# De l'approximation des systèmes dissipatifs au VER en espace - temps et plus loin


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Deux anecdotes à propos

de l'approximation des systèmes dissipatifs au VER en espace - temps et plus loin

- 2 anecdotes
- 1 conjecture
- un peu de physique

→ vers un dictionnaire

hydrodynamics ≠ dynamique de fluide

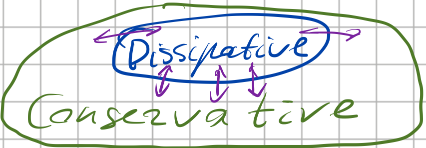
# What people do with dissipation

- Analytical description

Lagrangian → Variational calculus → EoM

I Approximation
- → trajectories
- → Lagrangian
- → effective behaviour

II Closure



## I | Approximation example

(Bersani, Caressa, dell'Isola)

$$\ddot{x} + \gamma \dot{x} + \omega^2 x = 0$$

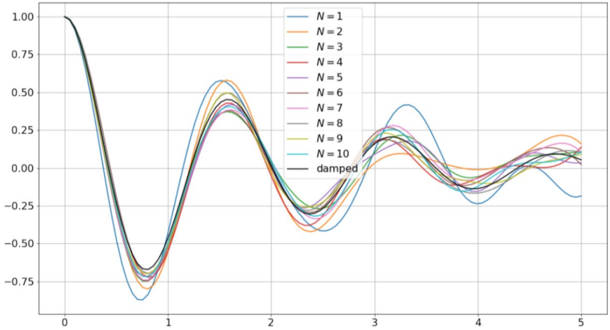
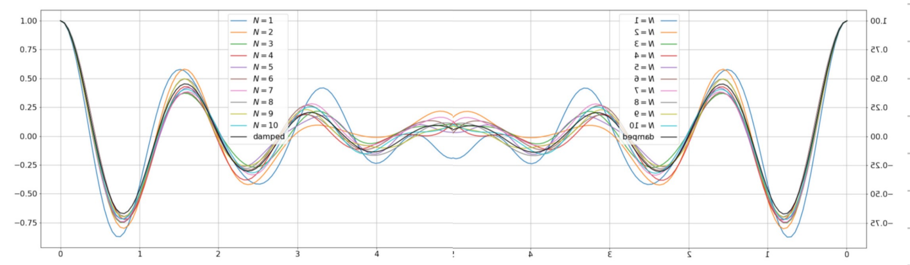
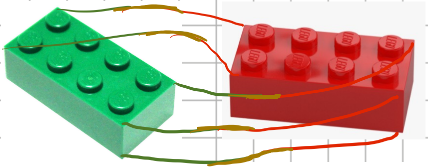



**Figure 3.** Approximation of a damped harmonic motion via an undamped one coupled with other $2N$ harmonic oscillators with different frequencies.

# Explanation — Fourier...

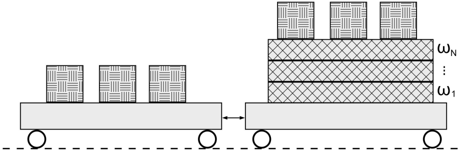



# Applications (AKA anecdotes)



→ Representative Space-time Volume Element

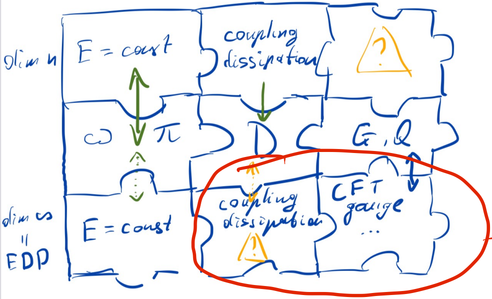
→ Damping

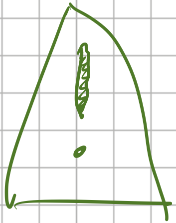


# II | Ideas for closure

GdR GDM 2021

## Instead of conclusion – big puzzle and questions



And what precisely about mechanics? What phenomena?

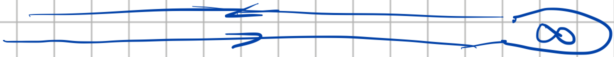
→ Thermo de Souriau (resp. Hamdouni, de Saxcé)

→ Calcul de Vinogradov (resp. Roubtsov)

# Feynman & Schwinger-Keldysh Path integral

$$I_F(J) := \langle Vac_{t=\infty} | U(J) | Vac_{t=-\infty} \rangle$$

$$I_{SK}(J_R, J_L) := Tr\{ U(J_R)\, \rho_0\, U^\dagger(J_L) \}$$



# Application (rather direct)

# Density function theory

Feynman
↳ DFT

Keldysh
↳ TD-DFT

OC-VS Conjecture
quasi-classical limit

ELSEVIER

Advances in Quantum Chemistry
Volume 21, 1990, Pages 293-302

## Integral Formulation of Density-Functional Theory

Weitao Yang

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The Hohenberg-Kohn-Sham density-functional theory is reformulated in terms of explicit relations between the electron density and the effective potential through the use of Feynman path integrals. In this formulation electron density is the only basic variable as in the Thomas-Fermi theory and orbitals are not needed. Possible applications to calculations in large molecules and the present limitations of the method are discussed.

# Some features

- Doubling

$$\mathcal{H} = \mathcal{H}_R \otimes \mathcal{H}_L$$

$$S_{SK} = S[\varphi_R] - S[\varphi_L]$$

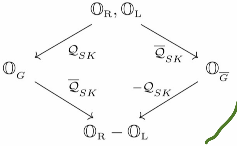
- Topological limit $\quad J_R, J_L \to J$

Doubling should be extended

**Keldysh supercharges** $\mathcal{Q}_{SK}$ and $\overline{\mathcal{Q}}_{SK}$ which are mutually anti-commuting, Grassmann odd, nilpotent operators with zero fermion number. They are defined by the graded commutators:

$$\begin{aligned} &[\mathcal{Q}_{SK}, \mathbb{O}_\mathrm{L}]_\pm = [\mathcal{Q}_{SK}, \mathbb{O}_\mathrm{R}]_\pm = \mathbb{O}_G, \quad [\mathcal{Q}_{SK}, \mathbb{O}_G]_\pm = 0, \quad [\mathcal{Q}_{SK}, \mathbb{O}_{\overline{G}}]_\pm = -(\mathbb{O}_\mathrm{R} - \mathbb{O}_\mathrm{L}). \\ &[\overline{\mathcal{Q}}_{SK}, \mathbb{O}_\mathrm{L}]_\pm = [\overline{\mathcal{Q}}_{SK}, \mathbb{O}_\mathrm{R}]_\pm = \mathbb{O}_{\overline{G}}, \quad [\overline{\mathcal{Q}}_{SK}, \mathbb{O}_{\overline{G}}]_\pm = 0, \quad [\overline{\mathcal{Q}}_{SK}, \mathbb{O}_G]_\pm = (\mathbb{O}_\mathrm{R} - \mathbb{O}_\mathrm{L}). \end{aligned} \tag{2.7}$$

We resort to a simple notation where $[A, B]_\pm$ denotes a commutator if either $A$ or $B$ is Grassmann even and an anticommutator otherwise. This can be represented diagrammatically as:




→ BRST

→ Equivariant cohomology

# Langevin system

$$m\ddot{x} + \frac{\partial U}{\partial x} + \nu \Delta_\beta x = N$$

viscous drag (e.g. $\beta \frac{d}{dt}$)

Gaussian noise

Keldysh: $x \to x_R, x_L$

$$x := -i\Delta_\beta^{-1}(x_R - e^{-i\delta_\beta \partial} x_L)$$

$$f_\psi := x_R - x_L$$

$$E_x := -m\ddot{x} - \frac{\partial U}{\partial x} - \nu \Delta_\beta x$$

$$L_{SK} = f_\psi E_x + i\nu f_\psi^2 + \bar{\psi}\left(\frac{\delta E_x}{\delta x}\right)\psi$$

Equivariant description

SK quadruplet $\{x, \psi, \bar{\psi}, f_\psi\}$. The action of SK supercharges

$$[\mathcal{Q}_{SK}, x]_\pm = \psi, \quad [\mathcal{Q}_{SK}, \psi]_\pm = 0, \quad [\mathcal{Q}_{SK}, \bar{\psi}]_\pm = -f_\psi, \quad [\mathcal{Q}_{SK}, f_\psi]_\pm = 0,$$
$$[\overline{\mathcal{Q}}_{SK}, x]_\pm = \bar{\psi}, \quad [\overline{\mathcal{Q}}_{SK}, \bar{\psi}]_\pm = 0, \quad [\overline{\mathcal{Q}}_{SK}, \psi]_\pm = f_\psi, \quad [\overline{\mathcal{Q}}_{SK}, f_\psi]_\pm = 0.$$

The KMS supercharges act on this basic multiplet as

$$[\mathcal{Q}_{KMS}, x]_\pm = 0, \quad [\mathcal{Q}_{KMS}, \psi]_\pm = 0, \quad [\mathcal{Q}_{KMS}, \bar{\psi}]_\pm = -i\Delta_\beta x, \quad [\mathcal{Q}_{KMS}, f_\psi]_\pm = -i\Delta_\beta \psi,$$
$$[\overline{\mathcal{Q}}_{KMS}, x]_\pm = 0, \quad [\overline{\mathcal{Q}}_{KMS}, \bar{\psi}]_\pm = 0, \quad [\overline{\mathcal{Q}}_{KMS}, \psi]_\pm = i\Delta_\beta x, \quad [\overline{\mathcal{Q}}_{KMS}, f_\psi]_\pm = -i\Delta_\beta \bar{\psi}.$$

Finally, we have the bosonic generator

$$[\mathcal{Q}_0, x]_\pm = 0, \quad [\mathcal{Q}_0, \psi]_\pm = 0, \quad [\mathcal{Q}_0, \bar{\psi}]_\pm = 0, \quad [\mathcal{Q}_0, f_\psi]_\pm = -i\Delta_\beta x.$$

The four non-local charges $\{\mathcal{Q}_0, \mathcal{Q}_{KMS}, \overline{\mathcal{Q}}_{KMS}, i\Delta_\beta\}$ form a Schwinger-Keldysh quartet of thermal (super-)translations:

$$[\mathcal{Q}_{SK}, \mathcal{Q}_0]_\pm = \mathcal{Q}_{KMS}, \quad [\overline{\mathcal{Q}}_{SK}, \mathcal{Q}_0]_\pm = \overline{\mathcal{Q}}_{KMS}, \quad [\mathcal{Q}_{SK}, \overline{\mathcal{Q}}_{KMS}]_\pm = -[\overline{\mathcal{Q}}_{SK}, \mathcal{Q}_{KMS}]_\pm = i\Delta_\beta,$$

— BRST

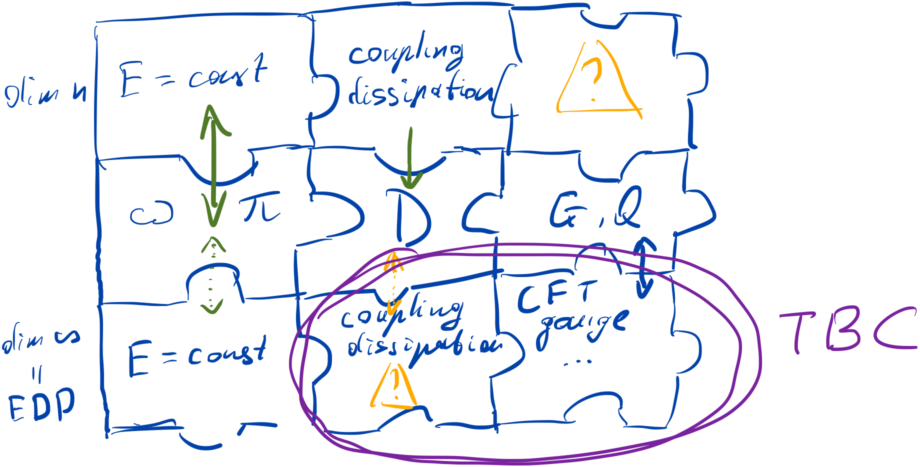
Faddeev-Popov

$$[\mathcal{Q}, x]_\pm = \psi, \qquad [\mathcal{Q}, \bar{\psi}]_\pm = -f_\psi + \phi_0 \Delta_\beta x,$$
$$[\mathcal{Q}, \psi]_\pm = \phi \Delta_\beta x, \quad [\mathcal{Q}, f_\psi]_\pm = \phi_0 \Delta_\beta \psi - \phi \Delta_\beta \bar{\psi} + \eta \Delta_\beta x,$$
$$[\overline{\mathcal{Q}}, x]_\pm = \bar{\psi}, \qquad [\overline{\mathcal{Q}}, \bar{\psi}]_\pm = \bar{\phi} \Delta_\beta x,$$
$$[\overline{\mathcal{Q}}, \psi]_\pm = f_\psi, \qquad [\overline{\mathcal{Q}}, f_\psi]_\pm = \bar{\phi} \Delta_\beta \psi,$$

— Cartan model of equivariant cohomology

$$L_{SK} = \left[\bar{Q}, \left[Q, \frac{m}{2}(\Delta_\beta x)^2 - U(x) - i\bar{\psi}\psi\right]\right]\Bigg|_{\substack{\varphi=\eta=\bar{\eta}=\bar{\varphi}=0 \\ \varphi_0=-i}}$$

# Instead of conclusion – big puzzle and questions

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Vielmols merci! E güeter!