergence end evolution of seeded magnetic fields may arise from **plasma shear flows** (e.g. Zhou et al 2022)

It is therefore **worth** to add **electromagnetic source**, namely by means of the (slightly extended) **Yang-Mills** action:

$$\boxed{\mathscr{S}_{YM} := -\frac{1}{4}\int_{\mathscr{B}} \mathcal{H}^{\mu\nu}\mathcal{F}_{\mu\nu}\ \omega_n}, \quad \mathcal{F}_{\mu\nu} := \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

where $A_\mu$ is the **electromagnetic potential**.

- **Electromagnetic variables**: Faraday tensor (primal variables) & Excitation tensor (dual variables) (e.g. Tonnelat 1959):

$$\mathcal{F}_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B^3 & -B^2 \\ E_2 & -B^3 & 0 & B^1 \\ E_3 & B^2 & -B^1 & 0 \end{pmatrix} \mathcal{H}^{\mu\nu} = \begin{pmatrix} 0 & D^1 & D^2 & D^3 \\ -D^1 & 0 & H_3 & -H_2 \\ -D^2 & -H_3 & 0 & H_1 \\ -D^3 & H_2 & -H_1 & 0 \end{pmatrix}$$

- **Constitutive laws** of **vacuum spacetime** in 3D description:

$$\left\{ \begin{array}{lcl} \mathbf{D} & = & \epsilon_0\ \mathbf{E} \\ \mathbf{H} & = & \mu_0^{-1}\ \mathbf{B} \end{array} \right. \longrightarrow \quad \mathcal{H} = \mathcal{H}(\mathcal{F}) \quad \longrightarrow \quad \mathscr{S}_{YM}$$

# ECEM-3 : Variation of the YM action

The **variation of electromagnetic action** is given by:

$$\begin{aligned}
\delta \mathscr{S}_{\mathrm{YM}} &= \int_{\mathscr{M}} \nabla_\mu \mathcal{H}^{\mu\nu} \; \delta A_\nu \; \omega_n + \int_{\mathscr{M}} \tilde{T}^{\mathrm{em}}_{\mu\nu} \; \delta g^{\mu\nu} \; \omega_n \\
&+ \int_{\mathscr{M}} \left[ \mathcal{H}^{\mu\nu} A_\nu - \frac{1}{2} \mathcal{H}^{\alpha\beta} \mathcal{F}_{\alpha\beta} \; \vartheta'(\tau) \; g^{\mu\nu} \aleph_\nu \right] \delta \aleph_\mu \; \omega_n
\end{aligned}$$

with an extended **electromagnetic energy-momentum**:

$$\tilde{T}^{\mathrm{em}}_{\mu\nu} := \frac{1}{8} \mathcal{H}^{\alpha\beta} \mathcal{F}_{\alpha\beta} \; g_{\mu\nu} + \frac{1}{4} \mathcal{H}^{\lambda\rho} \left( g_{\mu\lambda} \mathcal{F}_{\rho\nu} + \mathcal{F}_{\mu\lambda} g_{\rho\nu} \right) - \frac{1}{4} \mathcal{H}^{\alpha\beta} \mathcal{F}_{\alpha\beta} \; \vartheta'(\tau) \, \aleph_\mu \aleph_\nu$$

The additional term when compared to classical energy-momentum results from the **volume-form compatibility**.

**Remark**

*As for classical EM source, the extended electromagnetic energy-momentum* $\tilde{T}^{\mathrm{em}}_{\mu\nu}$ *breaks the isotropic symmetry.*

- **Action of GEM Vacuum Model**: Gravitation & Electromagnetism merged into **Einstein-Palatini** + **Yang-Mills** action:

$$\mathscr{S}_{GEM} := \frac{1}{2\chi}\int_{\mathscr{B}} g^{\beta\lambda}\ \Re_{\beta\lambda}\ \omega_n - \frac{1}{4}\int_{\mathscr{B}} \mathcal{H}^{\mu\nu}\mathcal{F}_{\mu\nu}\ \omega_n$$

- **Curvature & Euler variation**

$$\begin{cases} \mathcal{R} & := & g^{\beta\lambda}\ \Re_{\beta\lambda} = g^{\beta\lambda}\delta^{\alpha}_{\gamma}\ \Re^{\gamma}_{\alpha\beta\lambda} \\ \Re^{\gamma}_{\alpha\beta\lambda} & := & (\partial_\alpha \Gamma^{\gamma}_{\beta\lambda} + \Gamma^{\gamma}_{\alpha\mu}\ \Gamma^{\mu}_{\beta\lambda}) - (\partial_\beta \Gamma^{\gamma}_{\alpha\lambda} + \Gamma^{\gamma}_{\beta\mu}\ \Gamma^{\mu}_{\alpha\lambda}) \end{cases}$$

and its variation in EC spacetime (e.g. Lichnerowicz 1955, R 2003)

$$\delta\Re^{\gamma}_{\alpha\beta\lambda} = \nabla_\alpha(\delta\Gamma^{\gamma}_{\beta\lambda}) - \nabla_\beta(\delta\Gamma^{\gamma}_{\alpha\lambda}) + \aleph^{\mu}_{\alpha\beta}\ \delta\Gamma^{\gamma}_{\mu\lambda}$$

- **Faraday tensor & Euler variation** (e.g. Fraenkel 1997)

$$\mathcal{F}_{\mu\nu} := \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

and its variation in EC spacetime (non Abelian model !)

$$\delta\mathcal{F}_{\mu\nu} = \nabla_\mu(\delta A_\nu) - \nabla_\nu(\delta A_\mu) - A_\rho \delta\aleph^{\rho}_{\mu\nu}$$

**Theorem**

*Given a model of EC vacuum spacetime defined by the Lagrangian* $\mathscr{L} := \mathscr{L}_{EP} + \mathscr{L}_{YM}$, *then the covariant* **fields equations** *are given by (R 2024):*

$$\begin{cases} \mathbb{M}^{\nu} & := & \nabla_{\mu}\mathcal{H}^{\mu\nu} = 0 \\ \mathbb{E}_{\mu\nu} & := & \dfrac{1}{2\chi}\left(G_{\mu\nu} + \Lambda_{\aleph}\, g_{\mu\nu}\right) + \tilde{T}^{\mathrm{em}}_{\mu\nu} = 0 \\ \mathbb{L}^{\mu\nu}_{\nu} & := & \dfrac{1}{2\chi}\,\mathscr{D}^{\mu\nu}_{\nu} + \mathcal{H}^{\mu\nu}\, A_{\nu} - \dfrac{\vartheta'(\tau)}{2}\mathcal{H}^{\alpha\beta}\mathcal{F}_{\alpha\beta}\, g^{\mu\nu}\, \aleph_{\nu} = 0 \end{cases}$$

*with the* **distortion vector** *:* $\mathscr{D}^{\mu\nu}_{\nu} := [3 + 2\vartheta'(\tau)\mathcal{R}]\, g^{\mu\nu}\aleph_{\nu}$.

**Proof**: The variation $\delta\mathscr{S}_{GEM} := \int_{\mathscr{M}} \delta\mathscr{L}\ \omega_n + \mathscr{L}\,\delta\omega_n = 0$ induces

$$\delta\mathscr{S}_{GEM} = \int_{\mathscr{M}} \left(\mathbb{M}^{\nu}\ \delta A_{\nu} + \mathbb{E}_{\mu\nu}\ \delta g^{\mu\nu} + \mathbb{L}^{\mu\nu}_{\nu}\ \delta\aleph_{\mu}\right)\omega_n + \underbrace{\mathrm{B.\ Term}}_{=\ 0} = 0$$

**Compatibility** of $\nabla$, $\mathbf{g}$ and $\omega_n$ $\Longrightarrow$ Boundary Term = Divergence $\square$.

The main result is to have shown that it exists a mathematical possibility to define a **Cosmological Constant $\Lambda_{\aleph}$** scalar field on the **EC Vacuum spacetime** $\mathscr{B}$[3] which can rigorously be considered as a **Dark Energy** in the extended Einstein-Maxwell-**Cartan** equation $\mathbb{E}_{\mu\nu} = 0$.

[3] There is no need to assume an hypothetical extra Dark Energy.

# III. Cosmological Constant : Geometrical or Physical Necessity ?

# CC-1 : $\Lambda_{\aleph}$ is a geometrical variable

- **Geometric Cosmological Constant**: From **equation** $\mathbb{E}_{\mu\nu} = 0$

$$\frac{1}{2\chi}\left(G_{\mu\nu} + \Lambda_{\aleph}\ g_{\mu\nu}\right) + \tilde{T}^{\mathrm{em}}_{\mu\nu} = 0$$

- Compared to the original **Einstein equation**:

$$\frac{1}{2\chi}\left(\overline{G}_{\mu\nu} + \Lambda\ g_{\mu\nu}\right) + T^{\mathrm{em}}_{\mu\nu} = 0$$

The scalar $\Lambda_{\aleph}$ depending on **torsion** is a **Vacuum Spacetime Energy** resulting from EC geometry:

$$\boxed{\Lambda_{\aleph} := g^{\alpha\beta}\left[\aleph_\alpha \aleph_\beta - \left(\overline{\nabla}_\alpha \aleph_\beta + \overline{\nabla}_\beta \aleph_\alpha\right)\right]}$$

- **Some remarks**:
  1. $\Lambda_{\aleph} < 0$, acts as **Dark Energy** for spacelike torsion $(0, \aleph_i)$.
  2. **Isotropic symmetry** (Cosmological Principle) is broken due to EM and torsion in $G_{\mu\nu}$. **No Friedman's equations allowed !**

# CC-2 : $\Lambda_{\aleph}$ is an electromagnetic variable

1. **Linkage equation** : $\mathbb{L}^{\mu\nu}_{\nu} = 0$ is re-arranged as (since $\tau := g^{\alpha\beta}\aleph_\alpha\aleph_\beta$):

$$\left[\frac{1}{2\chi}\left[3 + 2\vartheta'(\tau)\,\mathcal{R}\right] - 2\vartheta'(\tau)\frac{\mathcal{H}^{\alpha\beta}\mathcal{F}_{\alpha\beta}}{4}\right] g^{\mu\nu}\aleph_\nu + \mathcal{H}^{\mu\nu}\;A_\nu = 0$$

2. **Physical Cosmological Constant** : For case where $\vartheta'(\tau) \simeq 0$, we have:

$$\boxed{\aleph_\mu = -\frac{2}{3}\chi\; g_{\mu\alpha}\;\mathcal{H}^{\mu\nu} A_\nu} \rightarrow \Lambda_{Chern-Simons} := g^{\alpha\beta}\left[\aleph_\alpha\aleph_\beta - (\overline{\nabla}_\alpha\aleph_\beta + \overline{\nabla}_\beta\aleph_\alpha)\right]$$

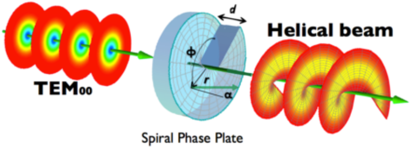
3. **Physical interpretation** :
   - **Chern-Simons current** better preserved than EM energy !

$$\mathcal{H}^{\mu\nu}\;A_\nu \quad\longrightarrow\quad \begin{pmatrix} \mathbf{B}\cdot\mathbf{A} \\ \mathbf{E}\times\mathbf{A} \end{pmatrix} \qquad \begin{matrix} \rightarrow \text{Magnetic Helicity} \\ \rightarrow \text{Spin Angular Momentum} \end{matrix}$$

   - **Yang-Mills Lagrangian**

$$\mathscr{L}_{YM} := -\frac{1}{4}\mathcal{H}^{\alpha\beta}\mathcal{F}_{\alpha\beta} \quad\longrightarrow\quad \mathscr{L}_{YM} := \frac{1}{2}\left(\mathbf{D.E} - \mathbf{B.H}\right)$$

**Physical interpretation** : $\mathcal{H}^{\alpha\beta} \; A_\beta$ is the 4-dim expression of **Spin Angular Momentum** $\mathbf{L}_{\text{spin}}$ (optics, ...): Transverse ElectroMagnetic wave (green axis)




**Moment of 3D-Poynting vector** $\mathbf{E} \times \mathbf{B}$ (e.g. Allen et al. 1992)

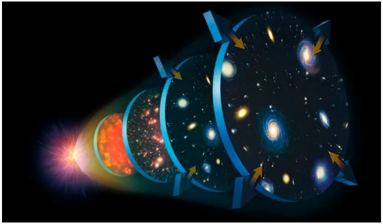
$$\begin{aligned} \mathbf{J} \;&:=\; \int_{\mathscr{M}} \mathbf{r} \times (\mathbf{E} \times \mathbf{B})\, dv \\ &=\; \sum_{i=1}^{i=3} \int_{\mathscr{M}} E_i\, (\mathbf{r} \times \nabla)\, A_i \;dv + \int_{\mathscr{M}} \mathbf{E} \times \mathbf{A} \;dv := \mathbf{L}_{\text{orbital}} + \mathbf{L}_{\text{spin}} \end{aligned}$$

for paraxial and non-paraxial EM field (Arrayas & Trueba 2018).

# IV. Final Remarks and Outlook

# FREM-1: Insights from experimental observations

**1998** : The Universe expands with acceleration. Introduction of an *ad hoc* **Dark Energy $\simeq \Lambda$ is required** to fit measured data in cosmology and astrophysics (cf. *Dark Energy Survey*).

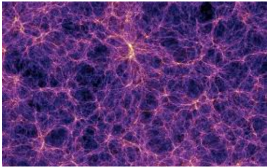


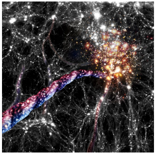
**2024** : Expansion occurs at **different rates**. James Webb and Hubble Space Telescopes confirmed that it is not due to a measurement error. One global Cosmological Constant is then not satisfactory.

**There is a need of an event-dependent $\Lambda_{\aleph} = \Lambda_{\aleph}(x^{\mu})$.** [4]

[4] *Illustration of Universe expansion. (Image credit: Mark Garlick/Science Photo Library via Getty Images)*

**Gravitation AND Magnetic Fields** are shown to be the **Major Forces** shaping out the large-structure of the cosmos **($\infty$ action-at-distance forces)**





- (Left) **Cosmic web** : Clusters of galaxies via filaments (e.g. **Vernstrom et al. 2021** "Discovery of **Magnetic Fields** Along Stacked Cosmic Filaments as Revealed by Radio and X-Ray Emission").
- (Right) **Spinning** filaments (e.g. **Wang et al. 2021** "Possible observational evidence for cosmic **Filament Spin**", ... Beck 2016).
- How about the **MF Initial Seeding** : Homogeneous *vs.* Scale-invariant stochastic *vs.* Helical *vs.* Non-helical (e.g. **Mtchedlidze et al. 2022** "Evolution of Primordial Magnetic Fields during Large-scale **Structure Formation**"). **Homogeneous MF distribution is related to inflation !**

**Cosmological Constant $\Lambda$** remains a **major problem** of the modern cosmological physics (mainly after 1998 and still nowadays)

- **1917**: **Einstein** introduced a **global $\Lambda$** to make the Universe static.
- **1929**: **Hubble** showed the **needless of $\Lambda$** and that the Universe is dynamic and **linearly** expands.
- **1998**: **Hypothetical $\Lambda$** (unknown $\simeq 70\%$ of Universe Energy cf. DES) was necessarily re-introduced to account for **accelerated expansion**.

**CONCLUSION**: A **Cosmological Constant** can be rigorously defined as:

- **Geometrical** $\Lambda_{\aleph}$ by using a **Generalized Continuum** for Vacuum Einstein-Cartan spacetime model.
- **Physical** $\Lambda_{\aleph} \rightarrow \Lambda_{Chern-Simmons}$ since identified as function of **Magnetic Helicity** and **Spin Angular Momentum** of Magnetized Universe.

# OUT-1 : Some Outlook

- **Friedmann equations with torsion** : Extending **Birkhoff theorem** for spherical symmetry and solving the **Killing equation**:

$$\begin{aligned} \mathcal{L}_\xi \; g_{\alpha\beta} &= \xi^\lambda \partial_\lambda g_{\mu\nu} + g_{\mu\lambda}\partial_\nu \xi^\lambda + g_{\lambda\nu}\partial_\mu \xi^\lambda \\ &= \xi^\gamma \; \nabla_\gamma g_{\alpha\beta} + g_{\gamma\beta} \; \nabla_\alpha \xi^\gamma + g_{\alpha\gamma} \; \nabla_\beta \xi^\gamma \\ &+ \xi^\gamma \left( g_{\alpha\nu} \; \aleph^\nu_{\gamma\beta} + g_{\nu\beta} \; \aleph^\nu_{\gamma\alpha} \right) = 0 \end{aligned}$$

might help to **extend** of Friedmann equations.

- **Defected Continuum Mechanics** : Generalized Continuum can also model Continuum with evolving defects (as $2^{\mathrm{nd}}$- Gradient **Elastoplasticity**):

$$\mathscr{S}_{CED} := \int_{\mathscr{B}} \mathscr{L} \left( g_{\alpha\beta}, \aleph^\gamma_{\alpha\beta}, \Re^\gamma_{\alpha\beta\lambda}, \cdots \right) \omega_n$$

Active diffeomorphisms along vector $\xi$ tangent to a flow $\xi$ allow us to derive the **conservation laws** which should account for $\omega_n$ **compatibility** (Ongoing).

**Hierarchy of GR-EM Physics models** (e.g. Kobe 1999)

| | | |
|---|---|---|
| **Quantum Field Theory** | Quantized particle field (Many quantum particles) | Quantized electromagnetic field (many photons) |
| | ↑ **2nd quantization** | ↑ **2nd quantization** |
| **Quantum Theory** | Schröndinger (or Dirac) equation | *Maxwell equations (Schröndiger equation of one photon)* |
| | ↑ **1st quantization** | ↑ **1st quantization** |
| **Classical Theory** | *Newton or Einsteinian Mechanics* | Geometrical Optics |

**Maxwell equations** $\simeq$ **Schröndiger equation** of **a single photon**, then $\in$ **Quantum Physics** when using **Weber vector W** (e.g. Mukunda & Sudarshan 1986, Bialynicki-Birula 1994, Sebens 2019):

$$\mathbf{W} := \frac{\mathbf{D}}{\sqrt{2\epsilon_0}} + i\,\frac{\mathbf{B}}{\sqrt{2\mu_0}} \Longrightarrow \frac{i}{c}\,\partial_0 \mathbf{W} = \nabla \times \mathbf{W} \iff \textbf{Maxwell Equations}$$

Move to Wave Function of clusters of photons.

" Il faudra, pour mesurer les **longueurs** et pour mesurer le **temps**, commencer par la physique toute entière et nous voici, dès l'abord, arrêtés, puisque nous ne pouvons pas songer, semble-t-il, à observer les phénomènes physiques, **si nous ne savons pas déjà mesurer les longueurs et le temps. ...** *(L'espace et le temps, Emile Borel, PUF 1949)* "



**"L'espace est bleu, des oiseaux volent dedans "**. Heisenberg a répondu par ces mots à une question posée par un de ses élèves : " *Et si l'espace après tout n'était que le champ d'application des opérateurs hermitiens ?* " *(cf. Lochak 1994)*

# Merci pour votre attention !

**Some personal references**:


- **Contribution à la modélisation géométrique et thermodynamique d'une classe de milieux faiblement continus**, *Archives for Rational Mechanics and Analysis* 141, 1997, pp 199-236.
- **A Geometric Approach to Thermomechanics of Dissipating Continua**, in *Progress in Mathematical Physics Series*, Birkhaüser, Boston, 2003.
- **Second Gradient Continuum : Role of Electromagnetism Interacting with the Gravitation on the Presence of Torsion and Curvature**, in *Generalized Models and Non-Classical Approaches in Complex Materials 1*, Springer, 2018, pp 675-694.
- **Covariance and Gauge Invariance in Continuum Physics**, *Progress in Mathematical Physics Series*, Birkhäuser, Boston, 2018.
- **Some Thoughts Concerning Spacetime Models in Cosmology : Role of Gravitation and Electromagnetism**, in *Proceedings of* $17^{th}$ *Marcel Grossman Meeting*, World Scientific, 20p, 2024.
- **Some Thoughts Concerning the Vacuum Spacetime and the Cosmological Constant : Gravitation and Electromagnetism**, *In preparation*, Birkhaüser, 2025 (330 references herein).