Masses négatives

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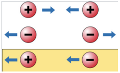
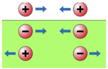
Negative masses

- Hermann BONDI (1957) : runaway motion

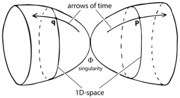


William BONNOR(1964, 1989)
"I regard the runaway (or self-accelerating) motion [. . .] so preposterous that I prefer to rule it out by supposing that inertial mass is all positive or all negative"

- Alternative candidates : theories of 2 parallel universes

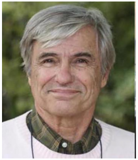


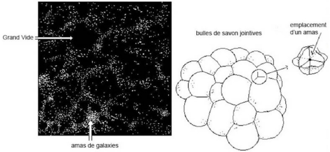
- Bimetric cosmological models
Nathan ROSEN
Mordehai MILGRON : bimetric theory MOND
Andreï SAKHAROV (1967)



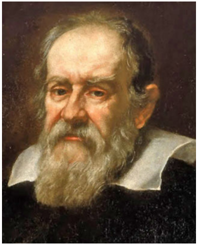
Recent bimetric models

- Frédéric HENRY-COUANNIER : dark gravity
  Sabine HOSSENFELDER, Thibault DAMOUR, Mohammed AL-FADHLI
- Jean-Pierre PETIT : JANUS model





Galilean and Newtonian Mechanics



- An **event** $\boldsymbol{X}$ of the space-time $\mathcal{M}$ is represented in a local chart by $X = \begin{pmatrix} t \\ x \end{pmatrix}$

  The 4-**velocity** $\overrightarrow{\boldsymbol{U}}$ is represented by $U = \frac{dX}{dt} = \frac{d}{dt}\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} 1 \\ v \end{pmatrix}$

- The **Galilean transformations** leave invariant the durations, the distances and the Uniform Straight Motion then they are affine with a linear part of the form :

  $P = \begin{pmatrix} 1 & 0 \\ u & R \end{pmatrix}$ where $u \in \mathbb{R}^3$ is the Galilean boost and R is a rotation.

  Their set is Galilei group

Theorem

$X \mapsto X'$ is such that $\partial X' / \partial X$ is Galilean **iff** $x' = (R(t))^T (x - x_0(t)), \qquad t' = t + \tau_0$

- The relation between local charts :
  $X \sim X' \Leftrightarrow X$ and X' are deduced one from each other by such a transition map is an equivalence relation

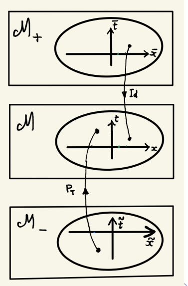
- An equivalence class under $\sim$ is called a **Galilean atlas** and its elements are called **Galilean charts** or Galilean coordinate systems or Galilean reference frames

Bi-matter space-time as a covering

- Negative masses are positive masses that **go back in time**
- We introduce a **covering** $\boldsymbol{\pi} : \mathcal{C} \mapsto \mathcal{M}$ with two disjoint sheets $\mathcal{M}_+$ and $\mathcal{M}_-$ called **sectors**, each of which populated with particles with or without mass
- **Censorship :** These two populations do not interact directly. In particular, photons emitted by particles of one sector cannot be received by particles of the other sector
- The only interaction between massive particles of both sectors is through the gravitation, a connection of $\mathcal{M}$ that is, as it were, the **theater of operations**

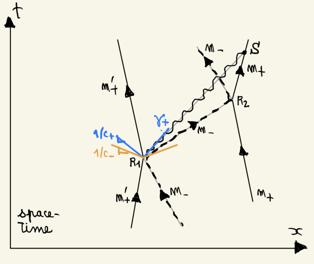
We claim that :

- $\mathcal{M}$, $\mathcal{M}_+$ and $\mathcal{M}_-$ are equipped respectively with **Galilean atlases** $\mathcal{A}$, $\mathcal{A}_+$ and $\mathcal{A}_-$
- For every local chart X of $\mathcal{A}$, there exists a local chart $\bar{X}$ of $\mathcal{A}_+$ such that the projection $\boldsymbol{\pi}$ is represented in these charts by the **identity** : $\qquad t = \bar{t}, \qquad x = \bar{x}$
- Likewise, for every local chart X of $\mathcal{A}$, there exists a local chart $\tilde{X}$ of $\mathcal{A}_-$ such that the projection $\boldsymbol{\pi}$ is represented in these charts by the **T-reversal** P_T : $\qquad t = -\tilde{t}, \qquad x = \tilde{x}$



Gravitation field equations

- In absence of massive particles, the space-time is flat and endowed with Minkowski metric $\eta = \begin{pmatrix} c^2 & 0 \\ 0 & -I \end{pmatrix}$
- **Why the speed of the light is chosen identical in both sectors ?**



In doing so, we avoid paradoxical situations such this :

If the speed of light is smaller in a sector —let say the positive one—, then, seen from this sector, a particle of the negative sector of larger velocity is a **tachyon**. According to relativity, it violates **causality**.

Gravitation field equations

- In the approximation of a **weak gravitational field**, the space-time is curved and the metric becomes

$$G = \eta + h = \begin{pmatrix} c^2 + 2\,\phi & 0 \\ 0 & -I \end{pmatrix} = \begin{pmatrix} c^2 & 0 \\ 0 & -I \end{pmatrix} + \begin{pmatrix} 2\,\phi & 0 \\ 0 & 0 \end{pmatrix}$$

and the gravity $g = -grad\,\phi$ satisfies the field equation $\Delta\,\phi = 4\,\pi\,k_N\,\rho$

- The 4-vector of **mass flux** $\overrightarrow{N}$ is obtained by push-forward

$$\overrightarrow{\boldsymbol{N}} = \boldsymbol{\pi}_*(\overrightarrow{\boldsymbol{N}}_+) + \boldsymbol{\pi}_*(\overrightarrow{\boldsymbol{N}}_-) = \boldsymbol{\pi}_*(\rho_+\,\overrightarrow{\boldsymbol{U}}_+) + \boldsymbol{\pi}_*(\rho_-\,\overrightarrow{\boldsymbol{U}}_-)$$

- For the positive sector, $N_+ = \pi_*(\bar{N}_+) = \pi_* \begin{pmatrix} \rho_+ \\ \rho_+ v_+ \end{pmatrix} = \begin{pmatrix} \rho_+ \\ \rho_+ v_+ \end{pmatrix}$ where $\rho_+ \geq 0$
- For the negative one, $N_- = \pi_*(\tilde{N}_-) = P_T \begin{pmatrix} \rho_- \\ \rho_- v_- \end{pmatrix} = \begin{pmatrix} -\rho_- \\ \rho_- v_- \end{pmatrix}$ where $\rho_- \geq 0$

Gravitation field equations

- The total flux of mass is represented in the local chart X by

$$N = N_+ + N_- = \begin{pmatrix} \rho_+ - \rho_- \\ \rho_+ v_+ + \rho_- v_- \end{pmatrix}$$

- The solution of the field equation is

$$\phi = \phi_+ + \phi_-, \qquad \phi_\pm = \mp \int \frac{k_N\, \rho_\pm(x', t)}{\| x - x' \|} \, \mathrm{d}^3 x'$$

- that leads to the decomposition of the gravity into

$$g = g_+ + g_-, \qquad g_\pm = -grad\, \phi_\pm = \mp \int \frac{k_N\, \rho_\pm(x', t)}{\| x - x' \|^2} \frac{x - x'}{\| x - x' \|} \, \mathrm{d}^3 x'$$

where, on a positive mass,
the effect of the acceleration g_+ is attractive and g_- is repulsive

Equations of motion

- In General Relativity, the **gravitation** is a connection.
  We call Galilean connections the symmetric connections associated to Galilei group.
  In a Galilean chart and in absence of Coriolis' effect, we have $\Gamma = \begin{pmatrix} 0 & 0 \\ -g\,dt & 0 \end{pmatrix}$
- the **linear 4-momentum** of a particle is $T = m\,U = m \begin{pmatrix} 1 \\ v \end{pmatrix} = \begin{pmatrix} m \\ p \end{pmatrix}$
- According to **Élie Cartan's** prescription, **the motion of a test-particle** is such that its linear 4-momentum is parallel-transported
  $\nabla_U T = \dot{T} + \Gamma(U)\,T = 0$
- For a particle of mass $m_+ > 0$ and velocity v in the positive sector
  $T = \pi_*(\bar{T}_+) = \pi_*(m_+ \bar{U}) = \begin{pmatrix} m_+ \\ m_+ v \end{pmatrix}$
  The equation of motion reads $m_+ \dot{v} = m_+ g$ then $\dot{v} = g_+ + g_-$
- For a particle of mass $m_- > 0$ and velocity v in the negative sector
  $T = \pi_*(\tilde{T}_-) = \pi_*(m_- \tilde{U}) = P_T \tilde{T}_- = \begin{pmatrix} -m_- \\ m_- v \end{pmatrix}$
  The equation of motion reads $m_- \dot{v} = -m_- g$ then $\dot{v} = -g_+ - g_-$
- **Anyway, the masses of the same sign attract each other (Newton's law), while masses of opposite signs repel each other (anti-Newton's law)**

The asymmetry between both sectors, a means to know in which sector we are living

- The most general form of a Galilean connection is

$$\Gamma = \begin{pmatrix} 0 & 0 \\ j(\Omega)\, d\,x - g\, d\,t & j(\Omega)\, d\,t \end{pmatrix}$$

  where $j(\Omega)$ is the skew-symmetric matrix
  associated to the 3-column vector Ω representing Coriolis' effects

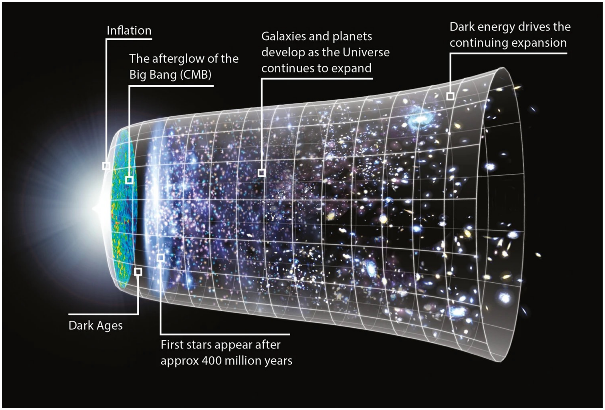
- In a Galilean chart, the equation of motion itemizes
  $\dot{m} = 0, \qquad \dot{p} = m\,(g - \Omega \times v) - \Omega \times p = 0$

- In the positive sector, it leads to $m_+ \dot{v} = m_+ (g - 2\,\Omega \times v)$
  then a particle of the positive sector is **sensitive** to Coriolis' force

- In the negative sector, it gives $m_- \dot{v} = -m_- g$
  then a particle of the negative sector is **insensitive** to Coriolis' effect

- **As, according to what the observations show, our mater is sensitive to Coriolis' effect, then we are living in the positive sector**

Cosmological model



The standard model

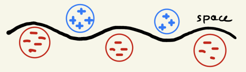
- Friedmann-Lemaître-Robertson-Walker (FLRW) metric $\boldsymbol{G}$ given in the comoving coordinates x^i and time t by $ds^2 = dt^2 - a^2(t)\,\gamma_{ij}(x)\,dx^i dx^j = dt^2 - a^2\left[\frac{dr^2}{1-k\,r^2} + r^2(d\vartheta^2 + \sin^2\vartheta^2 d\varphi^2)\right]$ k is the curvature parameter, a the scale factor and $H = \dot{a}/a$ the Hubble parameter
- The 4-velocity $\vec{\boldsymbol{U}}$ is such that $\boldsymbol{G}(\vec{\boldsymbol{U}}, \vec{\boldsymbol{U}}) = 1$. In the local chart of a comoving frame $U = \begin{pmatrix} \lambda_v \\ \lambda_v v \end{pmatrix}$ where $\parallel v \parallel_\gamma^2 = a^2 \gamma_{ij} v^i v^j$ and $\lambda_v = 1/\sqrt{1 - \parallel v \parallel_\gamma^2}$.
- The connection matrix reads $\Gamma(dX) = \begin{pmatrix} 0 & H\,a^2 dx^T \gamma \\ H\,dx & H\,I\,dt + \Gamma_s(dx) \end{pmatrix}$

Covering

- Likewise the Galilean limit case, we modelize two species of mater by a covering $\boldsymbol{\pi} : \mathcal{C} \mapsto \mathcal{M}$ with two disjoint sheets (or sectors) $\mathcal{M}_+$ and $\mathcal{M}_-$
- $\mathcal{M}$, $\mathcal{M}_+$ and $\mathcal{M}_-$ are equipped respectively with **atlases of comoving frames** $\mathcal{A}$, $\mathcal{A}_+$ and $\mathcal{A}_-$
- We define $\boldsymbol{\pi}$ in local charts by the identity and the T-reversal

Gravitation field equations

- Stress-energy tensor of a perfect fluid $T_{\alpha\beta} = (\rho + P)\, U_\alpha U_\beta - P\, G_{\alpha\beta}$
- Einstein equations $R_{\alpha\beta} - \frac{1}{2} R^\mu_\mu G_{\alpha\beta} = \Lambda\, G_{\alpha\beta} + 8\,\pi\, k_N\, T_{\alpha\beta}$
- The observation data suggests that the 3D space is flat ($k = 0$)



Because of the repellent effect, the positive and negative masses are clumped within distinct regions of the space, then the curvature is oscilating around zero and the mean value is very small at larger scale

- The ΛCDM is a parameterization of the Big Bang model
$H^2(a) = H_0^2 \left[\Omega_{b,0} \left(\frac{a_0}{a}\right)^3 + \Omega_{c,0} \left(\frac{a_0}{a}\right)^3 + \Omega_{\Lambda,0} \right]$
in which the universe contains today 3 major components :
    - ordinary matter (baryons) : $\Omega_{b,0} = 0.0486$
    - cold dark matter (CDM) : $\Omega_{c,0} = 0.2589$
    - cosmological constant (dark energy) : $\Omega_{\Lambda,0} = 0.6911$

Equation of motion

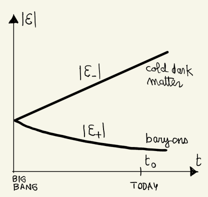
- We do not repeat details about the push-forward of the 4-momentum of particles of the two sectors
- In a nutshell, the 4-momentum $\vec{T}$ of a particle of mass $m = m_\pm$ at rest in the comoving frame is represented by $T = m\,U = \begin{pmatrix} E \\ p \end{pmatrix} = \begin{pmatrix} \pm\lambda_v m \\ \lambda_v m\, v \end{pmatrix}$ such that $E^2 - \parallel v \parallel_\gamma^2 = m^2$ and $p = \pm\, E\, v$
- The equation of motion $\nabla_U T = 0$ gives
  $\dot{E} + H\lambda_v a^2 v \cdot (\gamma\, p) = 0, \qquad \dot{p} + H\,\lambda_v (E\,v + p) = 0$

Qualitative analysis of the dynamics of both sectors :

- **Sector** $\mathcal{M}_+$ **:** $E = E_+ > 0$ and $p = E_+ v$
  $\dot{p} = -2\,H\,\lambda_v p \quad \Rightarrow \quad d/dt(1/2 \parallel p \parallel^2) = -2\,H\,\lambda_v \parallel p \parallel^2 < 0$
  and $\dot{E}_+ = -H\, m_+^{-1} \parallel p \parallel_\gamma^2 < 0$
  $E_+ > 0$ decreases, and if we neglect the time-variation of a and H, with a gradual reduction of the decline
- **Sector** $\mathcal{M}_-$ **:** $E = E_- < 0$ and $p = -E_- v$
  $\dot{p} = 0 \quad \Rightarrow \quad p$ remains constant
  $\dot{E}_+ = -H\, m_-^{-1} \parallel p \parallel_\gamma^2 < 0$
  $E_- < 0$ decreases and its magnitude $\mid E_- \mid$ increases with a rather constant growth

The dark matter

- Thanks to this cosmological model, we can develop
  a **scenario** able to explain the existence of the dark matter
- The most natural way to imagine the early universe is that at the Big Bang there was the **same number** of particles of equal and opposite mass
- **Hypothesis :** the cold dark matter is the matter of the sector of negative masses



- According to the qualitative analysis,
  the total energy $\mid \mathcal{E}_- \mid$ of negative masses increases at a constant growth rate
  while the total energy $\mid \mathcal{E}_+ \mid$ of positive masses decreases significantly
- leading to a long-term domination in energy of the sector of the cold dark matter over the baryons, as observed today

Conclusions

The gravitational interaction **violates the T-reversal symmetry** in two ways :

- The positive masses are sensitive to Coriolis' effect
  while the negative masses are not so.
- The magnitude of the energy of the positive masses decreases
  while the one of the negative masses increases.

Consequences :

- The first violation shows that we are leaving in the sector of positive masses
- The second one provides a credible scenario to explain the domination today of the energy of the cold dark matter over the baryons

Perspectives :

- This script seems to correspond to the data
  of the cosmological observations at least qualitatively

  and should be refined by integrating the equations numerically
  with the expectation to fit the observations

MERCI !