TDA in Engineering Applications

Francisco CHINESTA

Outline

1. Introduction
2. Applications

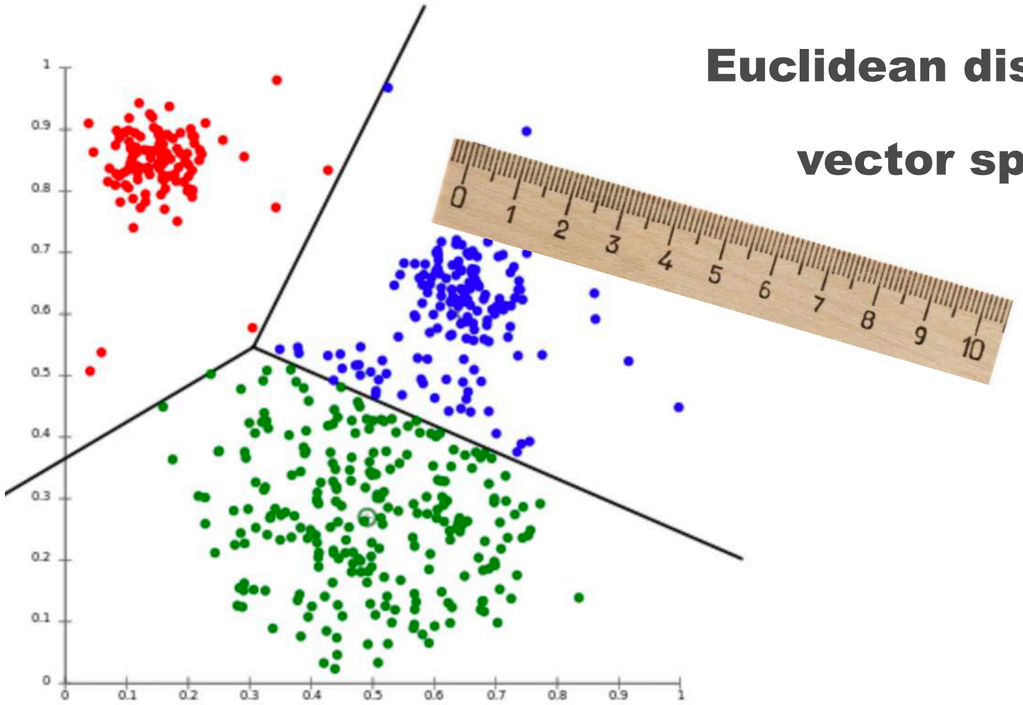
1. Introduction

2. Applications

Data in a vector space

Unsupervised

clustering



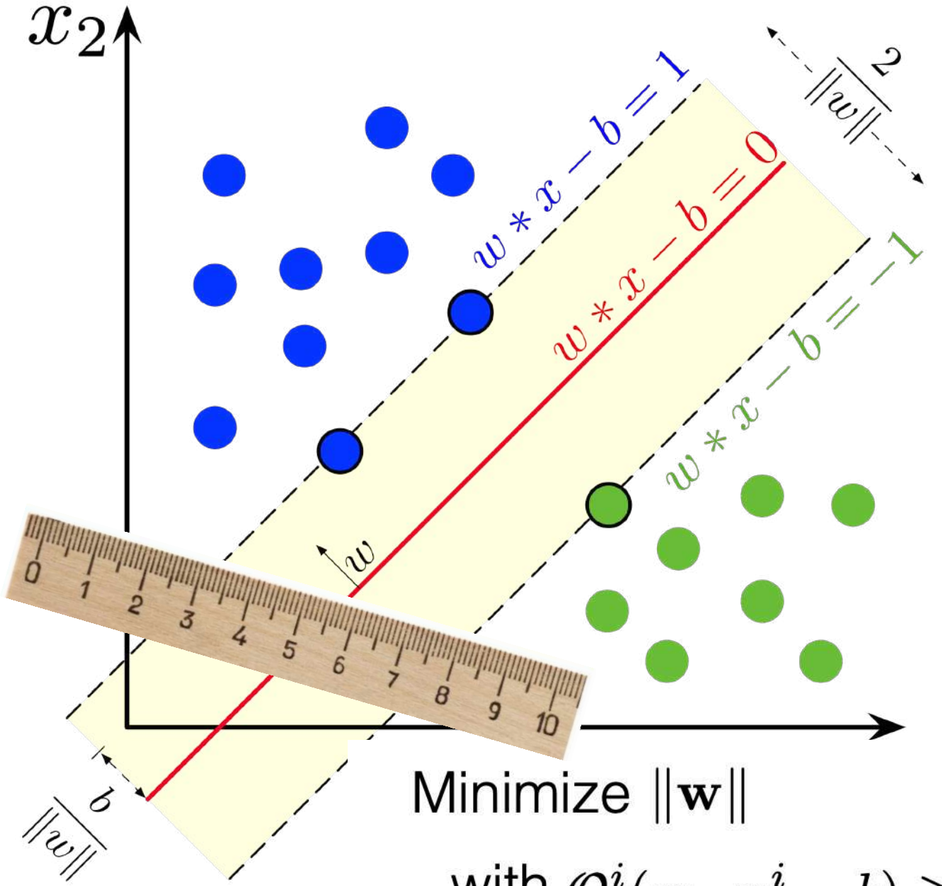
Euclidean distance in vector spaces

$$\mathbb{S} = \{\mathcal{S}_1, \cdots, \mathcal{S}_k\}$$

$$\arg\min_{\mathbb{S}} \sum_{i=1}^{k} \sum_{j \in \mathcal{S}_i} \|\mathbf{x}^j - \mu^i\|^2$$

Supervised

classification

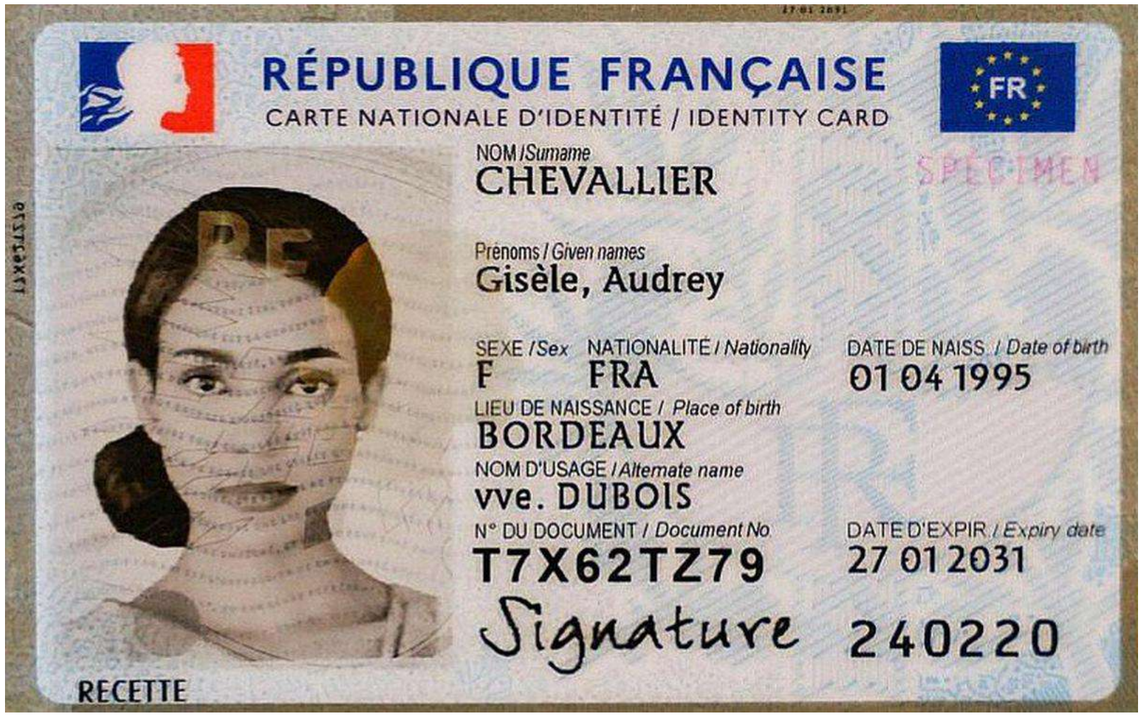


Minimize $\|\mathbf{w}\|$

with $\mathcal{O}^i(\mathbf{w} \cdot \mathbf{x}^i - b) \geq 1$

Classifier: $\mathbf{sgn}(\mathbf{w} \cdot \mathbf{x} - b)$

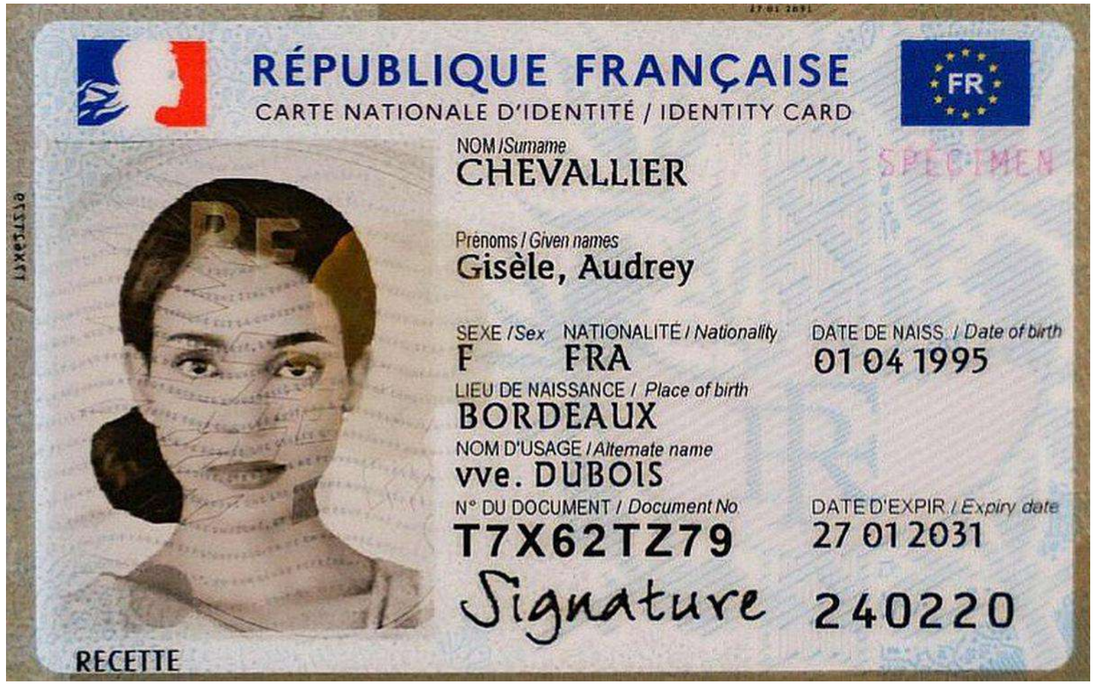
Complex Data

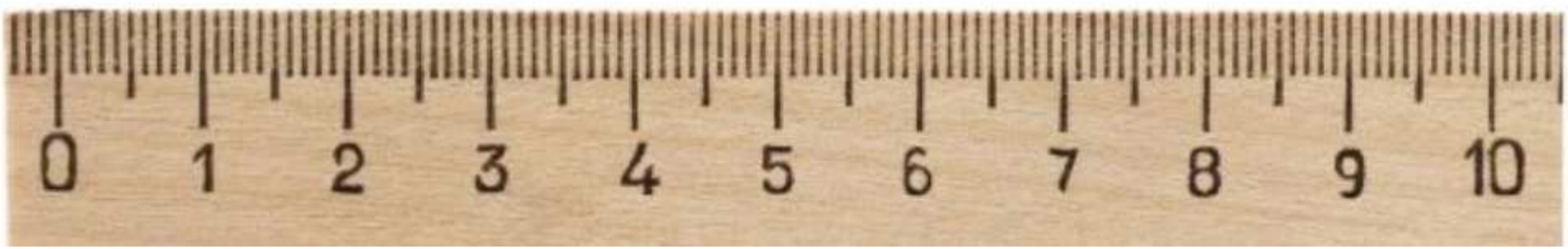


Data can contain multiple

heterogeneous information


RÉPUBLIQUE FRANÇAISE
CARTE NATIONALE D'IDENTITÉ / IDENTITY CARD
FR
SPÉCIMEN
NOM / Surname
MARTIN
Prénoms / Given names
Maëlis-Gaëlle, Marie
SEXE / Sex
F
NATIONALITÉ / Nationality
FRA
DATE DE NAISS. / Date of birth
13 07 1990
LIEU DE NAISSANCE / Place of birth
PARIS
NOM D'USAGE / Alternate name
usage NOM D'USAGE
N° DU DOCUMENT / Document No.
D2H6862M2
DATE D'EXPIR. / Expiry date
11 02 2030
Signature
546497

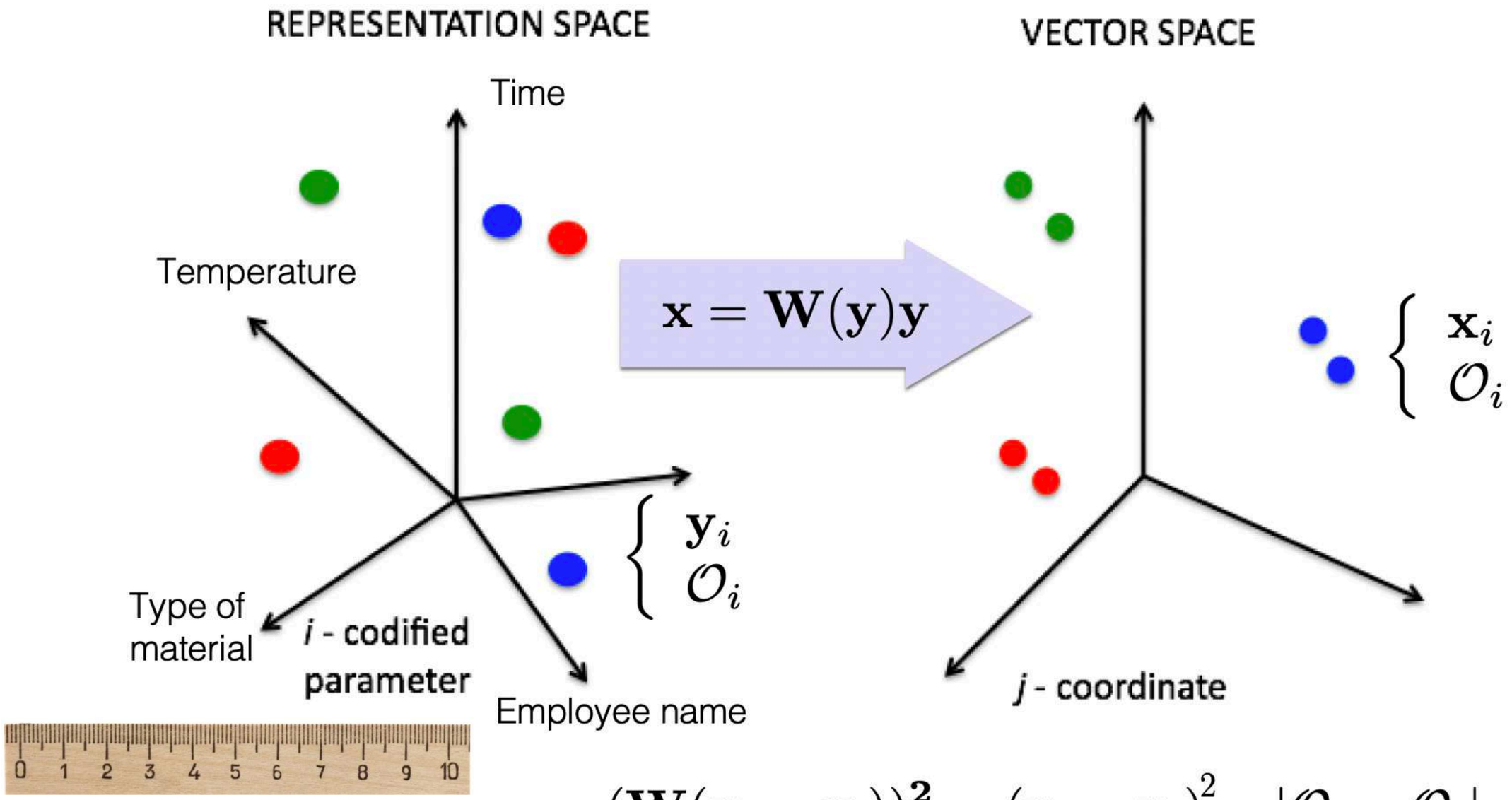

RÉPUBLIQUE FRANÇAISE
CARTE NATIONALE D'IDENTITÉ / IDENTITY CARD
FR
SPÉCIMEN
NOM / Surname
CHEVALLIER
Prénoms / Given names
Gisèle, Audrey
SEXE / Sex
F
NATIONALITÉ / Nationality
FRA
DATE DE NAISS. / Date of birth
01 04 1995
LIEU DE NAISSANCE / Place of birth
BORDEAUX
NOM D'USAGE / Alternate name
vve. DUBOIS
N° DU DOCUMENT / Document No.
T7X62TZ79
DATE D'EXPIR. / Expiry date
27 01 2031
Signature
240220
RECETTE


0 1 2 3 4 5 6 7 8 9 10

?

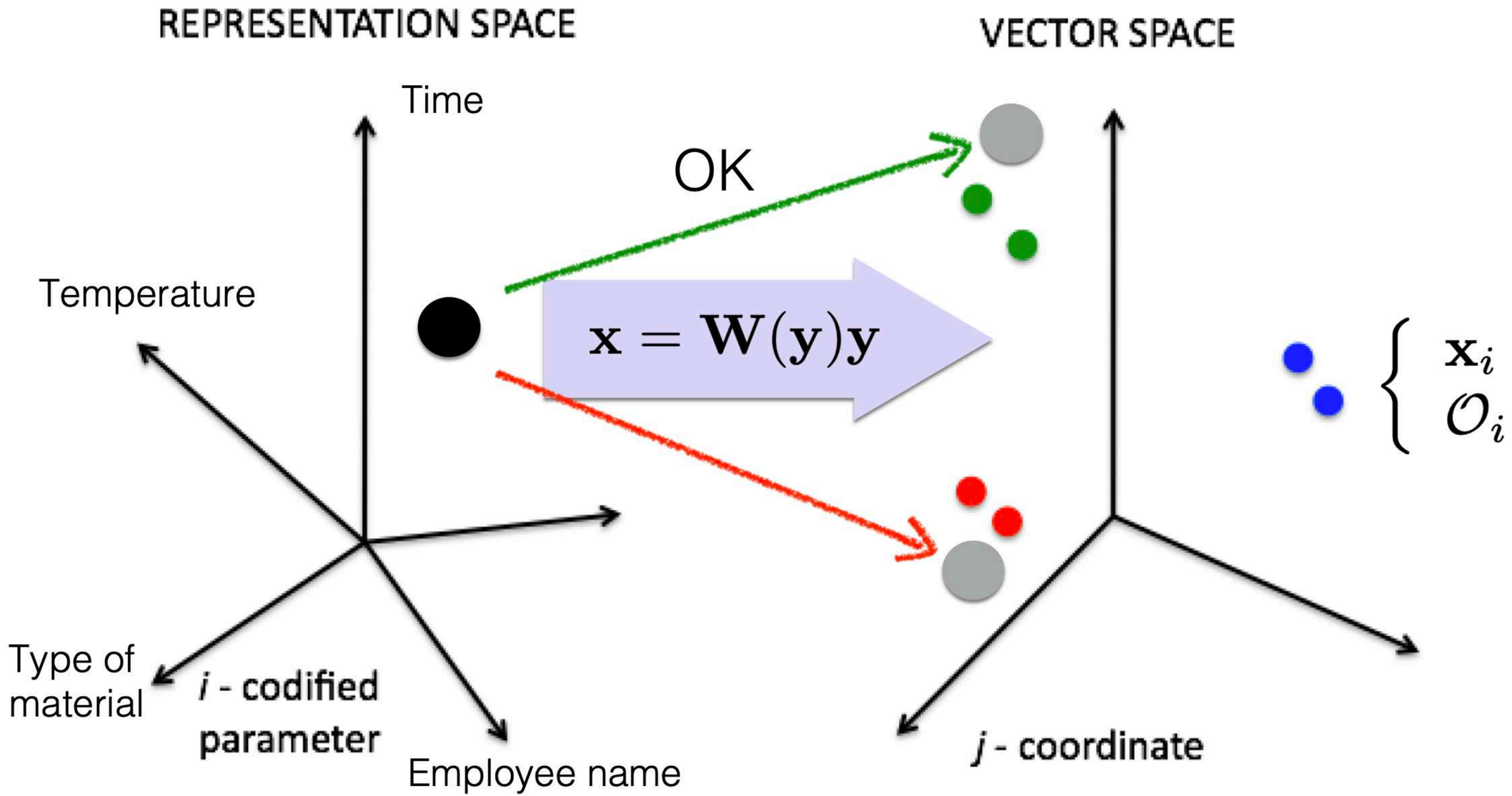
- **Learning metrics**
- **Data transformation**
- **Extracting data features**

Learning metrics: e.g. Code2Vect



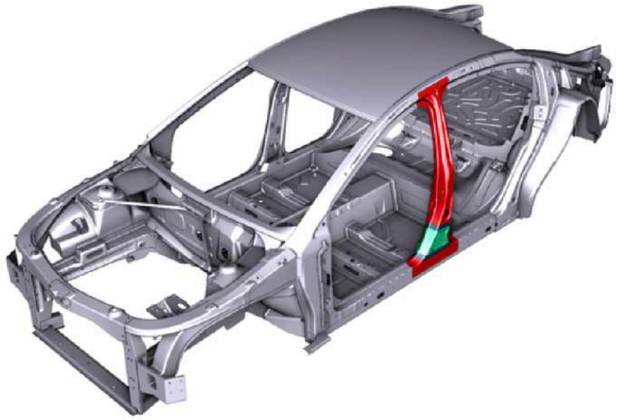
$$(\mathbf{W}(\mathbf{y_i} - \mathbf{y_j}))^{\mathbf{2}} = (\mathbf{x_i} - \mathbf{x_j})^2 = |\mathcal{O}_\mathbf{i} - \mathcal{O}_\mathbf{j}|$$

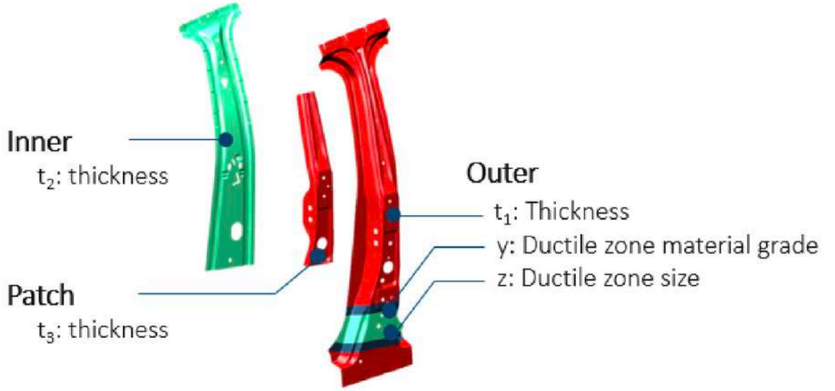
Code2Vect

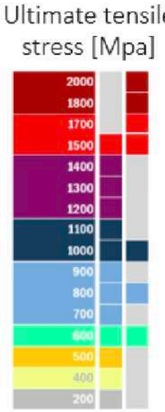

REPRESENTATION SPACE
VECTOR SPACE
Time
OK
Temperature
$\mathbf{x} = \mathbf{W}(\mathbf{y})\mathbf{y}$
$\mathbf{x}_i$
$\mathcal{O}_i$
Type of material
i - codified parameter
Employee name
j - coordinate

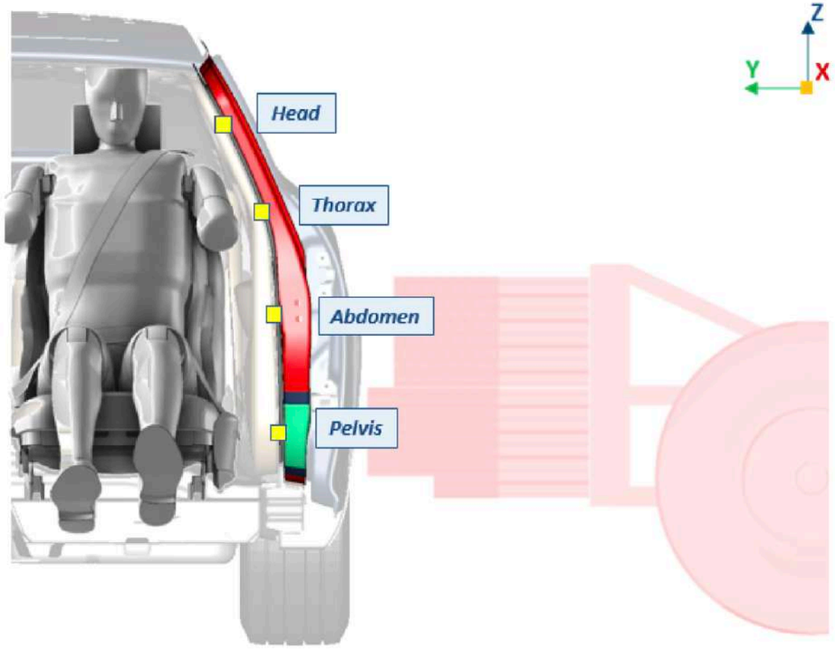
Code2Vect

Gestamp





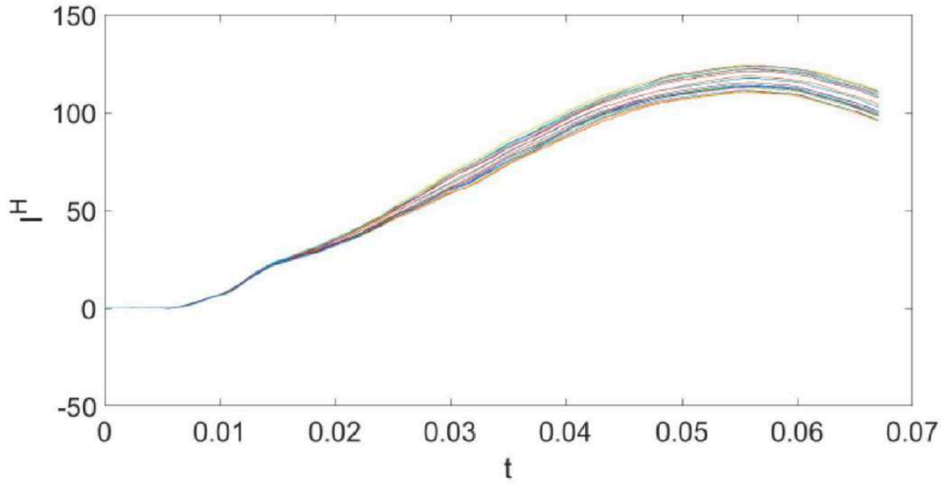


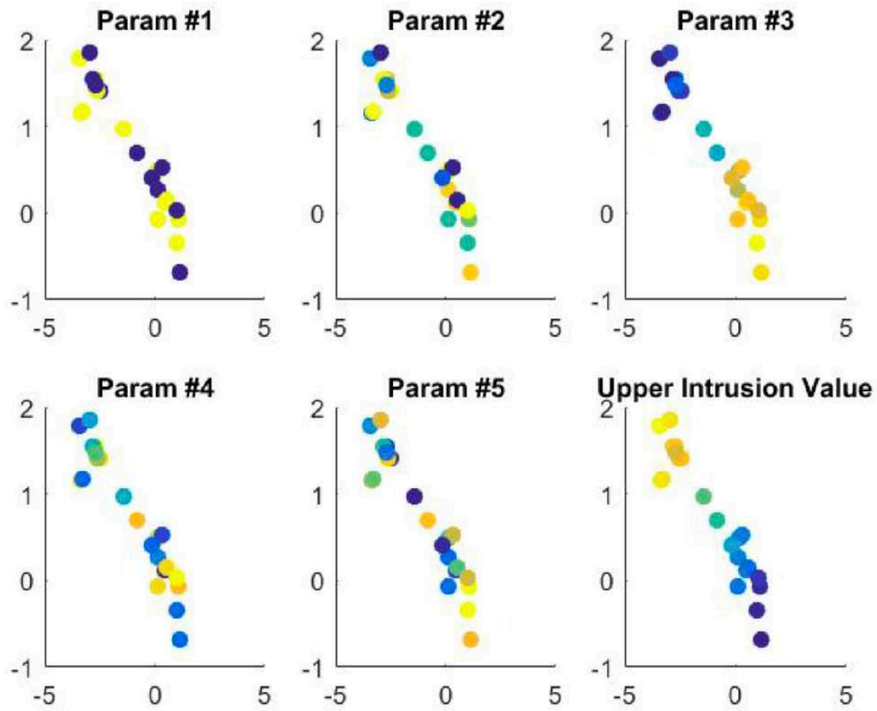


Code2Vect

DoE & High Fidelity simulations

Simulation	z	t_2	t_1	t_3	y
1	90	1.3	1.5	1.5	471
2	90	1.15	1.7	1.5	591
3	60	1.25	1.7	1.1	549
4	60	0.9	1.65	1.05	527
5	60	1.25	1.55	1.15	386
6	60	1.3	1.15	1.55	356
7	90	1.2	1.15	1.4	570
8	90	1.25	1.7	1	364
9	90	1.1	1.75	1.1	591
10	90	1	1.1	1.05	420
11	90	0.95	1.15	1.55	527
12	90	0.9	1.65	1.55	489
13	90	1.2	1.2	1.6	360
14	60	0.95	1.6	1.1	351
15	60	1.1	1.35	1.5	557
16	90	1.1	1.4	1.25	347
17	90	1.3	1.1	1.1	493
18	90	1.1	1.65	1.55	390
19	60	1.3	1.1	1.25	471
20	60	0.9	1.15	1.2	536
21	60	1	1.2	1.35	394
22	60	1.3	1.6	1.6	531





Beyond Euclidian Metrics

Apparently three trees, apparently !

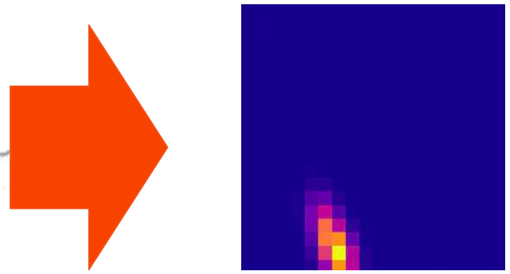


In what sense they are close?
What kind of resemblance?
How many parameters define them?
What is the adequate metric for comparing them?

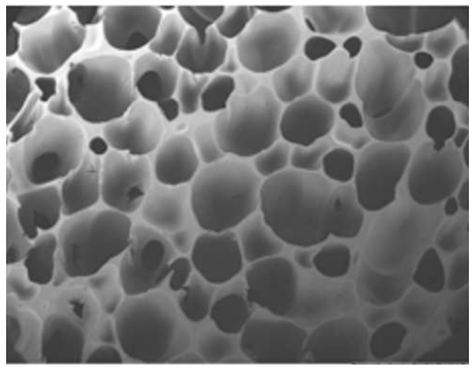
Data transformation, e.g. TDA

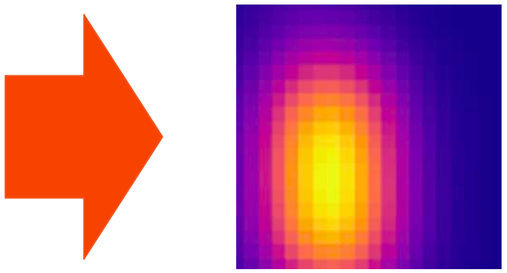
Topological Data Analysis - TDA

Time series



Complex images



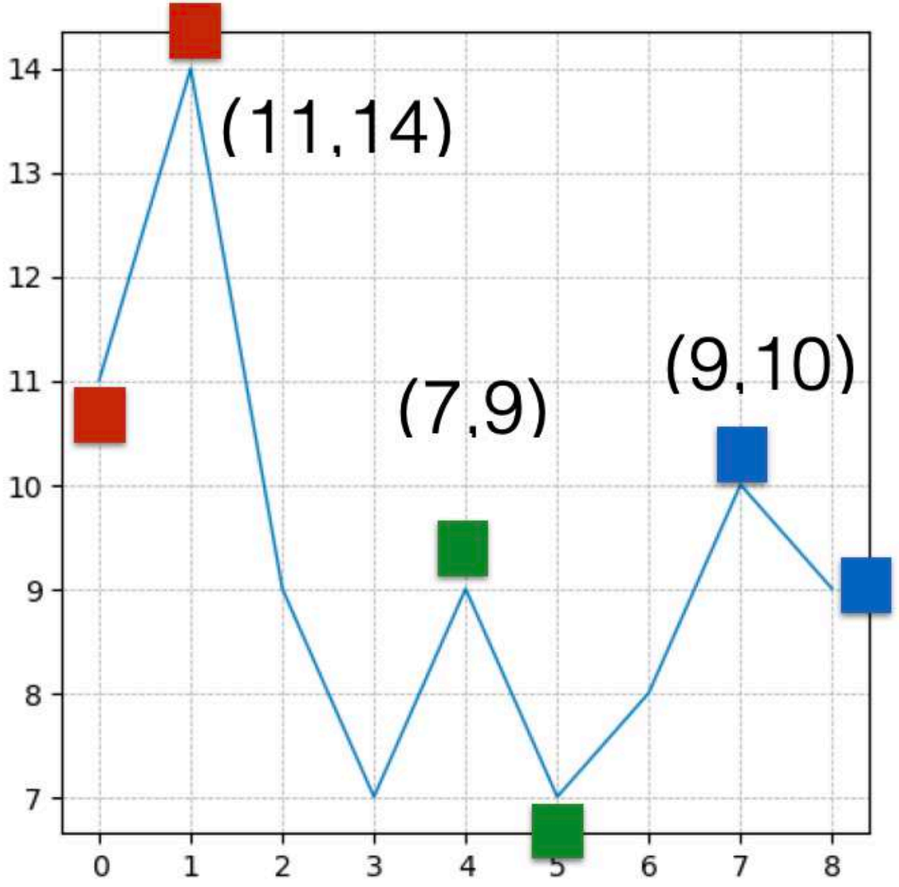


A sort of goal oriented

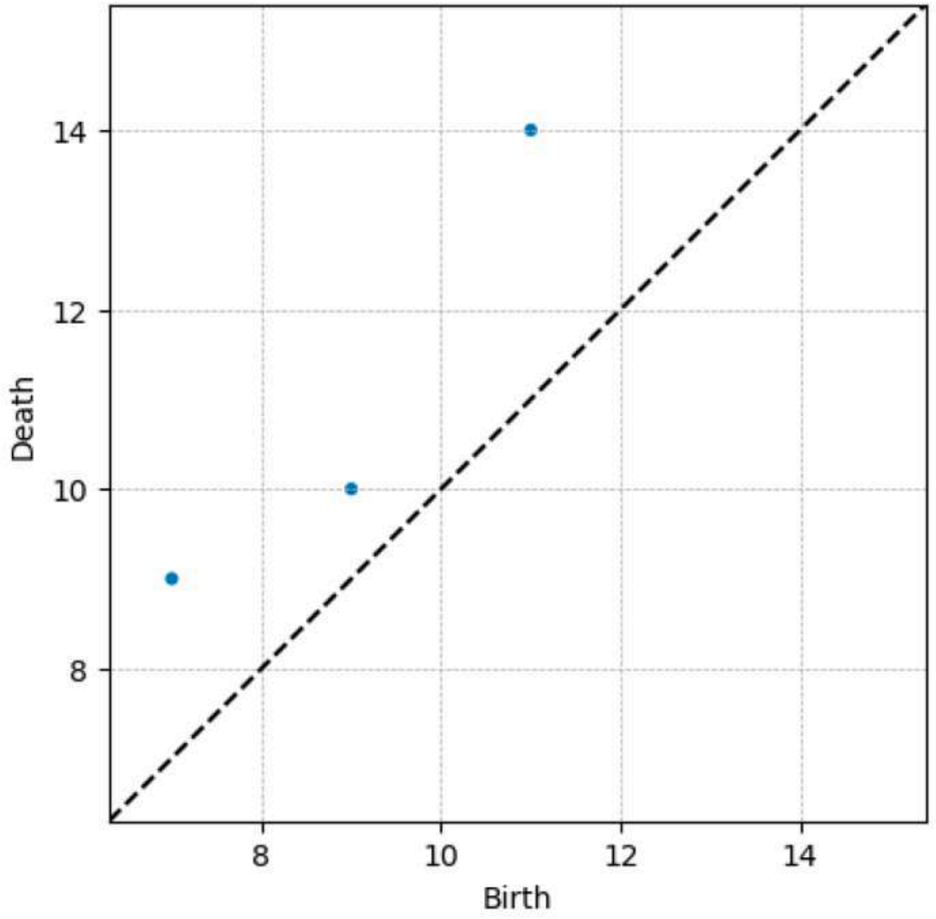
QR code / Passeport

BUT in a vector space

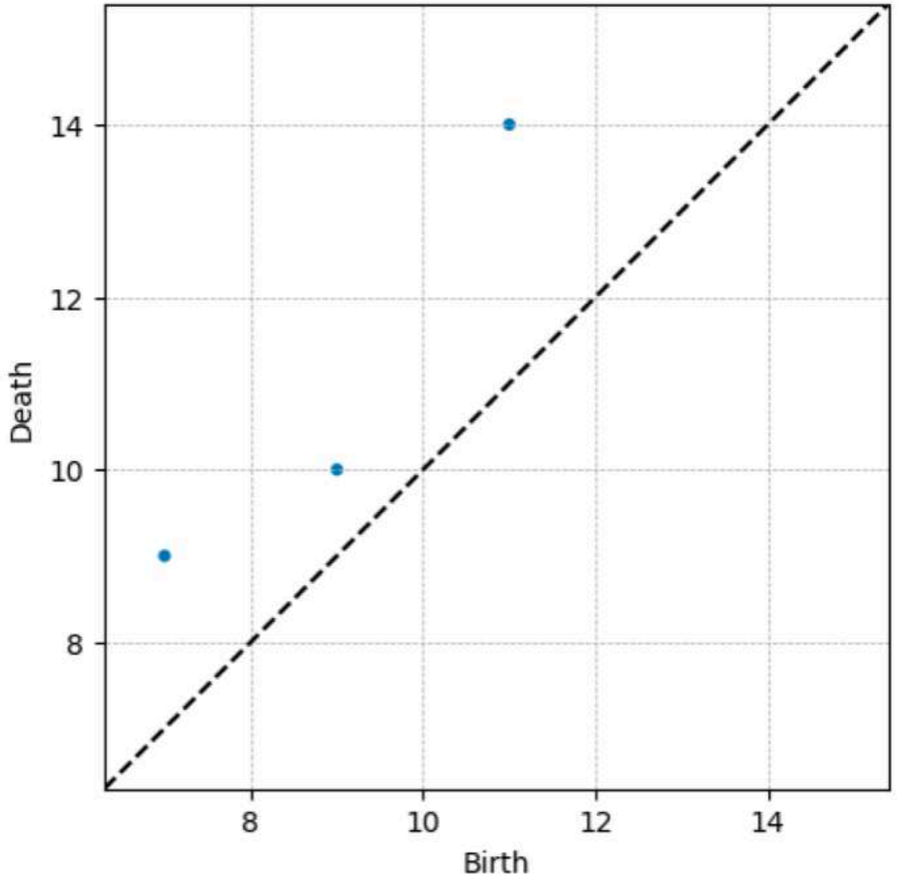
Pairing min-max



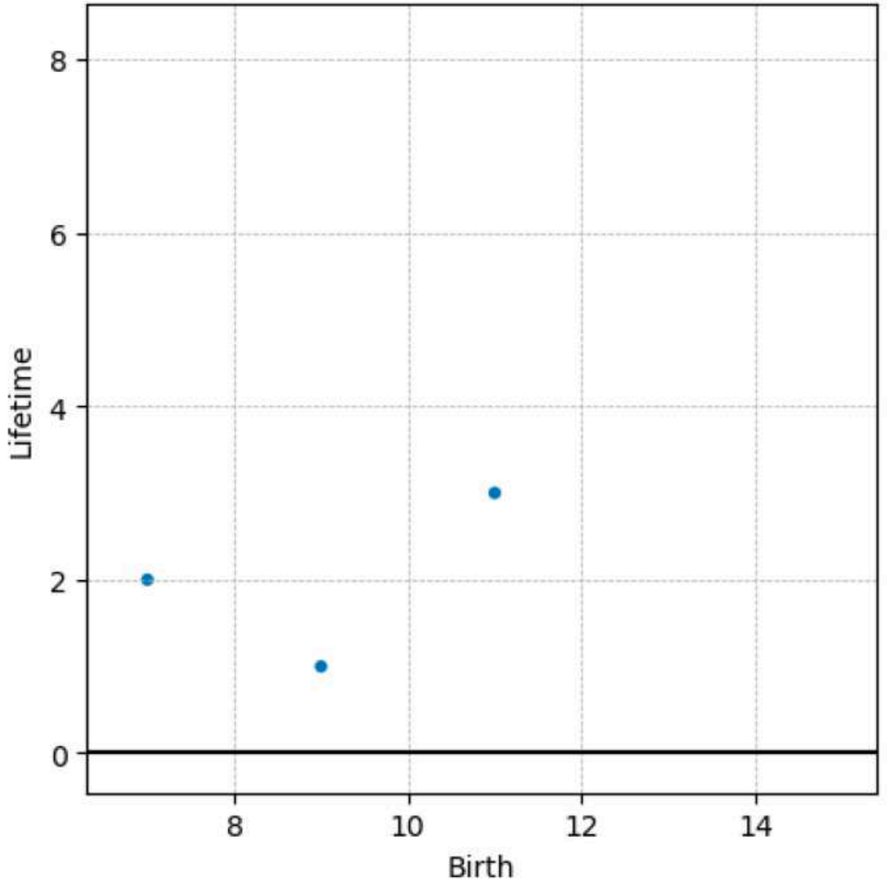
Persistence diagram



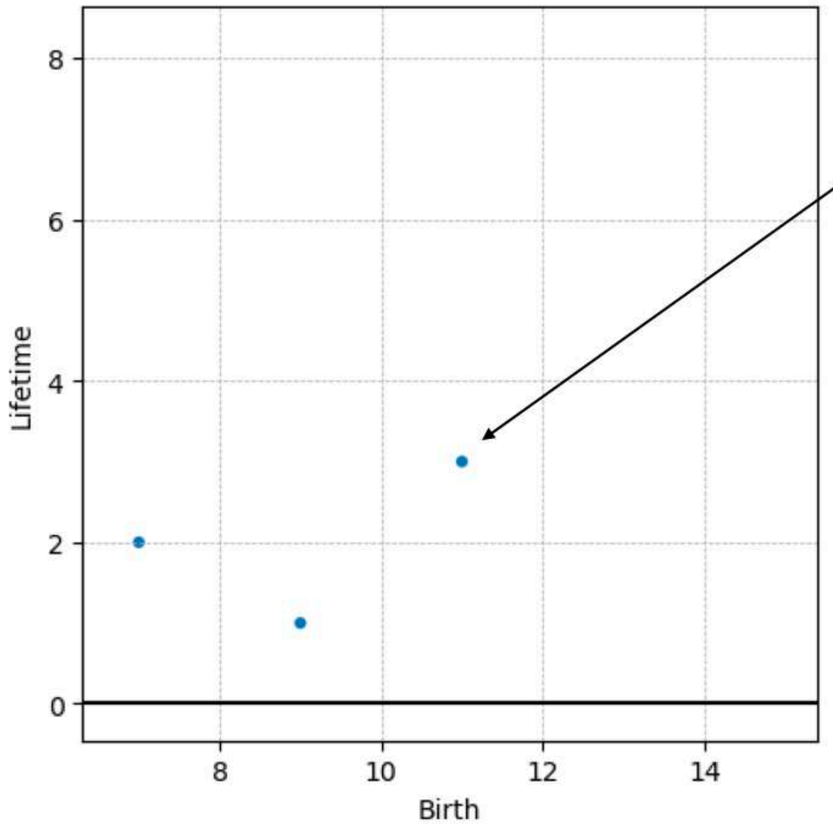
Persistence diagram



Lifetime diagram

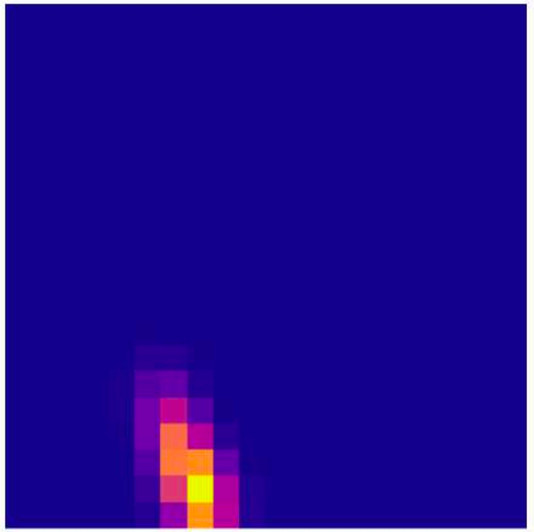


Lifetime diagram

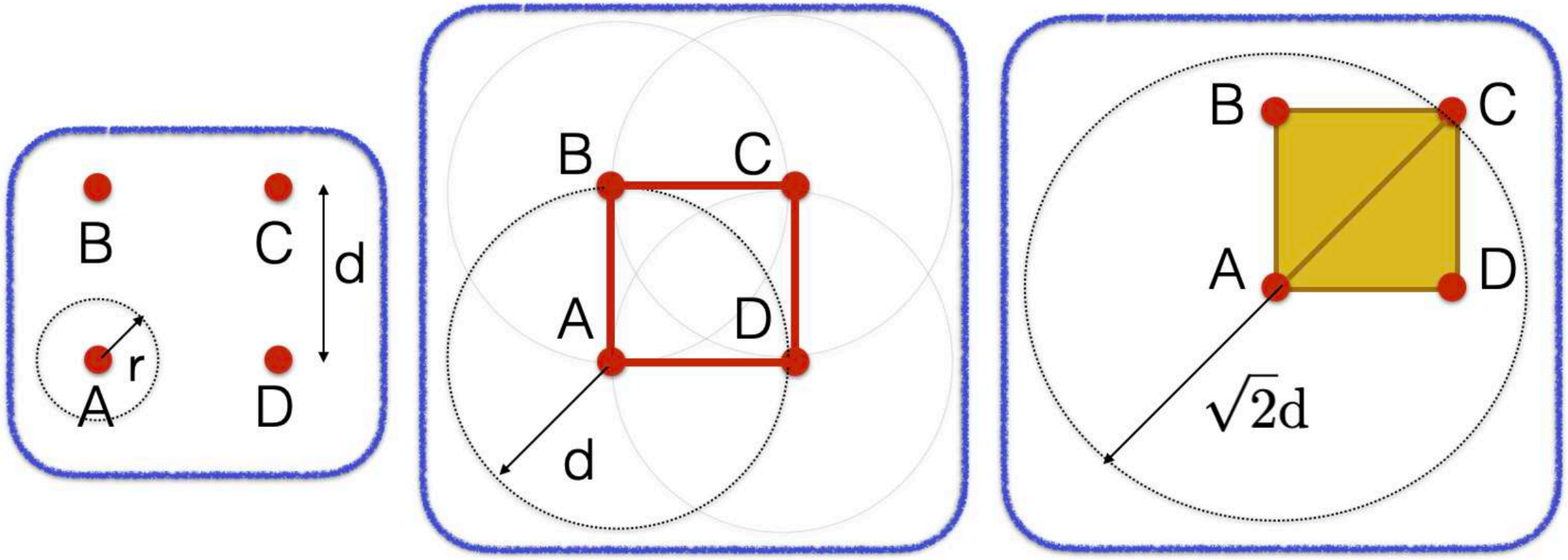


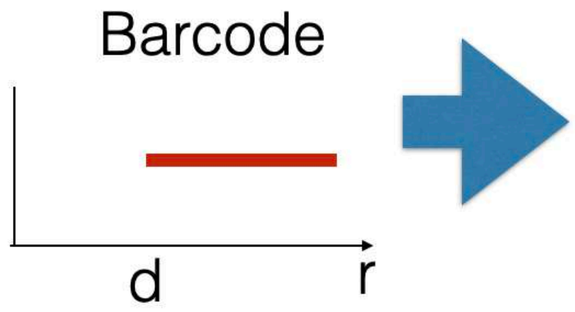
$$\rho_S(u,v) = \sum_{(x,y)\in\mathcal{T}(S)} w(x,y)\, g_{(x,y)}(u,v),$$

$$\mathcal{PI}_{P_i}(S) = \iint_{P_i} \rho_S(u,v)\, du\, dv.$$

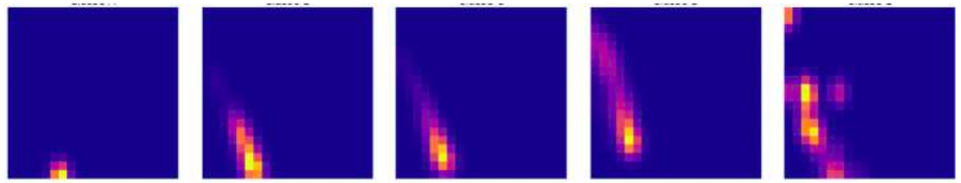


Persistence image

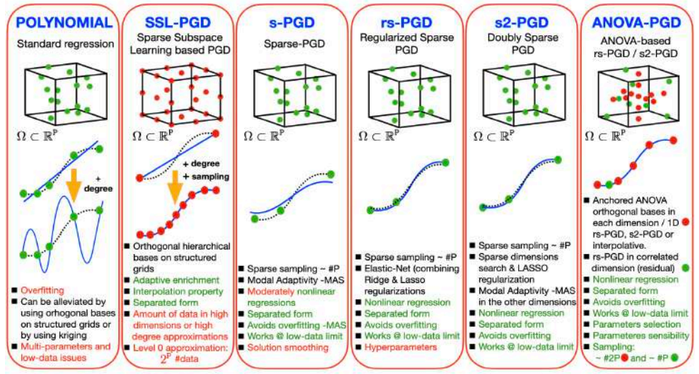




Persistence diagram,
Lifetime diagram
&
Peristence images



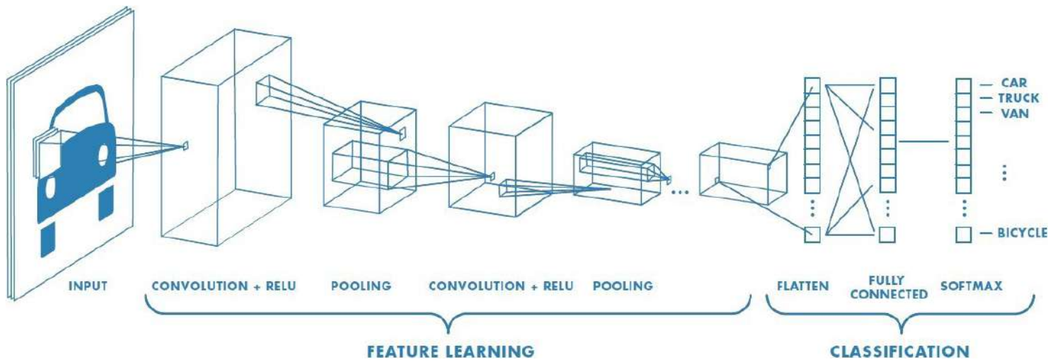
Regressions PGD-based



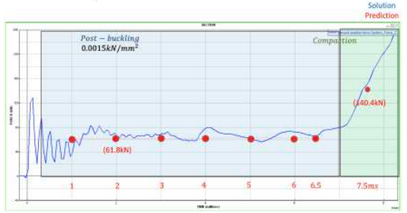
Reduction POD

$$\alpha_1, \alpha_2, \cdots$$

(convolutional) Neural Network



PERFORMANCES

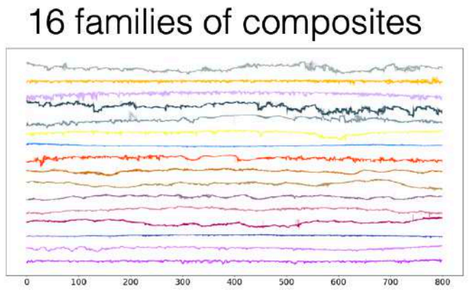


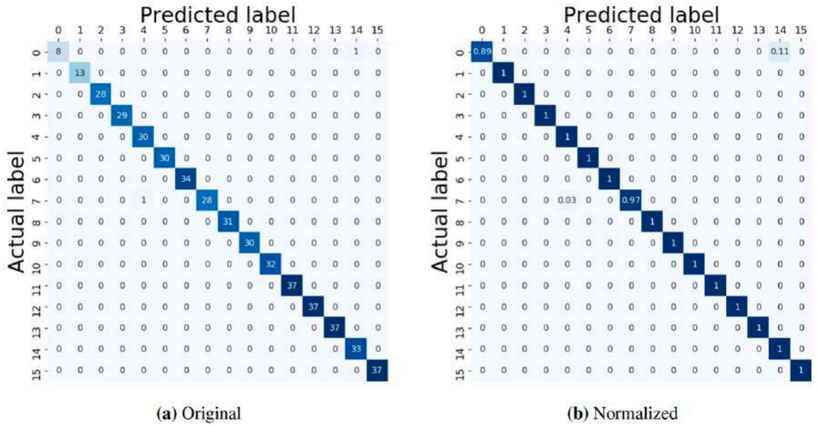
1. Introduction

2. Applications

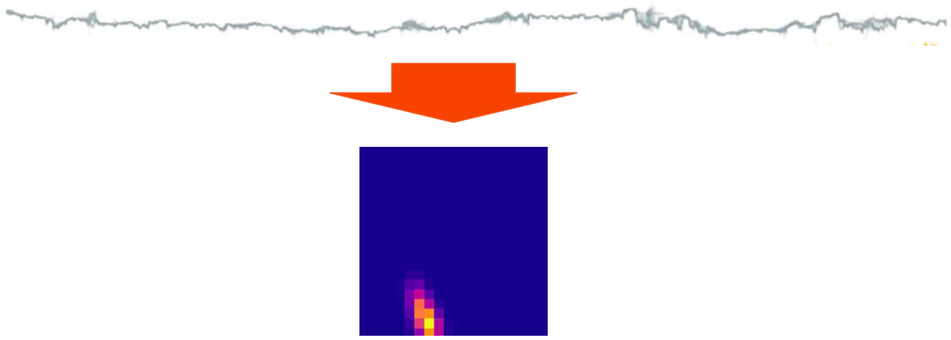
Composites processing

Rough surfaces clustering

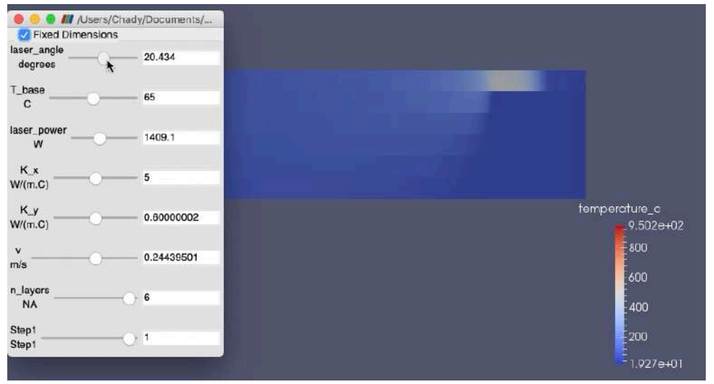


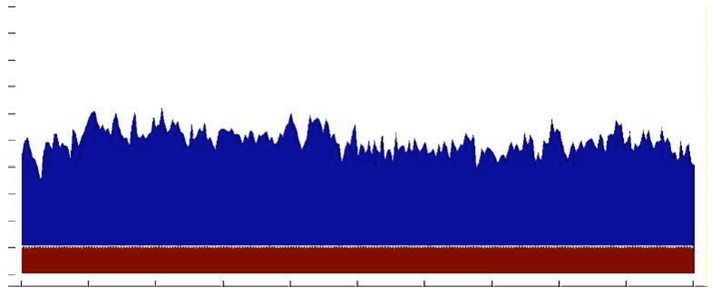


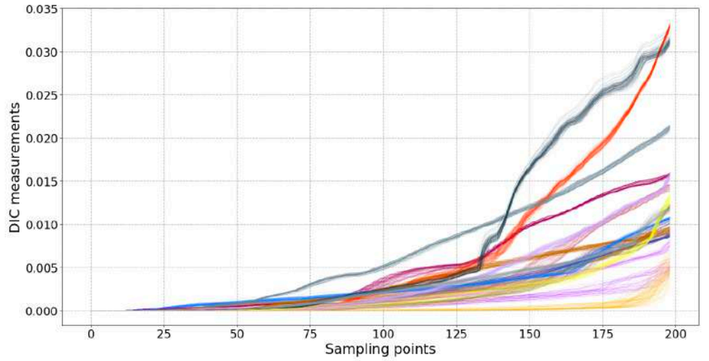
Rough surface description



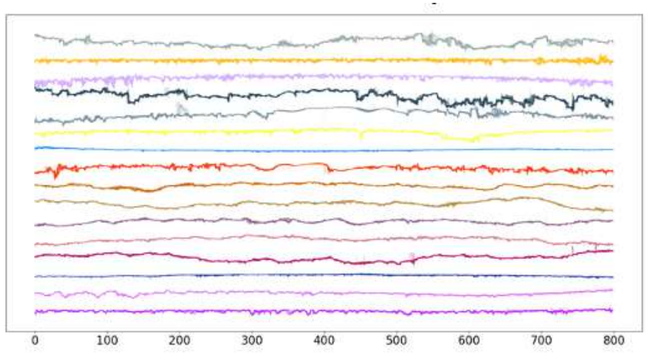
Rough surfaces consolidation analysis



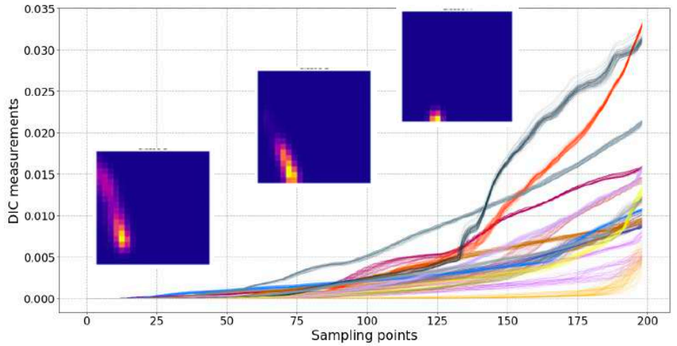




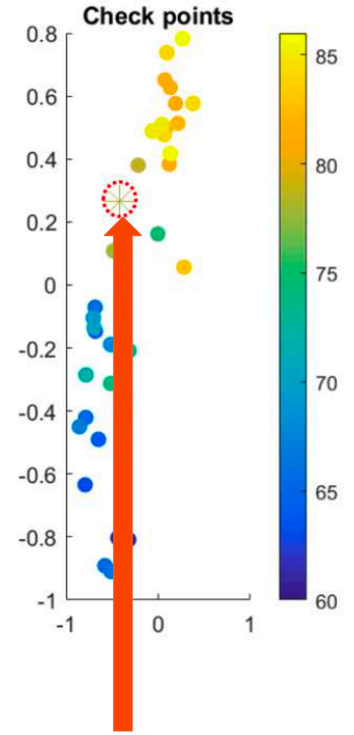
Offline



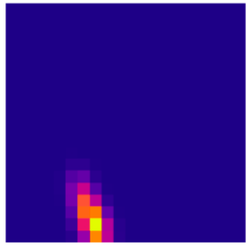
PGD & TDA



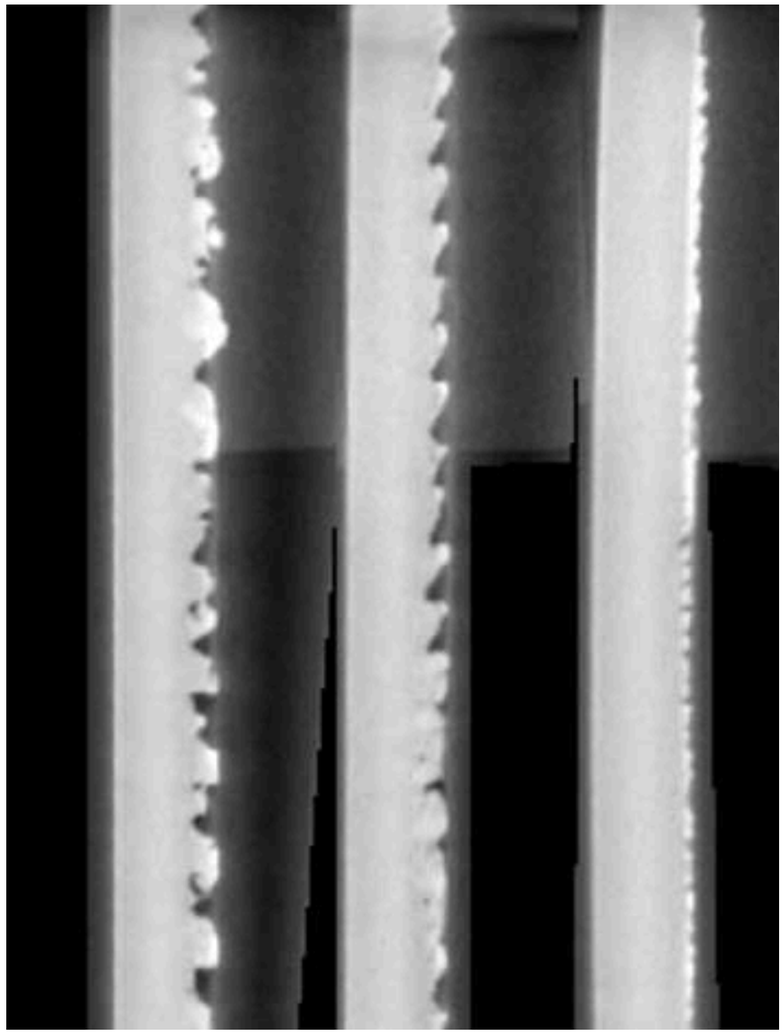
C2V

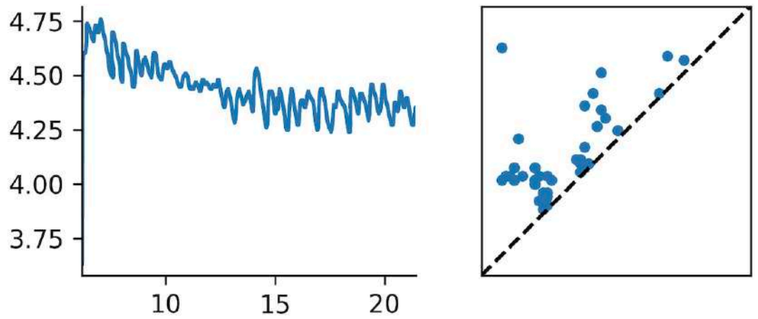


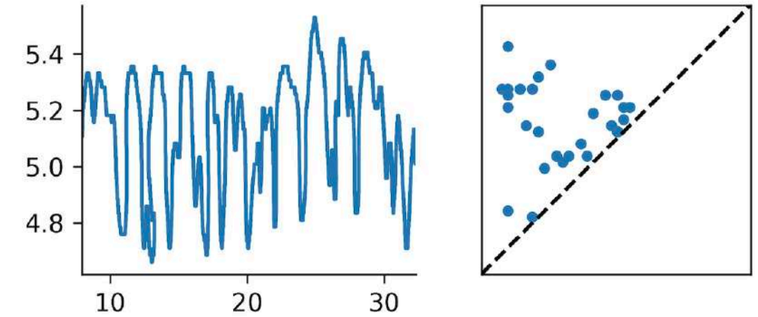
Online

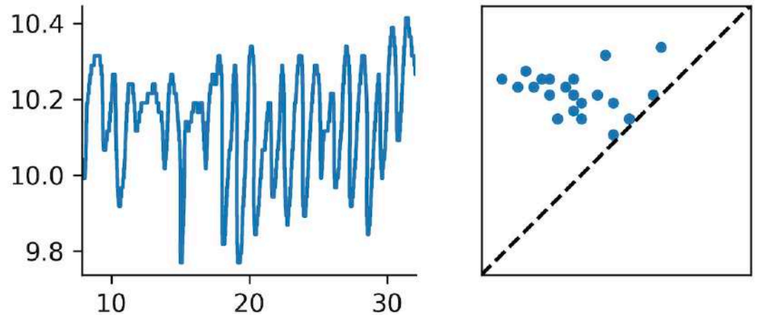


Bonding @ laser structured surfaces

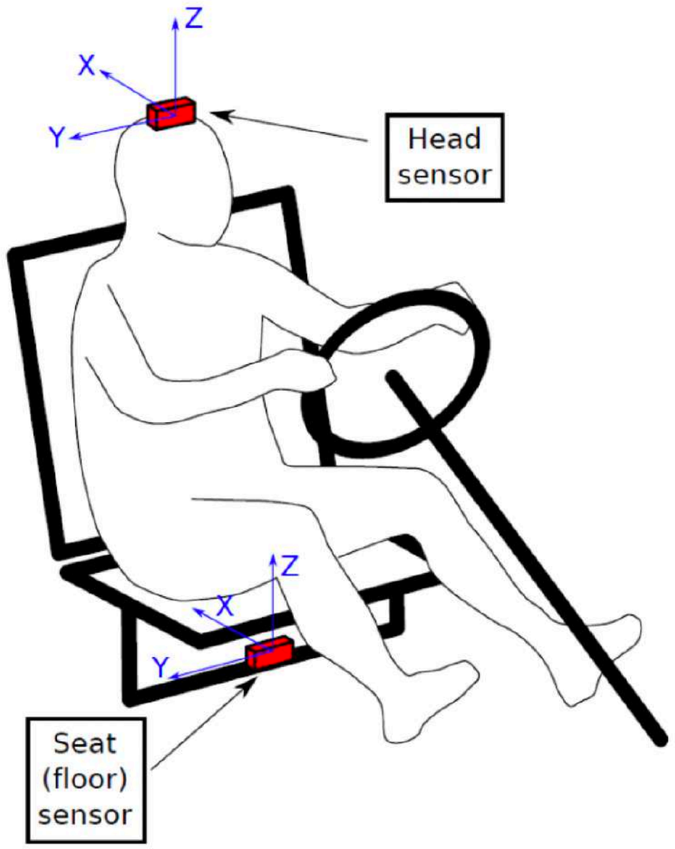


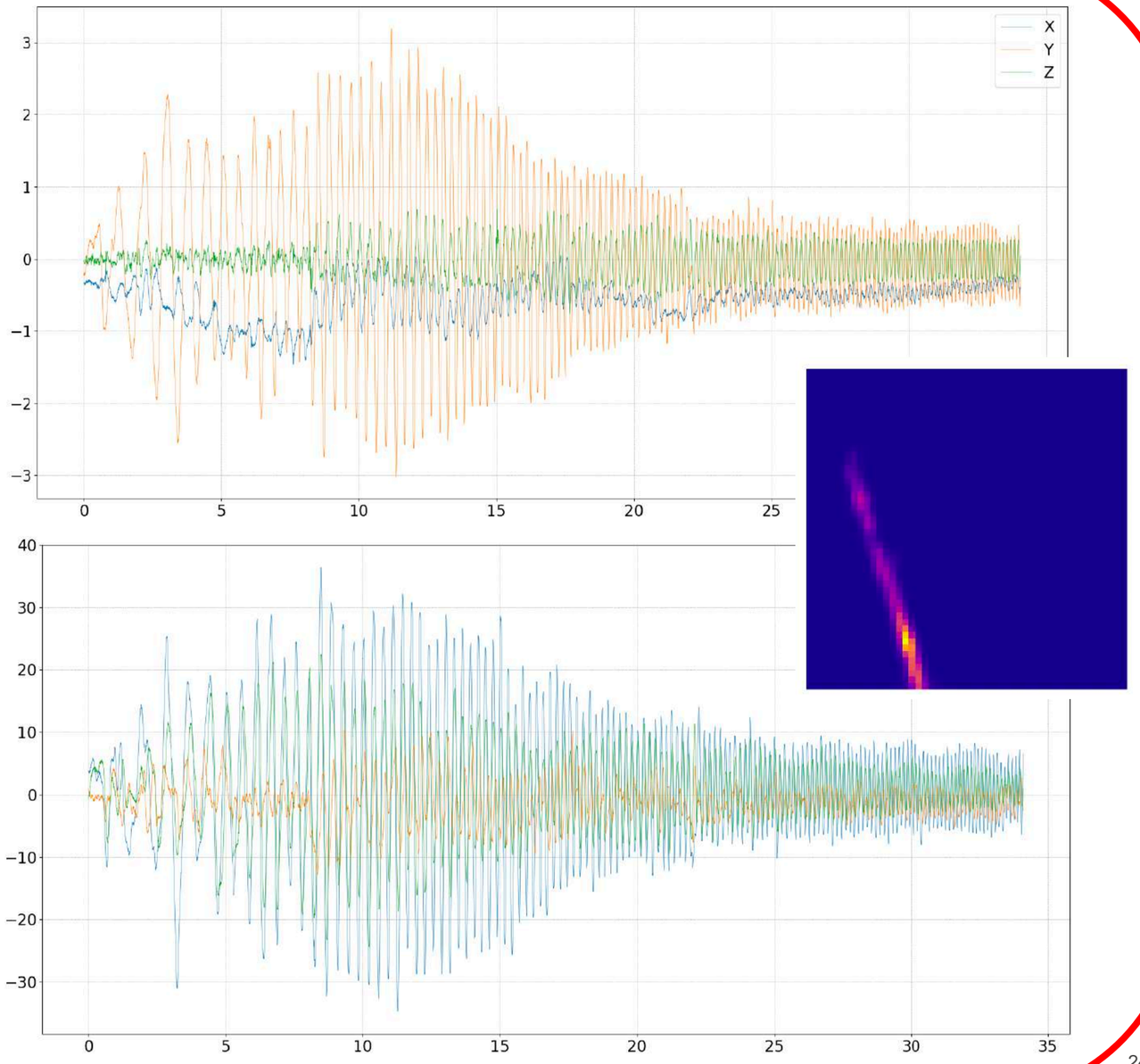




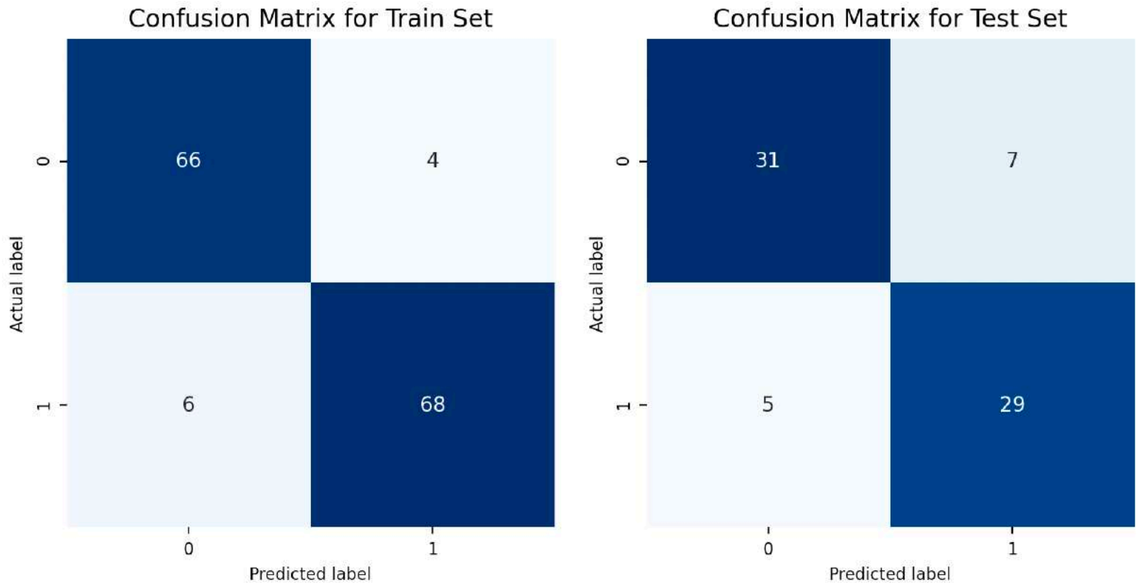


ADAS

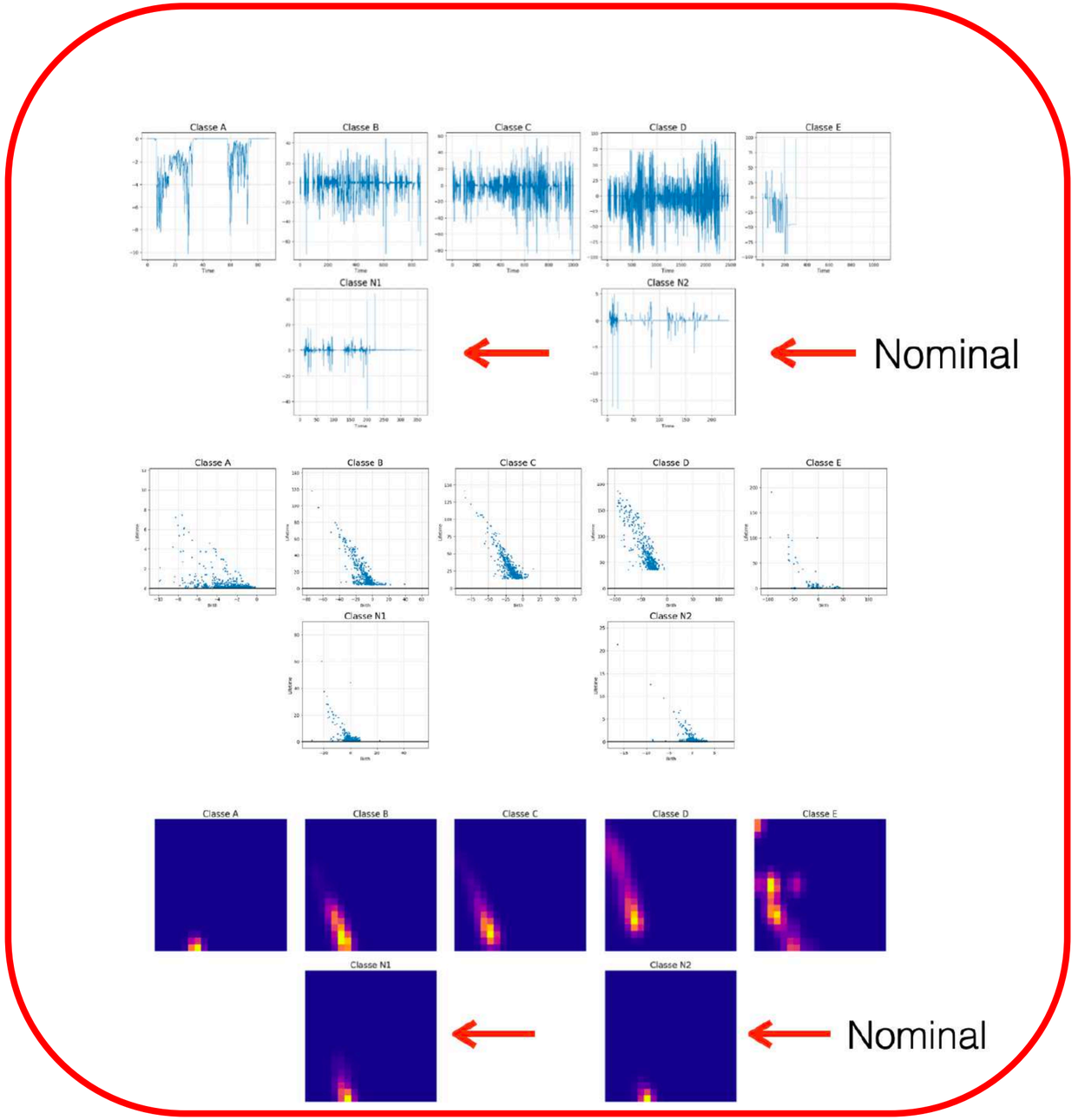



X
Y
Z
3
2
1
0
−1
−2
−3
0
5
10
15
20
25
40
30
20
10
0
−10
−20
−30
0
5
10
15
20
25
30
35

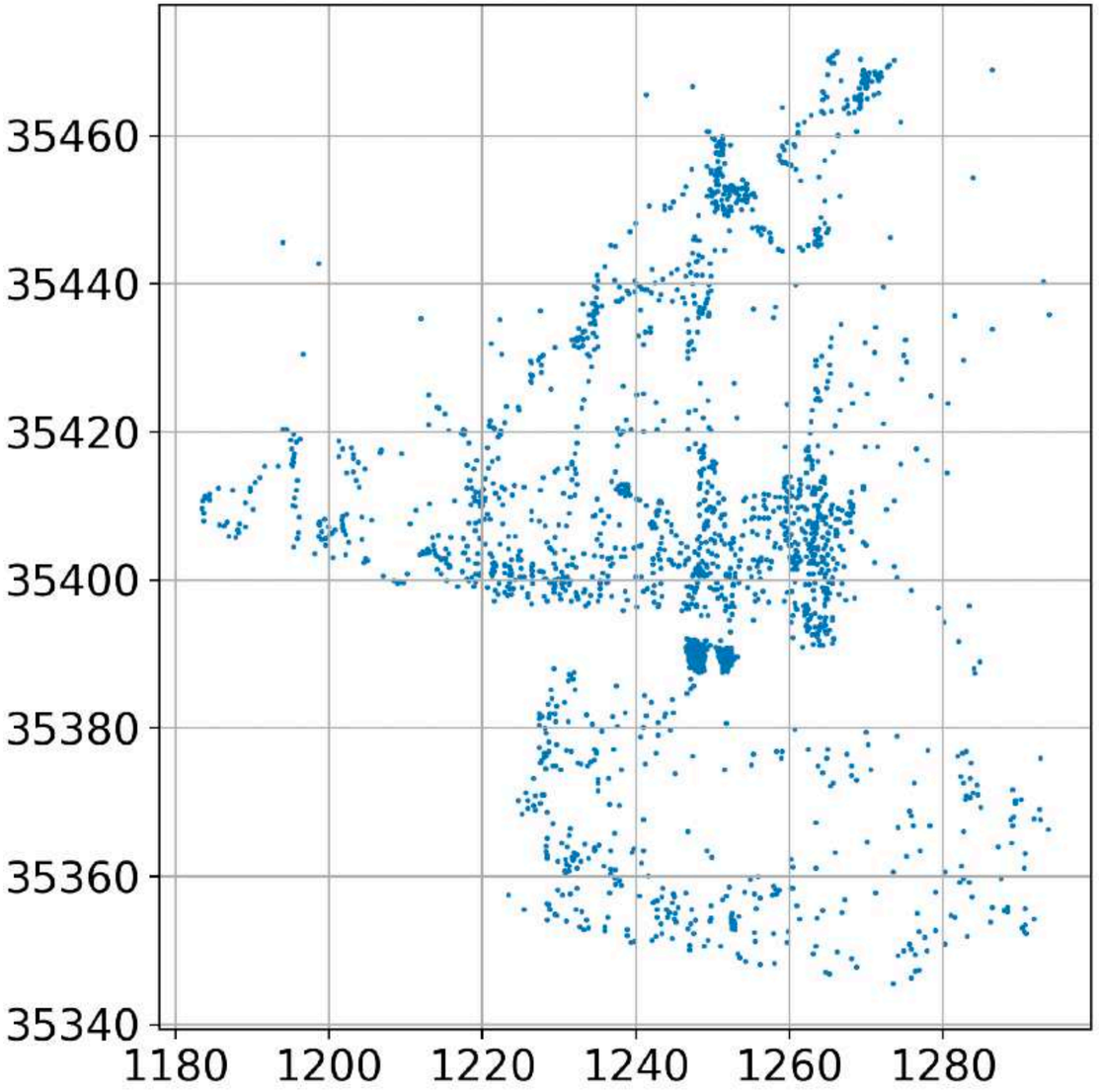
Class	Label
1	SAVRelaxedPassager
2	SAVTensePassager
3	SAVRelaxedDriver
4	SAVTenseDriver
5	RigidRelaxedPassager
6	RigidTensePassager
7	RigidRelaxedDriver
8	RigidTenseDriver

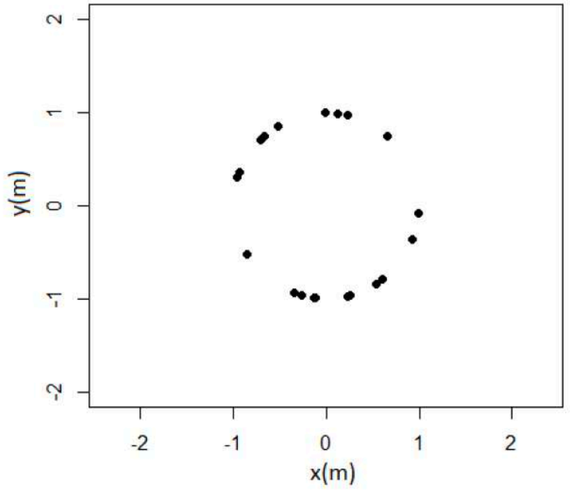


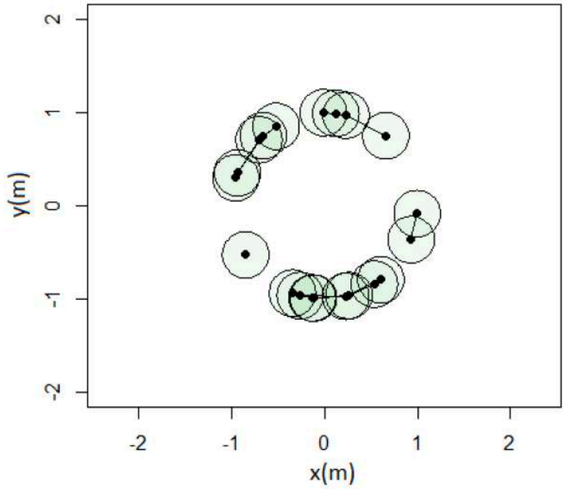
FAULTS

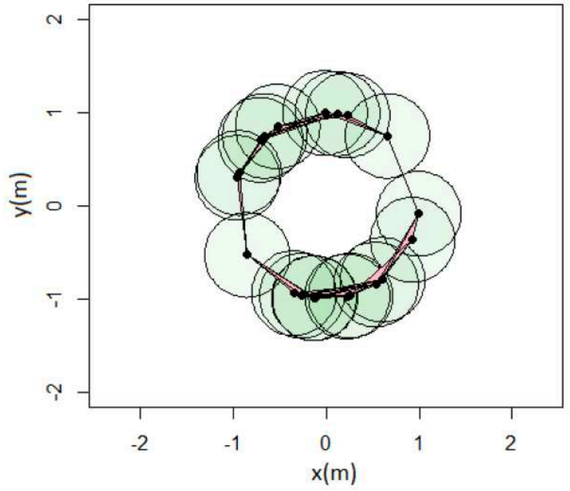

Classe A
Classe B
Classe C
Classe D
Classe E
Classe N1
Classe N2
Nominal
Classe A
Classe B
Classe C
Classe D
Classe E
Classe N1
Classe N2
Classe A
Classe B
Classe C
Classe D
Classe E
Classe N1
Classe N2
Nominal

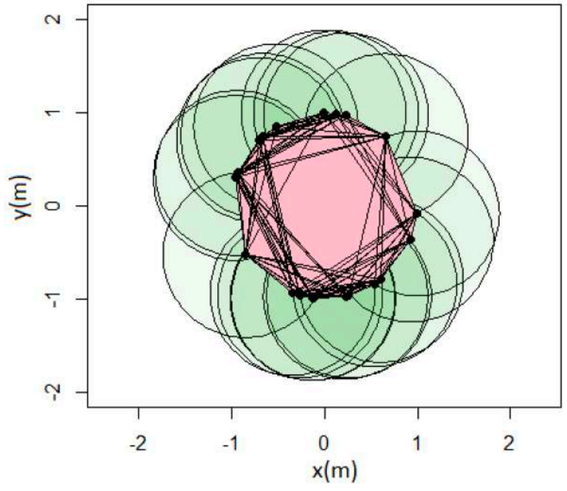
Robots

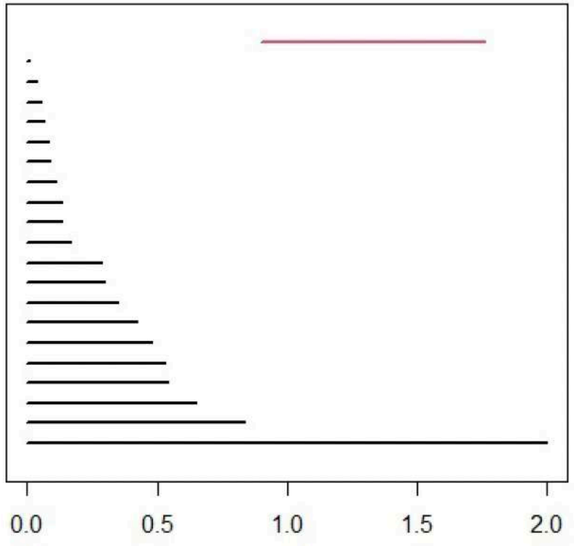

35460
35440
35420
35400
35380
35360
35340
1180
1200
1220
1240
1260
1280

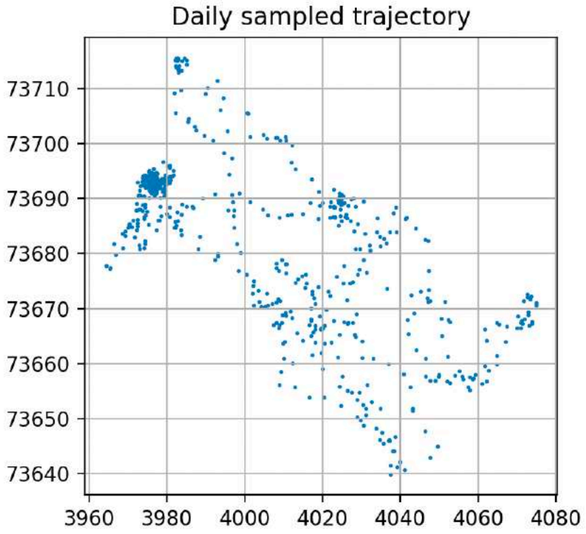

y(m)
x(m)
-2
-1
0
1
2

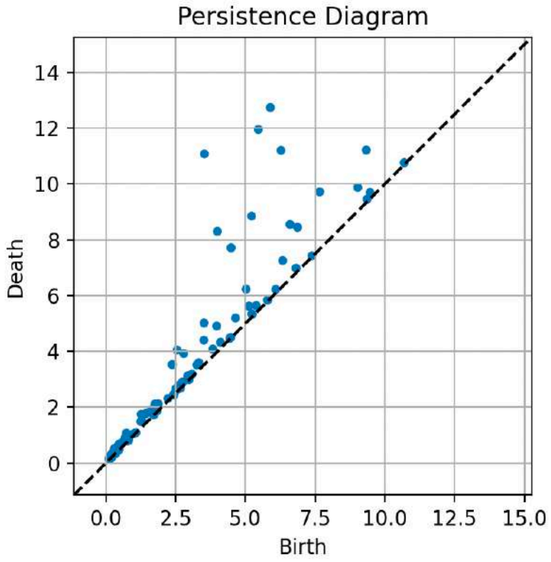

y(m)
x(m)
-2
-1
0
1
2

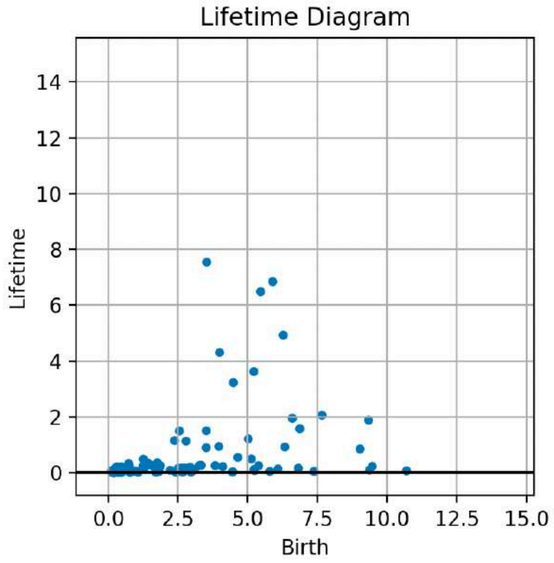

y(m)
x(m)
-2
-1
0
1
2

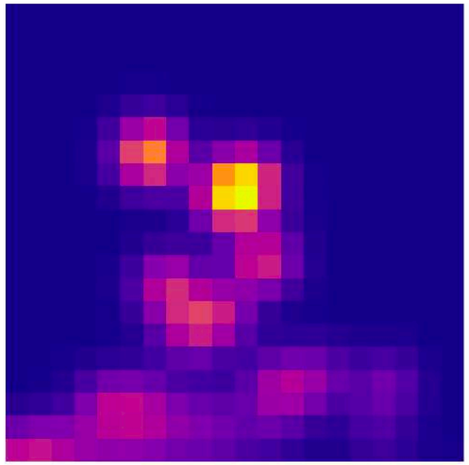

y(m)
x(m)
-2
-1
0
1
2

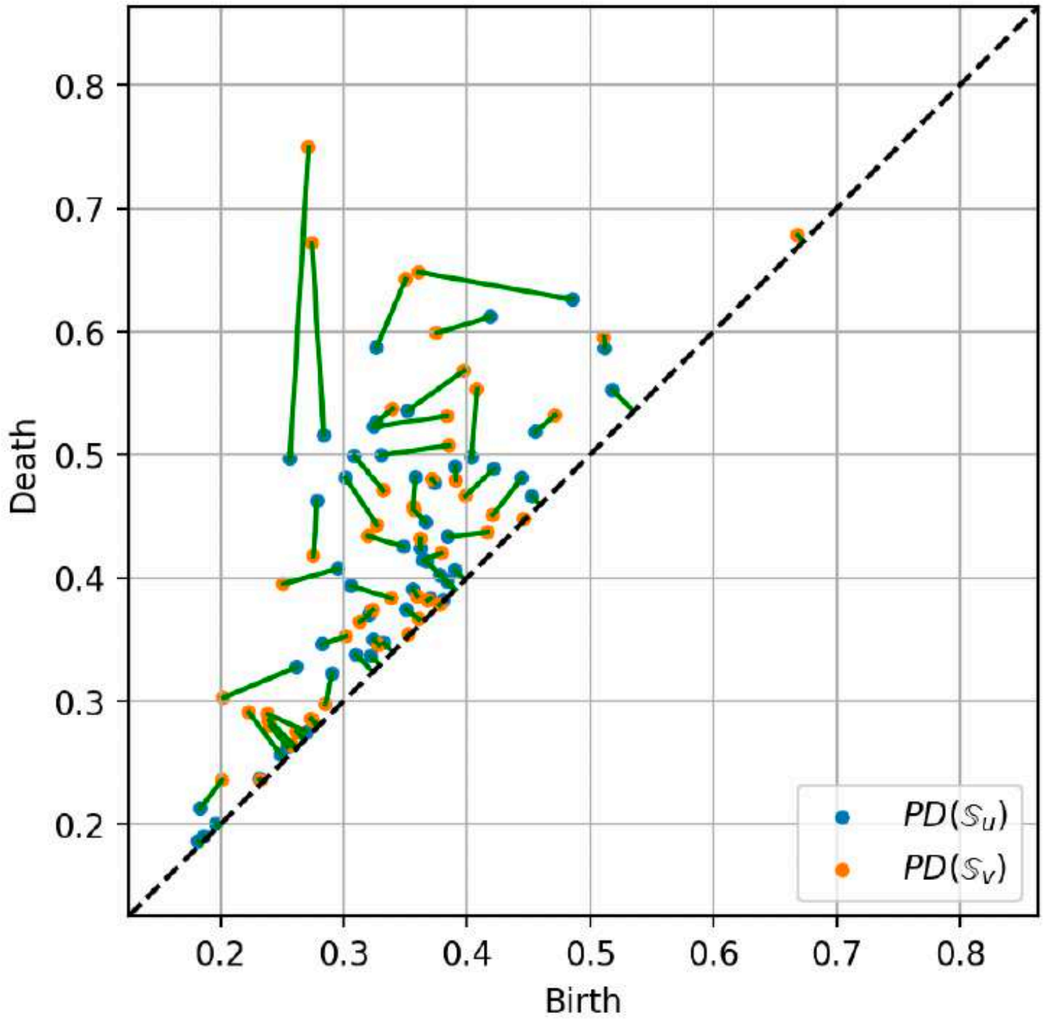

0.0
0.5
1.0
1.5
2.0

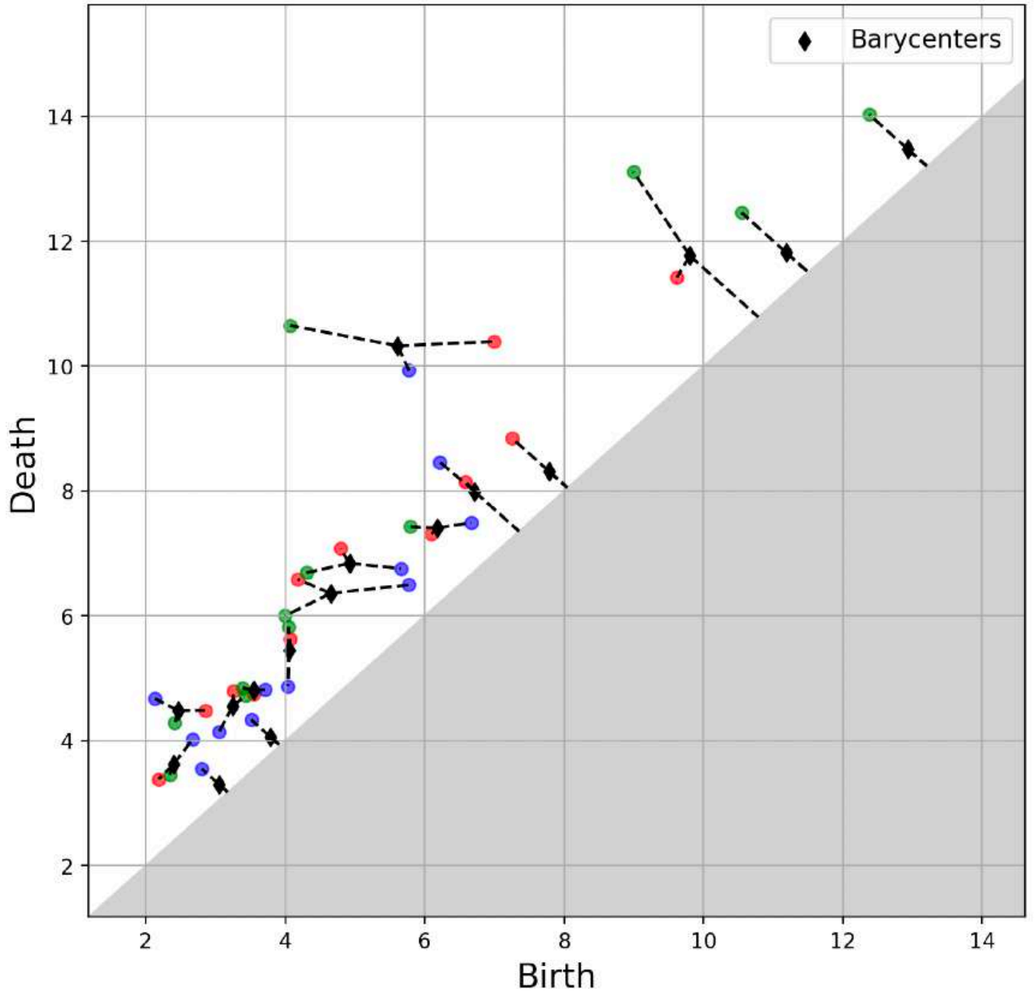

Daily sampled trajectory
73710
73700
73690
73680
73670
73660
73650
73640
3960
3980
4000
4020
4040
4060
4080

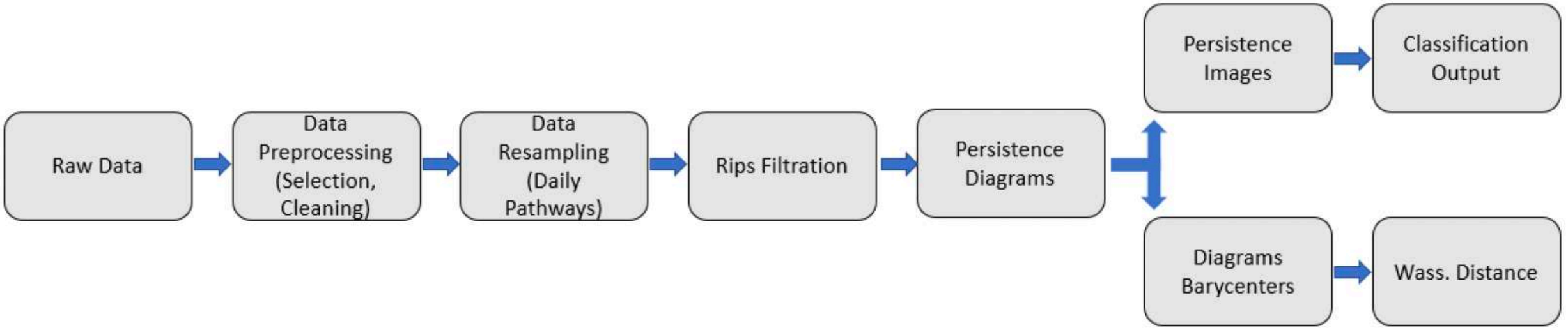

Persistence Diagram
Death
Birth
0
2
4
6
8
10
12
14
0.0
2.5
5.0
7.5
10.0
12.5
15.0

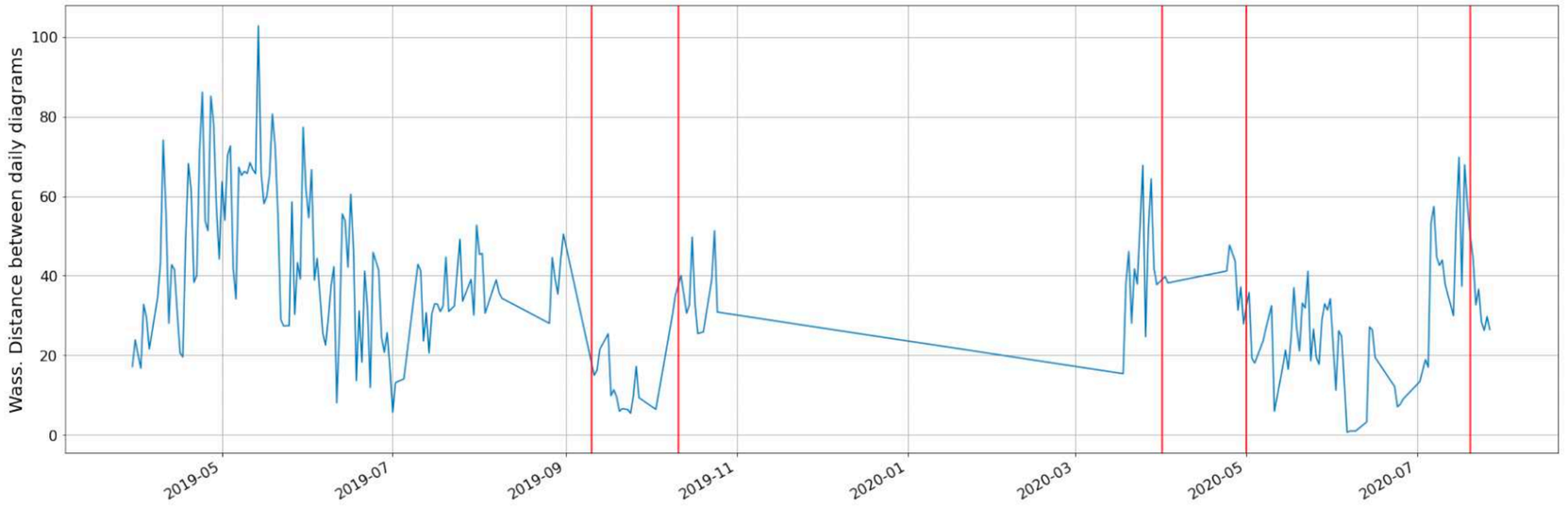

Lifetime Diagram
Lifetime
Birth
0
2
4
6
8
10
12
14
0.0
2.5
5.0
7.5
10.0
12.5
15.0

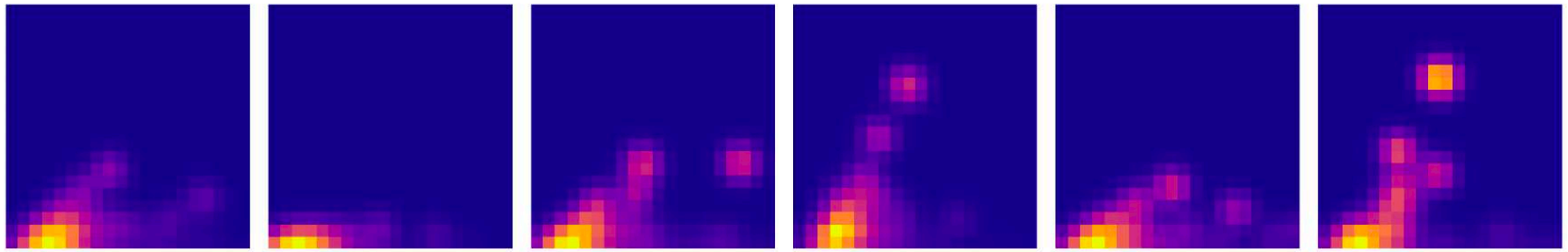

Persistence Image


0.8
0.7
0.6
0.5
0.4
0.3
0.2
Death
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Birth
$PD(\mathbb{S}_u)$
$PD(\mathbb{S}_v)$

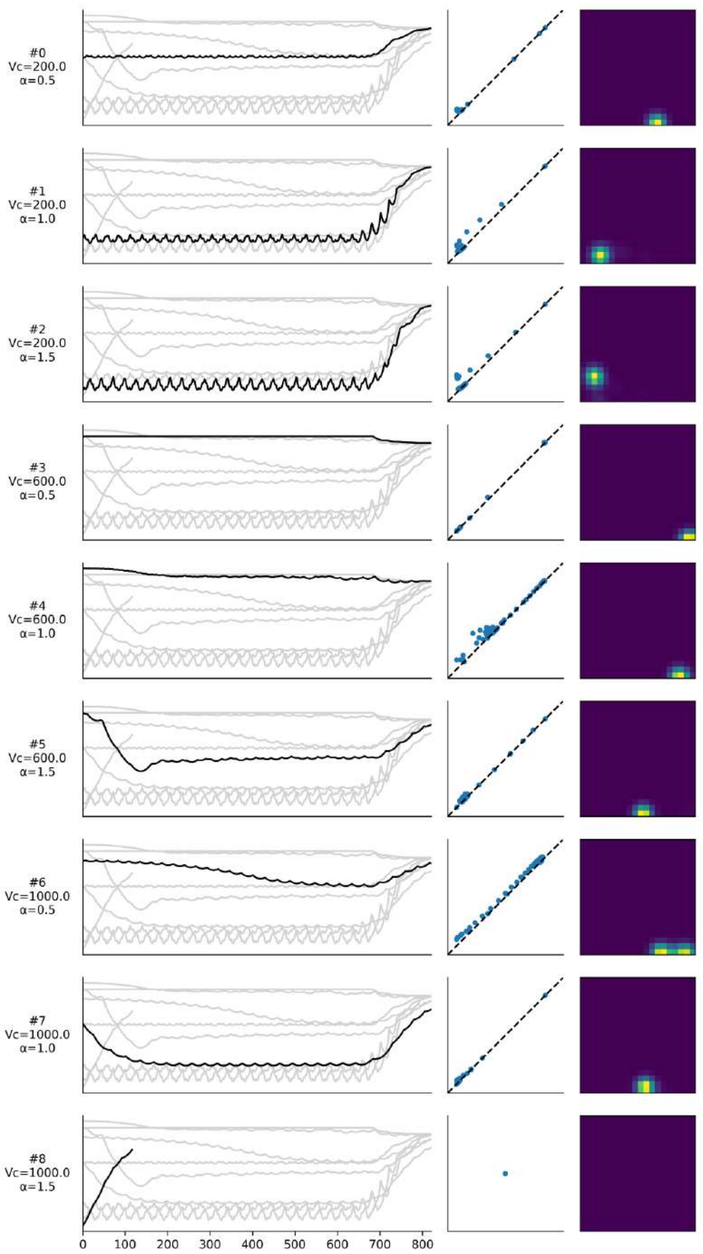

Barycenters
14
12
10
8
6
4
2
Death
2
4
6
8
10
12
14
Birth


Raw Data
Data Preprocessing (Selection, Cleaning)
Data Resampling (Daily Pathways)
Rips Filtration
Persistence Diagrams
Persistence Images
Classification Output
Diagrams Barycenters
Wass. Distance


Wass. Distance between daily diagrams
100
80
60
40
20
0
2019-05
2019-07
2019-09
2019-11
2020-01
2020-03
2020-05
2020-07



Surface quality in machining



POD on the images

$$PI = \sum_{i=1}^{R} \alpha_i(\mu_1, \mu_2) M_i$$

(μ_1, μ_2) *process parameters*

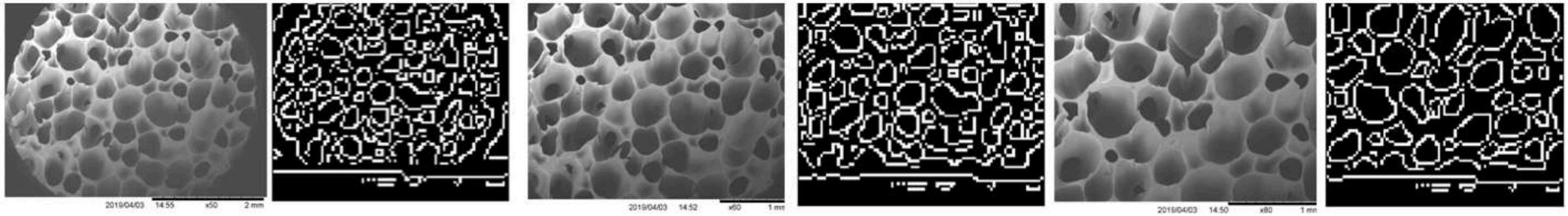
M_i *POD modes*

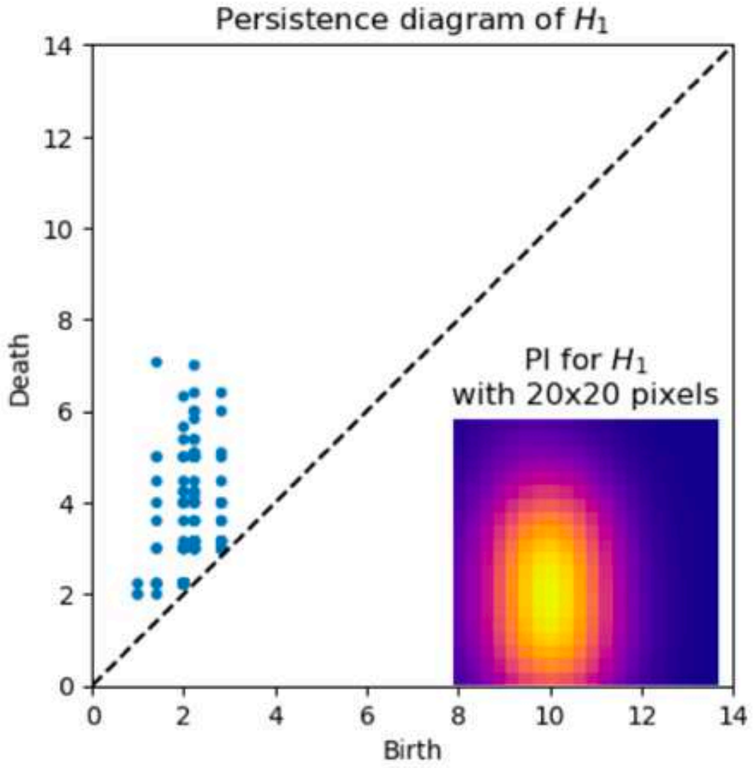
$$(\mu_1^*, \mu_2^*) \rightarrow \alpha_i^*, \ i = 1, ..., R \rightarrow PI^*$$

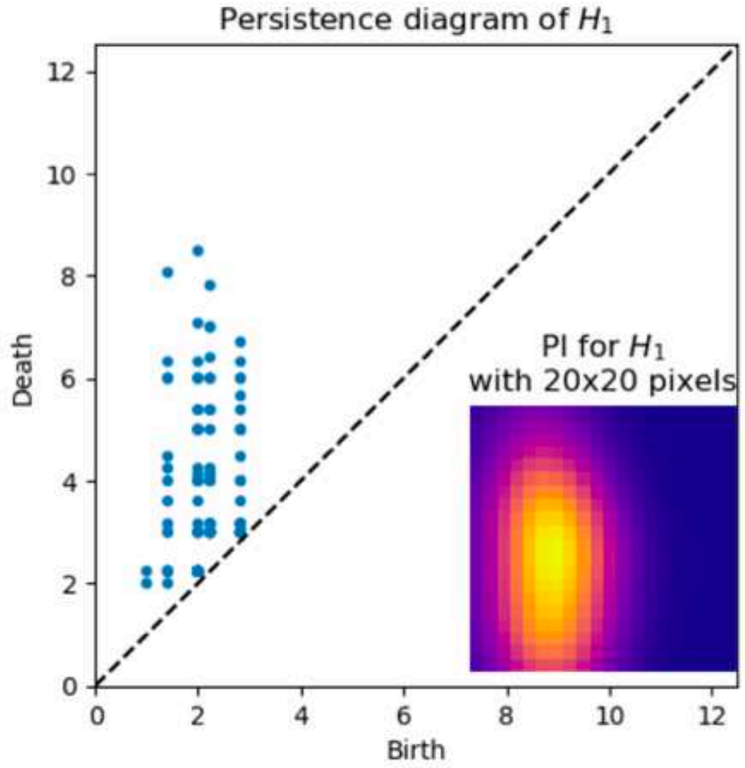
Look for the closest PI_j *to* PI^*

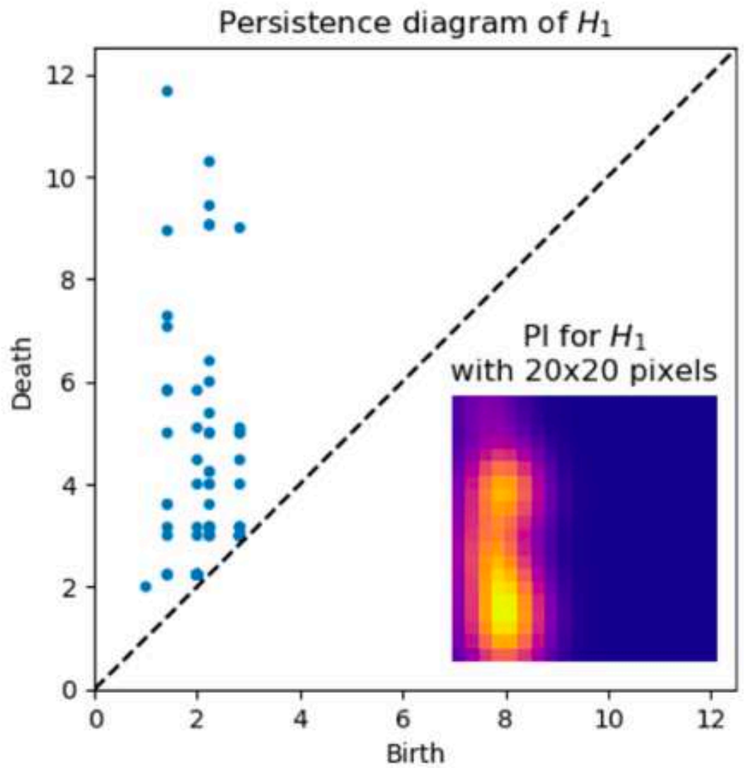
that represents the expected surface

Complex microstructures



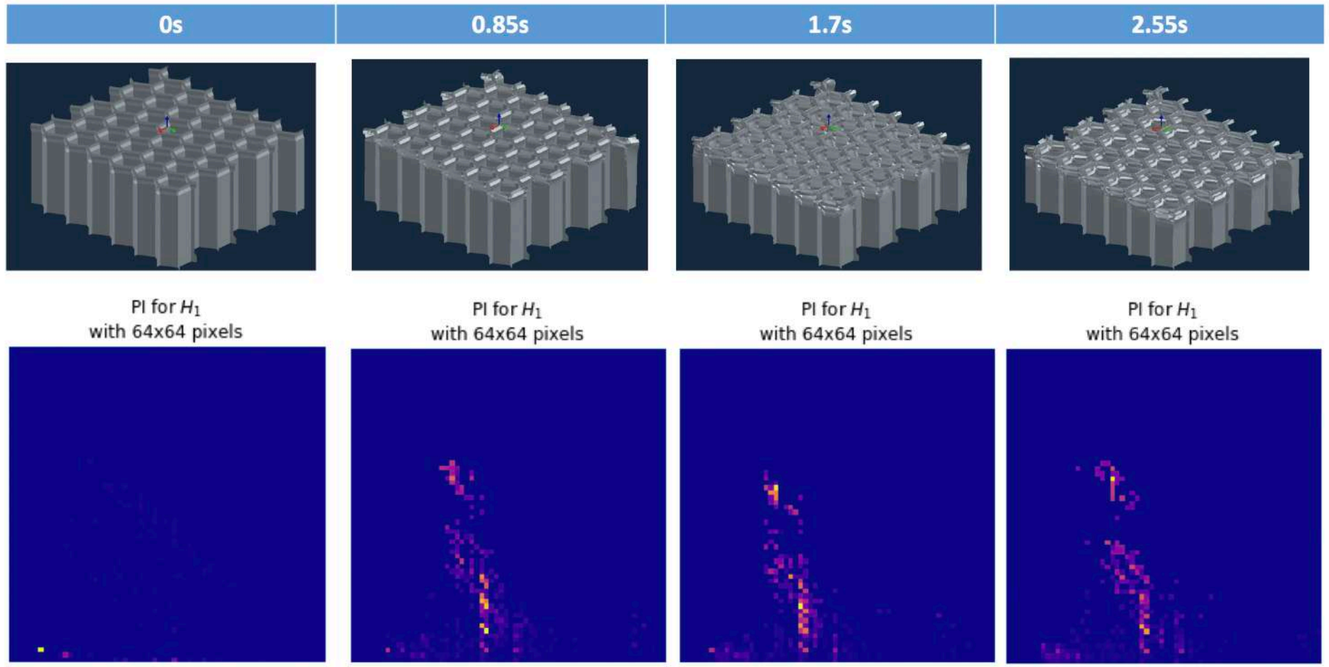


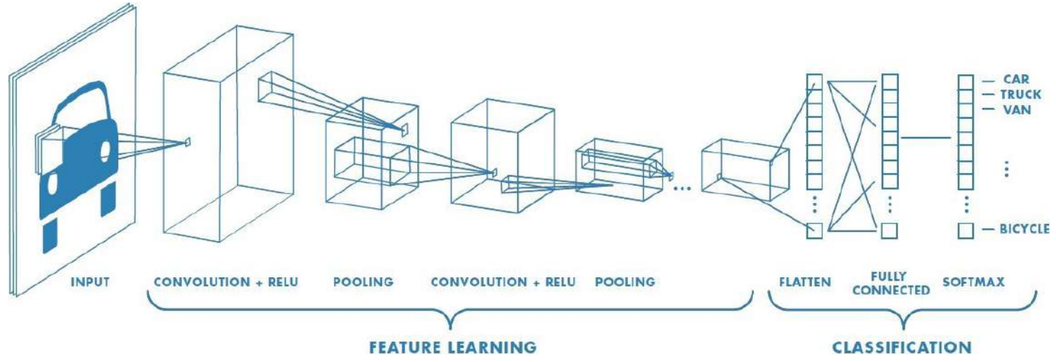


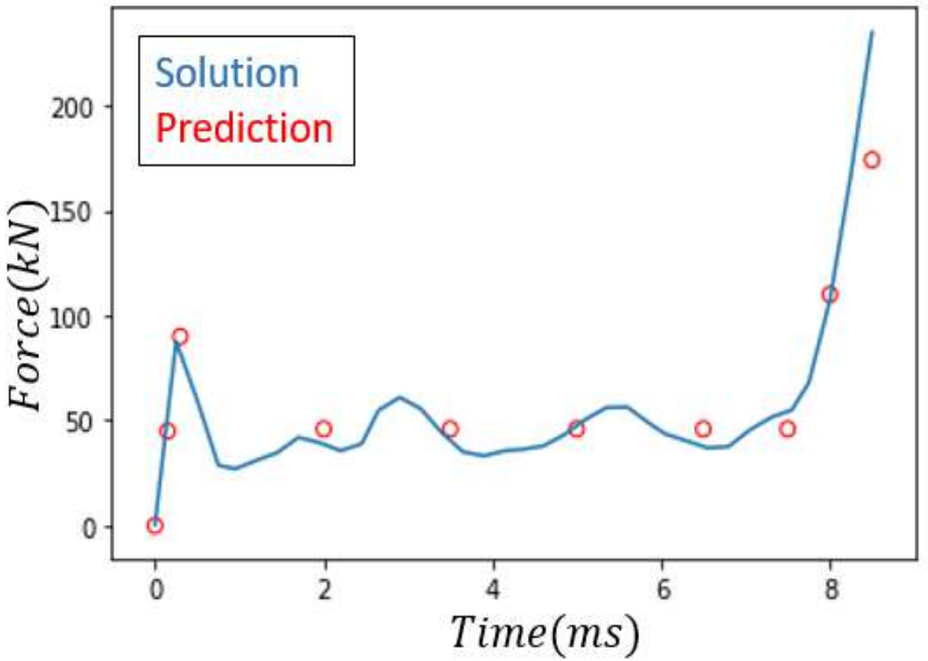


Complex behaviors

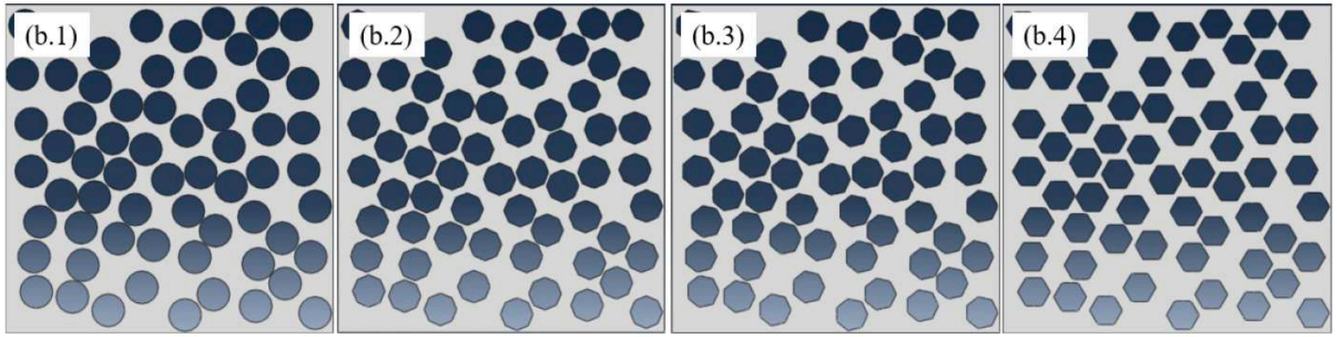
Data-driven modelling of honeycomb barriers



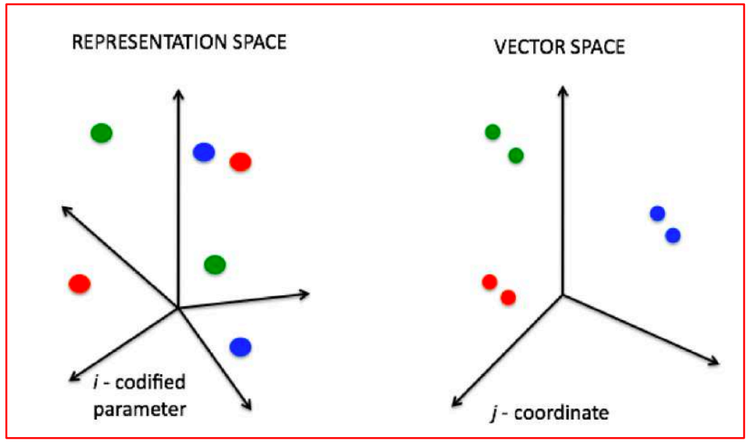


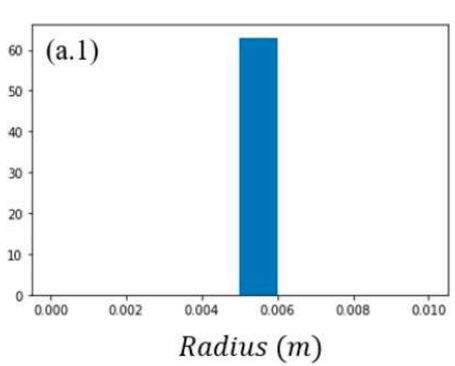


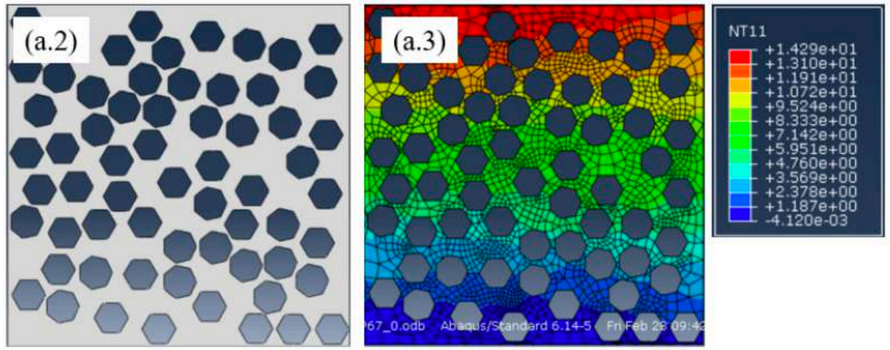
Complex behaviors (II)

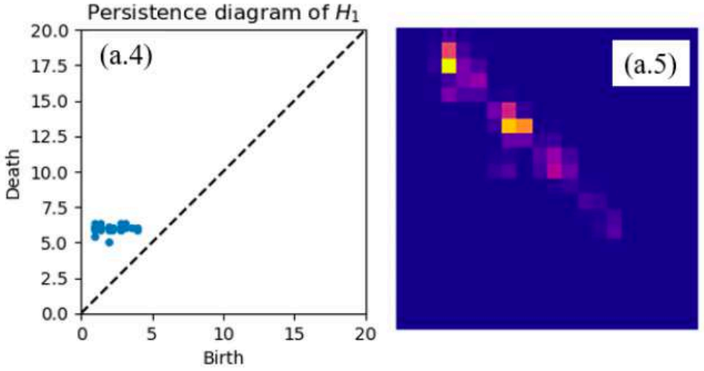


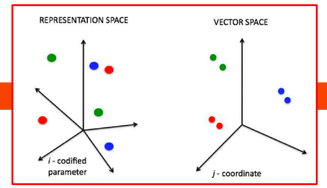
PCA

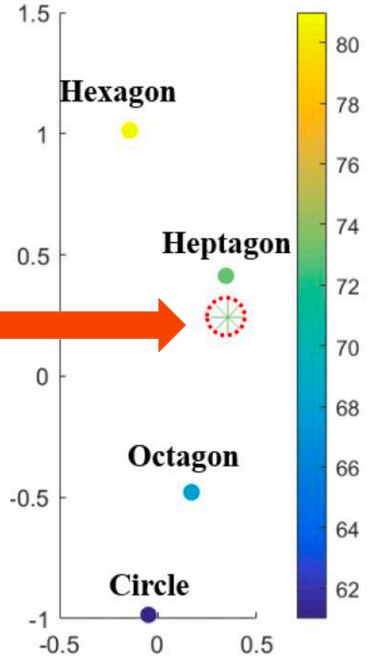




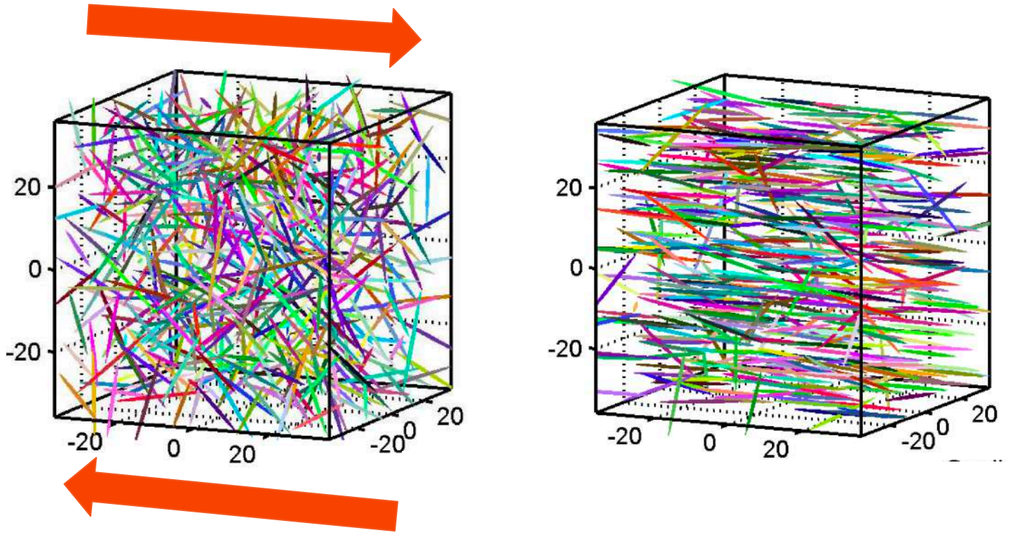


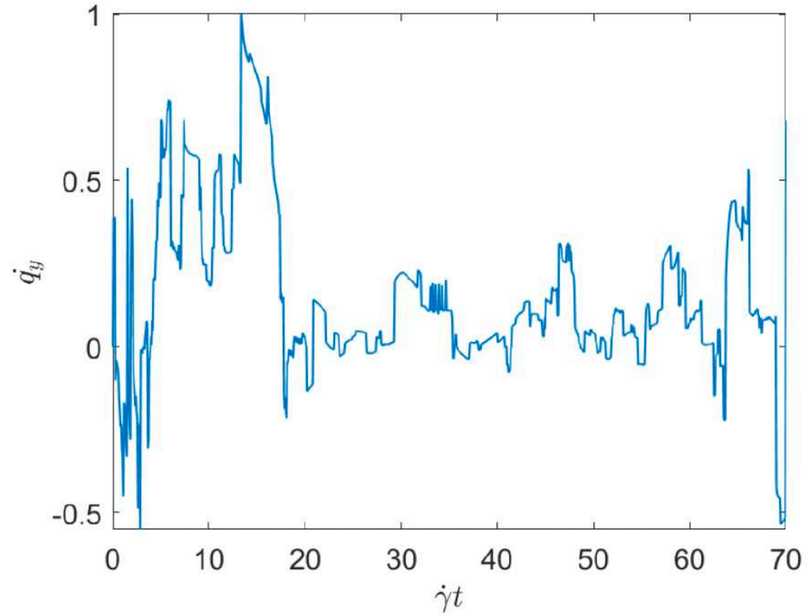






Rheology

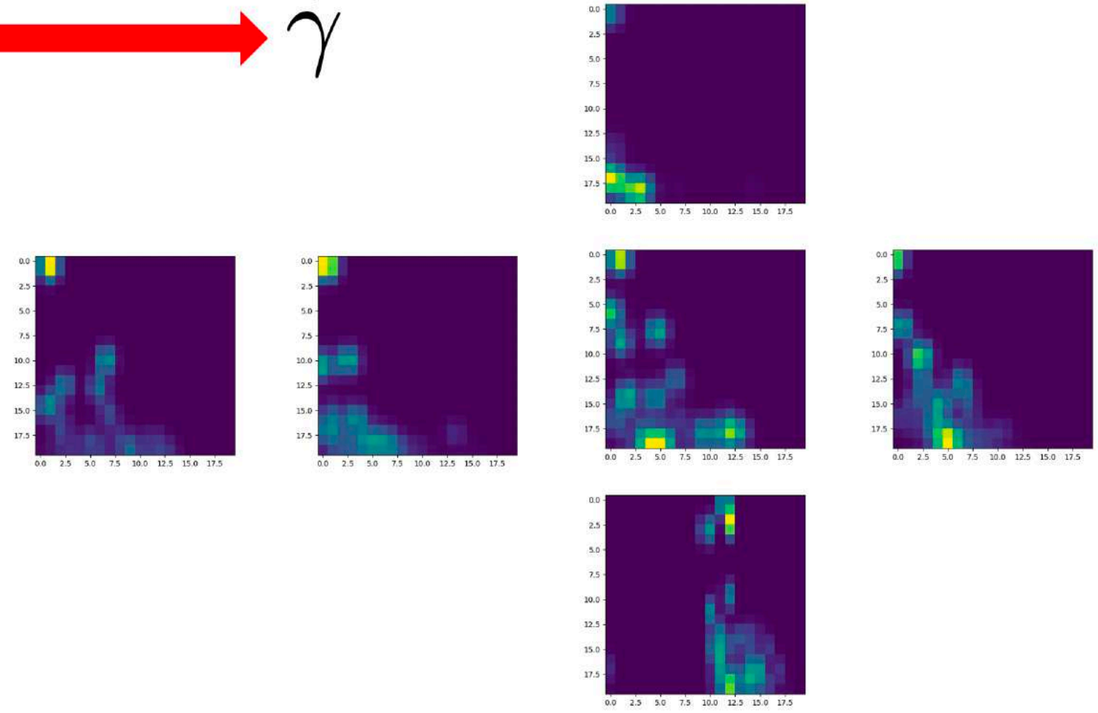




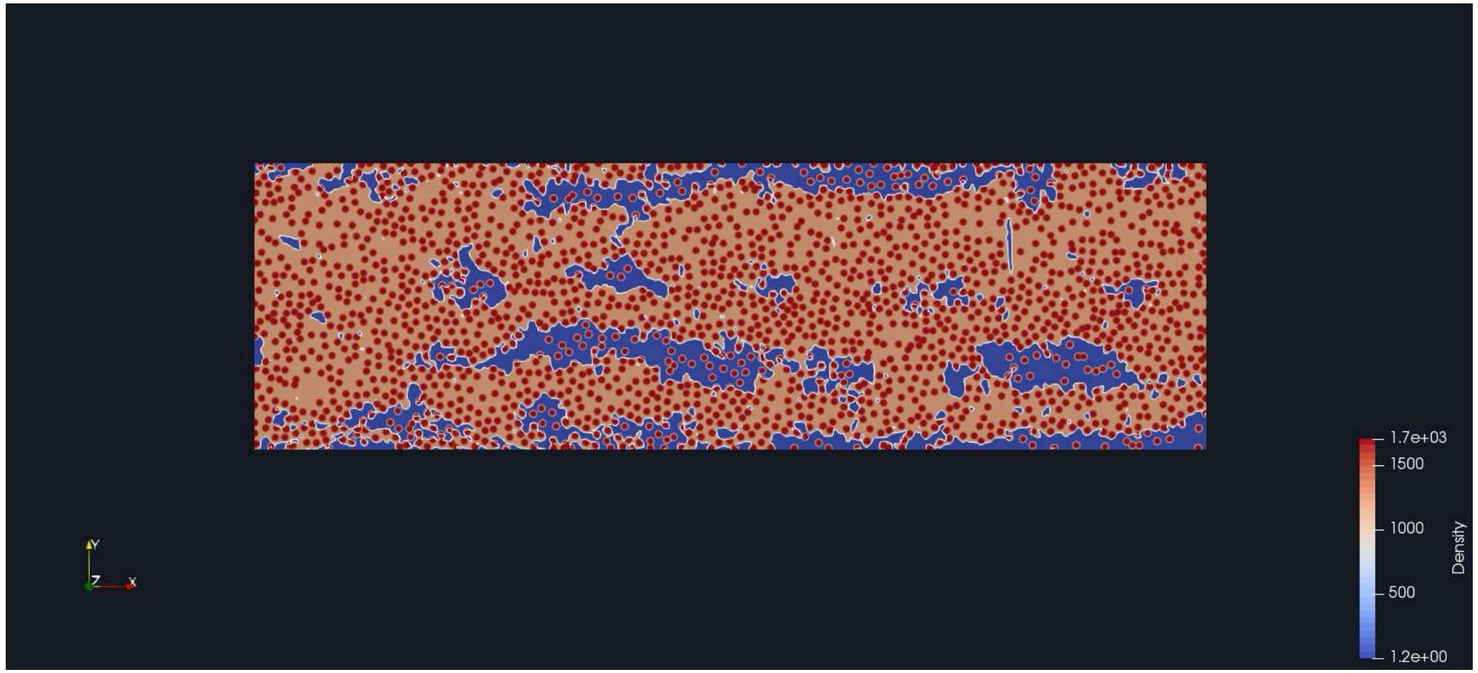
		Case 1: $5s^{-1}$, 14%	
Case 4: $1s^{-1}$, 18%	**Case 5:** $3s^{-1}$, 18%	**Case 2:** $5s^{-1}$, 18%	**Case 6:** $7s^{-1}$, 18%
		Case 3: $5s^{-1}$, 22%	

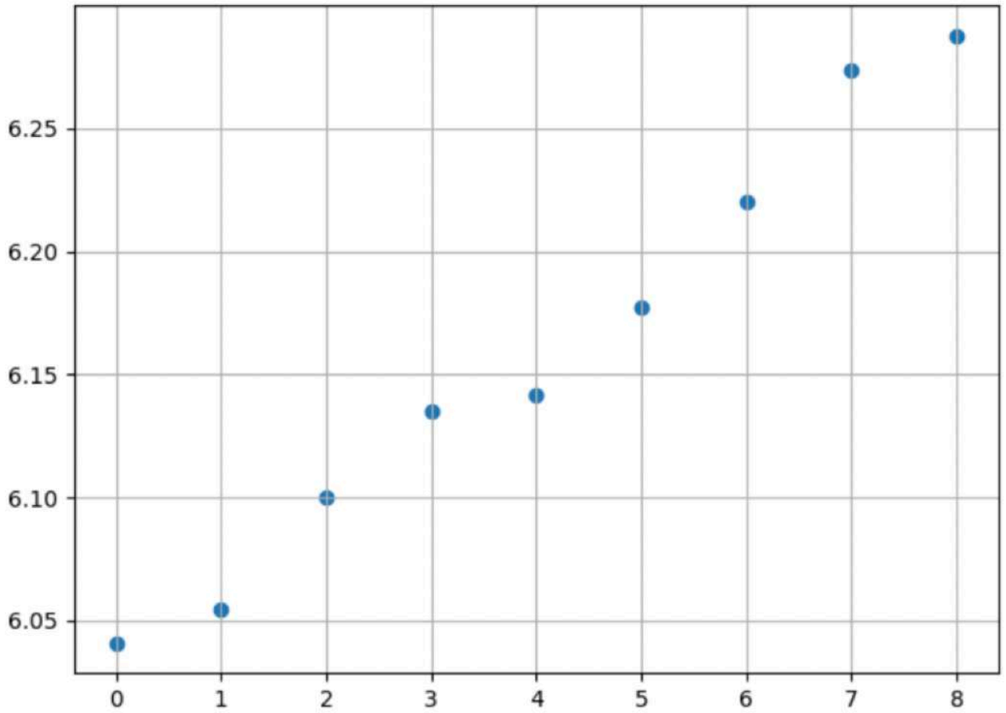
$\dot{\gamma}$

%



Rheology (II)

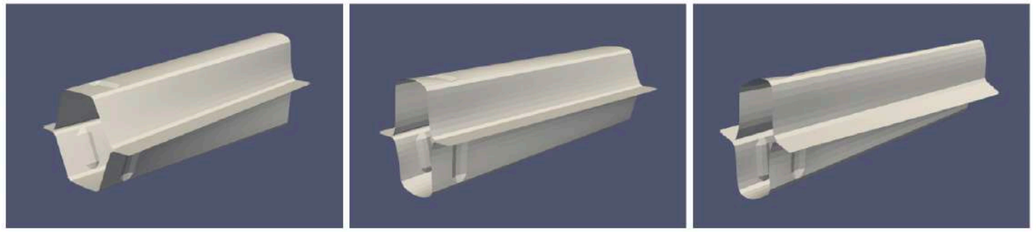

1.7e+03
1500
1000
500
1.2e+00
Density
Y
Z
X

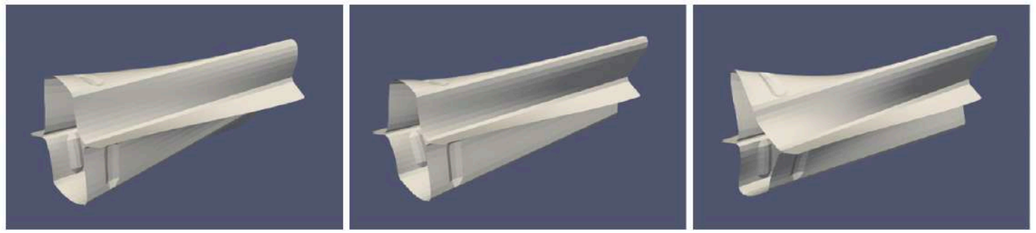

6.25
6.20
6.15
6.10
6.05
0
1
2
3
4
5
6
7
8

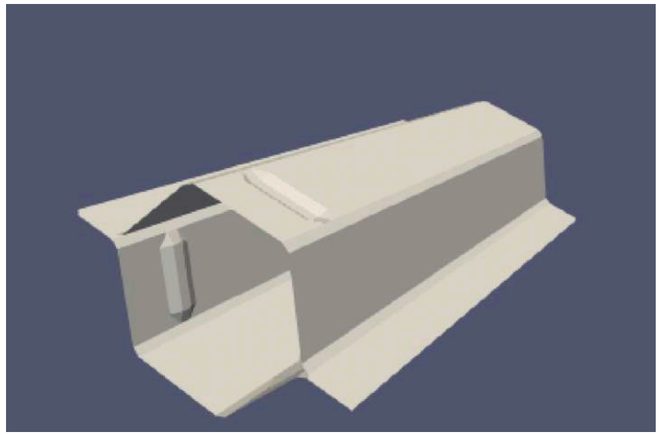
Parametric vibration modes clustering for advanced NVH

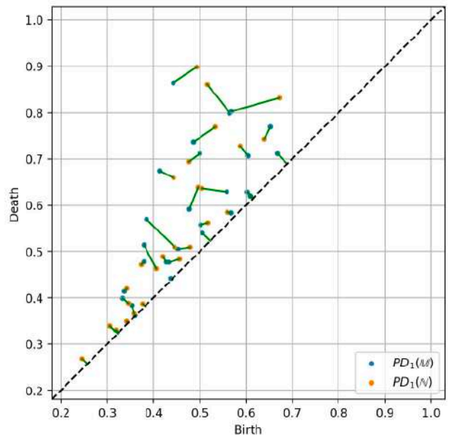
17 structure configurations: different thickness.

First 6th modes for a given structure thickness:









Use of alpha-filtration and Wasserstein distance (with respect to the undeformed geometry) for modes classification

Case	1st surf.	2nd surf.	3rd surf.	4th surf.	5th surf.	6th surf.
01	1	2	3	4	5	6
02	1	2	3	4	5	6
03	1	2	3	4	5	6
04	1	2	3	5	4	6
05	1	2	3	4	5	6
06	1	2	3	5	4	6
07	1	2	3	4	5	6
08	1	2	3	4	5	6
09	1	2	4	3	6	5
10	2	1	3	4	6	5
11	1	2	3	4	5	6
12	1	2	3	4	5	6
13	1	2	3	5	4	6
14	1	2	4	3	6	5
15	1	2	4	3	5	6
16	1	2	3	4	5	6
17	1	2	3	4	6	5

REFERENCES

- Nonlinear regression operating on microstructures described from Topological Data Analysis for the real-time prediction of effective properties. M. Yun, C. Argerich, E. Cueto, J.L. Duval, F. Chinesta. Materials, 13/10, 2335, 2020.
- Tape Surfaces Characterization with Persistence Images. T. Frahi, M. Yun, C. Argerich, A. Falco, F. Chinesta. AIMS Materials Science, 7/4, 364-380, 2020
- Empowering Advanced Driver-Assistance Systems from Topological Data Analysis. T. Frahi, F. Chinesta, A. Falco, A. Badias, E. Cueto, H.Y. Choi, M. Han, J.L. Duval. Mathematics.
- Empowering Advanced Parametric Modes Clustering from Topological Data Analysis. T. Frahi, A. Falco, B. Vinh Mau, J.L Duval, F. Chinesta. Applied Sciences.
- Monitoring weeder robots and anticipating their functioning by using advanced topological data analysis. T. Frahi, A. Sancarlos, M. Galle, X. Beaulieu, A. Chambard, A. Falco, E. Cueto, F. Chinesta. Frontiers in Artificial Intelligence.
- Study of concentrated short fibers suspensions in flows, using topological data analysis. R Mezher, J. Arayro, N. Hascoet, F. Chinesta. Entropy.