Continuum mechanics and weak field general relativity: what are the realities and limits of the space elastic behaviour?

par David Izabel

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1) Continuum mechanics and weak field general relativity: what are the realities and limits of the space elastic behaviour?

- The different approaches to the state of the art
- Beam-and-plate theory and analogy with the structure of general relativity
- From a tensor point of view:
- Deformation tensor/perturbation tensor of the metric
- Stress Tensor/Stress Energy Tensor
- General form of the theory of relativity / Hooke's law

2) Deformations of space in the case of gravitational waves

- Perturbation theory, linearized equation of Einstein, solution of this equation, order of magnitude of deformations measured by Ligo/Virgo
- Analogy between mechanical torsion of space and polarization of gravitational waves: consequences on G
- An equivalent Young's modulus of the elastic medium space
- Possible mechanical characteristics hidden in Einstein's constant κ
- Necessity to have an anisotropic model of space to be in accordance with the Poisson's ratio ν=1

3) Limits of the analogy between MMC /RG - Questioning of continuum mechanics in relation to these deformations of space

- Space as an anisotropic medium on a small scale?
- How are the deformations of space transmitted from one plane to another during the propagation of gravitational waves? Local plasticization of an equivalent crystalline medium?
- If the RG has to be modified, the modification must be very small: is geometric torsion a good candidate?
- The contribution of defect theory and its analogy with Einstein Cartan's general relativity, a way to explain the propagation of gravitational waves in space? a local dislocation of the medium?
- Are the polarizations of gravitational waves in the case of GR with torsion a means of supplementing the tensor of plane strains of the space medium observed in the case of GR without geometrical torsion?

4) Conclusion

1) Continuum mechanics and weak field general relativity: what are the realities and limits of the space elastic behaviour?

- The different approaches to the state of the art
- Beam-and-plate theory and analogy with the structure of general relativity
- From a tensor point of view:
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  - Stress Tensor/Stress-Energy Tensor
  - General form of the theory of relativity / Hooke's law

State of the art : T. Damour: why an elastic analogic space-time material?space-time like jelly

Source T.Damour presentation

T.D "Spacetime is **an elastic structure** that is distorted by the presence within it of mass-energy"

T.D "Space = jelly"

DI: So This « jelly » is the equivalent analogic elastic material fulling space time

State of the art : other approaches to study the analogy between general relativity and theory of elasticity

Two main approaches

Solution 1

Start from the elasticity equations in 3 dimensions and transform them into 4 dimensions Hooke’s law

Solution 2

Start from the general relativity equation and add terms or transform the equations to cover elasticity behaviour

Example of solution 1: Elasticity 3d => 4d

Let be a material point of a coordinate material (x_1, x_2, x_3) ;

Let be the position of this material point after a displacement (x'_1, x'_2, x'_3) ;

Such as:

$$x'_i = x_i + u_i$$

With for the strain vector (or relative displacement):

$$u_i = u_i(t, \vec{x}) = u_i(t, x_1, x_2, x_3)$$

The nonlinear strain tensor is written:

$$u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} + \frac{\partial u_l}{\partial x_i}\frac{\partial u_l}{\partial x_k}\right)$$

The interval between two material points after deformation is then written:

$$dl'^2 = dl^2 + 2u_{ik}dx_i dx_k \equiv C_{ik}dx^i dx^k$$

With:

$$dl'^2 \equiv \sqrt{dx'^2_1 + dx'^2_2 + dx'^2_3}$$

$$dl^2 \equiv \sqrt{dx_1^2 + dx_2^2 + dx_3^2}$$

$$C_{ik} \equiv \delta_{ik} + 2u_{ik}$$

This leads the authors to the following metric constructed from the Lamé coefficients:

[178] Cherubini, C.; Filippi, S. (2015) «An Analog of Einstein's General Relativity Emerging from Classical Finite Elasticity Theory: Analytical and Computational Issues»

$$g_{\mu\nu} = \begin{bmatrix} \frac{1}{\rho_R^{(0)}} & 0 & 0 & 0 \\ 0 & -\frac{2}{(2\mu+\lambda)}\frac{1}{\Xi^{(0)}} & 0 & 0 \\ 0 & 0 & -\frac{1}{K}\sqrt{\frac{\rho_R^{(0)}(2\mu+\lambda)\Xi^{(0)}}{2K}} & 0 \\ 0 & 0 & 0 & -\frac{1}{K}\sqrt{\frac{\rho_R^{(0)}(2\mu+\lambda)\Xi^{(0)}}{2K}} \end{bmatrix}$$

An Analog of Einstein's General Relativity Emerging from Classical Finite Elasticity Theory: Analytical and Computational Issues

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Abstract. The "analogue gravity formalism", an interdisciplinary theoretical scheme developed in the past for studying several non relativistic classical and quantum systems through effective relativistic curved space-times, is here applied to largely deformable elastic bodies described by the nonlinear theory of solid mechanics. Assuming the simplest nonlinear constitutive relation for the elastic material given by a Kirchhoff-St Venant strain-energy density function, it is possible to write for the perturbations an effective space-time metric if the deformation is purely longitudinal and depends on one spatial coordinate only. Theoretical and numerical studies of the corresponding dynamics are performed in selected cases and physical implications of the results obtained are finally discussed.

Example of solution 2: GR => Elasticity compatibility

In **[62], [179]** and **[183]** it is proposed to add to the momentum energy tensor and elastic strain density tensor of spacetime:

$$G_{\mu\nu} = T_{e\mu\nu} + \kappa T_{\mu\nu}$$

Elastic potential energy (space strained)

Where $T_{e\mu\nu}$ is the energy/effective momentum tensor obtained from the "elastic" potential energy term in the Lagrangian

In **[65]** and **[128]** the same approach is carried out by completing the momentum energy tensor:

$$R_{\mu}^{\nu} = \frac{8\pi G}{c^4}\left[T_{\mu}^{\nu} + \sigma_{\mu}^{\nu} - \frac{1}{2}\delta_{\mu}^{\nu}(T+\sigma)\right]$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}\left[T_{\mu\nu} + \sigma_{\mu\nu}\right]$$

[62] A. Tartaglia (1995), Four Dimensional Elasticity and General Relativity (Springer, Boston).

[65] M. Beau (2015), Ann. Fond. Louis Broglie 40 1.1850083-21 « On the nature of space-time, cosmological inflation, and expansion of the universe »

[128] M. R. Beau (2014) « Théorie des champs des contraintes et des déformations en relativité générale et expansion cosmologique ». - Foundations of Physics manuscript arXiv:1209.0611v2 p4 and Annales de la Fondation Louis de Broglie, Volume 40, 2015

[179] Levrino, Luca; Tartaglia, Angelo (2012) « From the elasticity theory to cosmology and vice versa »

[183] Tartaglia, Angelo; Radicella, Ninfa (2009) «From Elastic Continua To Space-time»

The beam theory of Timoshenko as a general relativity in one dimension

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https://doi.org/10.1007/s12043-020-01954-5

Cas of the Pure bending : M= cte

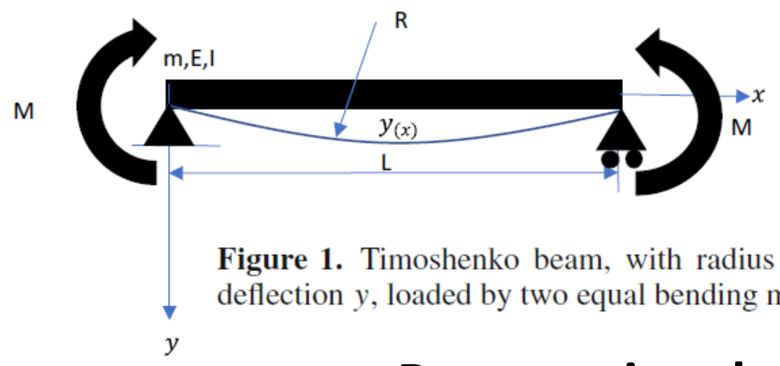


Figure 1. Timoshenko beam, with radius of curvature R, deflection y, loaded by two equal bending moments M.

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} = \frac{1}{R}$$

$$M_{(x)} = M = -\frac{EI}{R}$$

$$U = \frac{kgm}{s^2} \times m$$

By reporting the expression above in the energy bending expression

In static (deflection $y_{(x)}$ independent of the time), the fundamental relation connecting the curvature (1/R) at the bending moment $M_{(x)}$ and at the second derivative of the deflection $y_{(x)}$ can be written as following:

$$U = \frac{1}{2}\int_0^L \frac{(EI)^2}{R^2 EI} dx$$

Bending rigidity

$$\frac{1}{\frac{1}{m^2}\left(\frac{kgm}{s^2}\right)m^4} \times \frac{U}{m} = \frac{s^2}{kgm} \times \frac{U}{m^3}$$

As the curvature is constant, we obtain so:

$$U = \frac{1}{2}\frac{EIL}{R^2}$$

1/m²

Energy by unit of length

in the case of the pure bending, we obtain: :

Curvature

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{U}{L}\right)$$

In 1 D

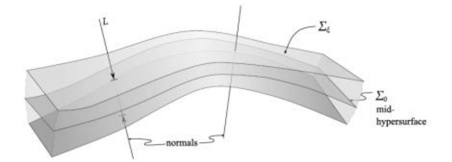
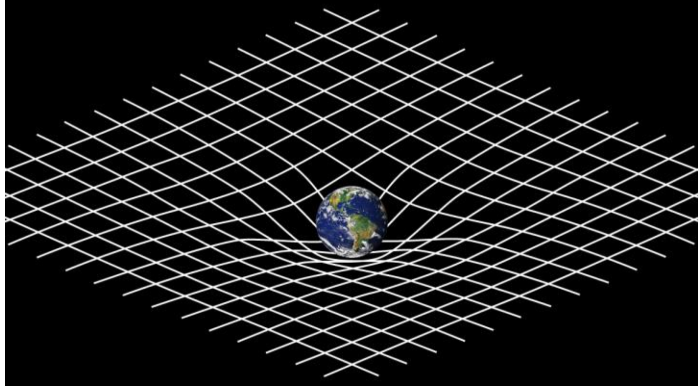


Fig. 4. The cosmic fabric is treated as a stack of three-dimensional hypersurfaces Σ_ξ each parameterized by $\xi \equiv x^4 = \text{const}$, and L is its thickness.

T. Tenev and M.F. Horstemeyer propose a Young's Modulus Y and Poisson's ratio ν of the space-time

The Timoshenko's plate theory

Relation energy curvature of a plate



Gravity can be thought of as the movements of particles through curved space-time. Credit: NASA

Solid Mechanics Perspective

Mid-hypersurface of a hyperplate called "cosmic fabric".

The world volume of the cosmic fabric's mid-hypersurface

Intrinsic curvature of the fabric's mid-hypersurface

Intrinsic curvature of the fabric's world volume

Volumetric strain ε^{3D}, such that $\varepsilon^{3D} = -\Phi/c^2$

Shear waves traveling at the speed of light

Matter induces prescribed strain causing the fabric to bend

Action integral outside of inclusions,

$$S = \frac{L^2 Y}{24} \int R\sqrt{|g|}\, d^4x$$

Elastic constants:

$Y = 6c^7/2\pi\hbar G^2, \quad \nu = 1$

$$U_{flexion} = \frac{D}{2}\int_0^a \int_0^b \left[\left(\frac{1}{R_x}\right)^2 + \left(\frac{1}{R_y}\right)^2 + 2(1-\nu)\left\{\left(\frac{1}{R_{xy}}\right)^2\right\} + 2\nu\left\{\frac{1}{R_x}\frac{1}{R_y}\right\}\right] dxdy$$

With for the bending rigidity D:

For a plate of width b = 1 m :

$$I = \frac{bh^3}{12} = \frac{1 \times h^3}{12}$$

Curvature

Area energy density

In 2 D

$$\left[\left(\frac{1}{R_x}\right)^2 + \left(\frac{1}{R_y}\right)^2 + 2(1-\nu)\left\{\left(\frac{1}{R_{xy}}\right)^2\right\} + 2\nu\left\{\frac{1}{R_x}\frac{1}{R_y}\right\}\right] = \frac{2(1-\nu^2)}{EI} \times \frac{U}{ab}$$

$1/m^2$

$$\frac{1}{\frac{1}{m^2}\left(\frac{kgm}{s^2}\right) m^4/m} \times \frac{U}{m^2} = \frac{s^2}{kgm} \times \frac{U}{m^3}$$

Curvature in general relativity

Curvature = Second derivative of the metric $g_{\mu\nu}$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \rightarrow R := g^{\alpha\beta} R_{\alpha\beta}$$

$$R_{\alpha\beta} = \frac{\partial \Gamma^{\mu}{}_{\alpha\beta}}{\partial x^{\mu}} - \frac{\partial \Gamma^{\mu}{}_{\alpha\mu}}{\partial x^{\beta}} + \Gamma^{\mu}{}_{\alpha\beta}\Gamma^{\nu}{}_{\mu\nu} - \Gamma^{\nu}{}_{\alpha\mu}\Gamma^{\mu}{}_{\nu\beta}$$

$$\Gamma^{\alpha}{}_{\mu\nu} := \frac{1}{2} g^{\alpha\sigma}\left(\frac{\partial g_{\sigma\nu}}{\partial x^{\mu}} + \frac{\partial g_{\mu\sigma}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}}\right)$$

In 4 D

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Curvature in plate theory

Curvature = Second derivative of the deflection w

$$\frac{1}{R_x} = \frac{\partial^2 w_{(x,y)}}{\partial x^2}$$

$$\frac{1}{R_y} = \frac{\partial^2 w_{(x,y)}}{\partial y^2}$$

$$\frac{1}{R_{xy}} = \frac{\partial^2 w_{(x,y)}}{\partial x \partial y}$$

In 2 D

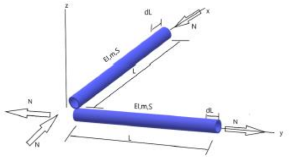
$$\frac{1}{z^2}\left\{\varepsilon_{xx}; \varepsilon_{yy}; \frac{\varepsilon_{xy}}{2}\right\}\begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 2(1-\nu) \end{pmatrix}\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \frac{\varepsilon_{xy}}{2} \end{pmatrix} = \frac{24(1-\nu^2)}{EI} \times \frac{dU}{dxdy}$$

Other expressions of the strength of material (normal effort and shear load added) Ligo/Virgo

Sollicitation	Relation between displacement, rotation and strain	Displacement, rotation	Expression of the energy
Bending moment M(x)	$\frac{d^2y}{dx^2} = -\frac{M}{EI} = \frac{1}{R}$	$y_{v(x)}$ Vertical deflection due to bending	$\frac{1}{R^2} = \frac{2}{EI} \times \frac{U}{L}$
Torque of twisting T(x)	$T = GI_t \frac{d\theta}{dx}$	$\theta_{(x)}$ Twisting rotation	$\frac{1}{R_T^2} = \frac{2}{GI_t} \times \frac{U}{L}$
Normal effort N(x)	$N = ES\frac{du}{dx} = ES\varepsilon = \sigma S$	$u_{(x)}$ Lengthening shortening (strain)	$\left(\frac{du}{dx}\right)^2 = \frac{2}{ES}\frac{U}{L}$
Shear force V(x)	$V = GS_r\frac{dy}{dx} = GS_r\gamma_{(x)}$	$\gamma_{(x)}$ Shear force distortion	$\left(\frac{dy}{dx}\right)^2 = \frac{2}{GS_r} \times \frac{U}{L}$

Measured in the Ligo/Virgo Arms

2 D = 2 x 1D



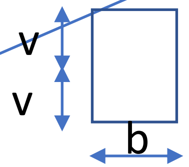
Mechanical conversion of κ: links with strength of material (Timoshenko's beam in bending at 1 dimension) (DI's Book and Pramana publication)=> **generalization Hooke's law at all the other solicitations: shear and tensile/compression N^{-1} like κ**

Solicitations	Curvature/energy formula	Dimensional equations	
Bending moment of $M_{(x)}$	$\frac{1}{R^2}=\frac{2}{EI}\times\frac{U}{L}=\frac{2}{YI}\times\frac{U}{L}$	$\frac{1}{\mathrm{m}^2}=\frac{\mathrm{s}^2}{\mathrm{kgm}^3}\times\frac{U}{\mathrm{m}}$	At 1 dimension
Twisting torque $T_{(x)}$	$\frac{1}{R_T^2}=\frac{2}{GI_t}\times\frac{U}{L}=\frac{2}{\mu I_t}\times\frac{U}{L}$	$\frac{1}{\mathrm{m}^2}=\frac{\mathrm{s}^2}{\mathrm{kgm}^3}\times\frac{U}{\mathrm{m}}$	At 1 dimension
Normal effort $N_{(x)}$	$\left(\frac{du}{dx}\right)^2=\frac{2}{ES}\times\frac{U}{L}=\frac{2}{YS}\times\frac{U}{L}$	$1=\frac{\mathrm{s}^2}{\mathrm{kgm}}\times\frac{U}{\mathrm{m}}$	At 1 dimension
Shear effort $V_{(x)}$	$\left(\frac{dy}{dx}\right)^2=\frac{2}{GS_r}\times\frac{U}{L}=\frac{2}{\mu S_r}\times\frac{U}{L}$	$1=\frac{\mathrm{s}^2}{\mathrm{kgm}}\times\frac{U}{\mathrm{m}}$	At 1 dimension
General relativity	$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+g_{\mu\nu}=-\frac{8\pi G}{c^4}T_{\mu\nu}$	$\frac{1}{\mathrm{m}^2}=\frac{\mathrm{s}^2}{\mathrm{kgm}}\times\frac{U}{\mathrm{m}^3}$	At 4 dimensions

Like κ

1D

Second derivative of the metric $g_{\mu\nu}$



I= $b(2v)^3/12$

$$U=\frac{1}{2}\int_0^L \frac{M^2}{EI}dx$$

$$\frac{d^2y}{dx^2}=\frac{M}{EI}=\frac{1}{R}$$

σ = M v /I

σ = ε E

EI

L

$$\frac{1}{R^2}=\left(\frac{d^2y}{dx^2}\right)^2=\left(\frac{M}{EI}\right)^2=\frac{\sigma^2}{E^2v^2}=\frac{\varepsilon^2}{v^2}=\frac{2}{EI}\frac{U}{L}$$

Beam theory: Relation curvature 1/R, strain ε, stress σ and energy density U/L in strength of material for a material of Young's modulus E =Y and inertia I

1) GR and simplified elasticity theory (beam in strength of material) shows compatibilities in terms of curvature and mechanic aspects
2) GR mixture of curvature and torsion

Summary : = equivalence between Timoshenko in static and Einstein

Theory	Formula curvature = K density of energy	Dimension	Unknown
Timoshenko (elastic beam in static)	$\left(\frac{\varepsilon_x}{z}\right)^2 = \frac{1}{R^2} = \frac{2}{EI}\frac{U}{L}$	1	$y_{(x)}$
Timoshenko (elastic plate in static)	$\left[\left(\frac{1}{R_x}\right)^2 + \left(\frac{1}{R_y}\right)^2 + 2(1-\nu)\left\{\left(\frac{1}{R_{xy}}\right)^2\right\} + 2\nu\left\{\frac{1}{R_x}\frac{1}{R_y}\right\}\right] = \frac{2(1-\nu^2)}{EI} \times \frac{U}{ab}$ $\frac{1}{z^2}\left[(\varepsilon_{xx})^2 + (\varepsilon_{yy})^2 + 2(1-\nu)\left\{\frac{1}{4}\varepsilon_{xy}\right\} + 2\nu\{\varepsilon_{xx}\varepsilon_{yy}\}\right] = \frac{24(1-\nu^2)}{EI} \times \frac{dU}{dxdy}$	2	$w_{(x,y)}$ ε_{xy} ε_{xx} ε_{yy}
Einstein (General relativity)	$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$	4	$g_{\mu\nu}$

Curvature (ε) = K x energy density

$1/m^2$

$s^2/(kg.m)=N^{-1}$

$J/m^3=U/V = (kgm^2/s^2) / m^3$

Parallelism strength of material and general relativity :mechanical conversion of κ

GR curvature is linked with 1/R²

Strength of material curvature

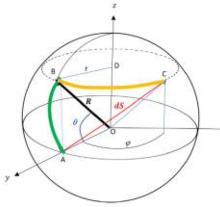
Strain energy density

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{W_{ext(total)}}{L}\right) = K\left(\frac{W_{ext(total)}}{L}\right) = \frac{2}{EI}\left(\frac{U}{L}\right) \quad (10d)$$

$$G^{\mu\nu} = -\frac{8\pi G}{c^4}T^{\mu\nu} = -\kappa\, T^{\mu\nu} = -\kappa\, M^{\mu\nu} \quad (10e)$$

Energy density

Mechanical parallelism



$$g^{\mu\nu} = \begin{bmatrix} \frac{1}{R^2} & 0 \\ 0 & \frac{1}{R^2\cos\theta^2} \end{bmatrix} \Rightarrow g^{\mu\nu} = \begin{bmatrix} g^{\theta\theta} & g^{\theta\varphi} \\ g^{\varphi\theta} & g^{\varphi\varphi} \end{bmatrix}$$

$$R = g^{\mu\nu}R_{\mu\nu} = g^{\theta\theta}R_{\theta\theta} + g^{\varphi\varphi}R_{\varphi\varphi} = \frac{1}{R^2}\times 1 + \frac{1}{R^2\cos\theta^2}\times\cos\theta^2 = \frac{2}{R^2}$$

GR applied at a sphere gives the curvature obtain for the beam 1/R²

	Curvature	= K	Energy density
Beam	$\left(\frac{du}{dx}\right)^2$	$= \frac{2}{ES}$	$\frac{U}{L}$
	$\left(\frac{dy}{dx}\right)^2$	$= \frac{2}{GS_r}$	$\frac{U}{L}$
	$\frac{1}{R^2}$	$= \frac{2}{EI}$	$\frac{U}{L}$
Plate	$\frac{1}{z^2}\left[(\varepsilon_{xx})^2 + (\varepsilon_{yy})^2 + 2(1-\nu)\frac{1}{4}\{\varepsilon_{xy}\}^2 + 2\nu\{\varepsilon_{xx}\varepsilon_{yy}\}\right]$	$= \frac{24(1-\nu^2)}{Eh^2}$	$\frac{dU}{dxdyh}$
GR	$G_{\mu\nu} + \Lambda g_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu}$	$= 2.0766\ 10^{-43}$ N^{-1}	$T_{\mu\nu}$

Main conclusion κ should depends on mechanical characteristics (Y= E, ν)

Therefore, mechanical parameters are hidden in these values

The equivalences clearly established: stress tensor σ_{ij} / stress energy tensor T_{ij}

Appendix A. Demonstration of the equivalence between the stress tensor $\sigma_{\mu\nu}$ and the stress energy tensor $T_{\mu\nu}$ [49]

In the theory of elasticity, resulting from the continuum mechanics, the relation between the stress tensor σ_{ij}, (with $T_i = \sigma_{ij} n_j$ where $\vec{T}$ is a stress vector attached to the facet of normal vector $\vec{n}$), and the applied force Q_i on a surface S_j can be written as follows:

$$Q_i = \sigma_{ij} S_j. \tag{A1}$$

In the field of variational approach, the stress tensor can be written as follows:

$$\sigma_{ij} = \frac{\Delta Q_i}{\Delta S_j} \quad \text{with } \Delta S_j \to 0, \tag{A2a}$$

where ΔS_i is an area.

So, with m as the mass, ρ as the density of mass energy, V as the volume and a_i as acceleration, we have:

$$\sigma_{ij} = \frac{\Delta Q_i}{\Delta S_j} = \frac{\Delta(m \times a_i)}{\Delta S_j} = \frac{\Delta(\rho \times V \times a_i)}{\Delta S_j}. \tag{A2b}$$

Assuming that the variation of the force is due only to the variation of volume V as a function of time t we obtain with, $a_i = \frac{v_i}{\Delta t}$,

$$\sigma_{ij} = \frac{\Delta(\rho \times V \times a_i)}{\Delta S_j} = \rho \frac{1}{\Delta S_j}\left(\frac{\Delta V}{\Delta t}\right) v_i. \tag{A3a}$$

Thus, we get with $V = \Delta x_i \times \Delta x_j \times \Delta x_k$

$$\sigma_{ij} = \rho \frac{1}{\Delta S_j}\left(\frac{\Delta x_i \times \Delta x_j \times \Delta x_k}{\Delta t}\right) v_i. \tag{A3b}$$

We can replace S_j by its value:

$$\Delta S_j = \Delta x_i \times \Delta x_k. \tag{A4}$$

So the new expression of the stress tensor is

$$\sigma_{ij} = \rho \frac{v_i}{\Delta t}\left(\frac{\Delta x_i \times \Delta x_j \times \Delta x_k}{\Delta x_i \times \Delta x_k}\right). \tag{A5}$$

After simplification we obtain

$$\sigma_{ij} = \rho v_i \left(\frac{\Delta x_j}{\Delta t}\right). \tag{A6}$$

By definition of speed v_j, we have

$$v_j = \left(\frac{\Delta x_j}{\Delta t}\right). \tag{A7}$$

We finally obtain the expression of the stress tensor at low speed as a function of energy density ρ and based on the multiplication of the velocities v_i and v_j:

$$\sigma_{ij} = \rho v_i v_j. \tag{A8}$$

Demonstration of the Pramana paper

The stress energy tensor results from the product of the energy density and the multiplication of the four-velocities (four dimensions of the space–time) resulting from the general relativity:

$$T_{\mu\nu} = \rho u_\mu u_\nu \tag{A9}$$

Classic speed vectors

$$\sigma_{ij} = \rho v_i v_j.$$

$$T_{\mu\nu} = \rho u_\mu u_\nu$$

Density

Special relativity, four speed vectors

Transversalism between GR and elasticity theory: mechanical conversion of κ (form of mechanical expression function of Y)

If we compare and analyze GR and elasticity theory on a tensor point on view, in link with strain energy U, the “transversalism” between General Relativity and elasticity theory appears, it becomes clear that κ should depend of mechanical parametra and especially Y=E and ν

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = -\frac{8\pi G}{c^4}T^{\mu\nu} \qquad (5a)$$

All type of gravitational field

$$\partial^\lambda \partial_\lambda \bar{h}_{\mu\nu} = \Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu} \qquad (21)$$

Weak gravitation field

$$\left[\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right)\right]\varepsilon^{ij} = \frac{2(1+\nu)}{E}U_{ij} = \frac{(1+\nu)}{E}\sigma_{ij}\varepsilon^{ij} \qquad (43c)$$

Elastic medium

$$\partial^\lambda \partial_\lambda \left(\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\bar{\varepsilon}\right) = \Box\left(\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\bar{\varepsilon}\right) = -\frac{8\pi G}{c^4}T_{\mu\nu}$$

This formula is similar to T. Damour’s formula in his book “if Einstein had told me”

$$D_{(g)} = \kappa T$$

$$U_{ij} = \frac{1}{2}\sigma_{ij}\varepsilon^{ij}. \Rightarrow U_{ij} = \frac{E}{2(1+\nu)}\left[\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right)\right]\varepsilon^{ij} \Rightarrow \left[\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right)\right]\varepsilon^{ij} = \frac{2(1+\nu)}{E}U_{ij} = \frac{(1+\nu)}{E}\sigma_{ij}\varepsilon^{ij}.$$

$$\sigma_{ij} = \frac{E}{(1+\nu)}\left(\varepsilon_{ij} + \frac{\nu}{(1-2\nu)}\varepsilon_{kk}\delta_{ij}\right)$$

Universe homogeneous and isotropic

Hooke’s law in isotrope medium

Conclusion about the mechanical approach of κ

Reason 1 parallelism curvature = K energy density

General relationship between the strain tensor and elastic strain energy

$$\left[\left(\varepsilon_{ij} + \frac{v}{1-2v}\varepsilon_{kk}\delta_{ij}\right)\right]\varepsilon^{ij} = \frac{2(1+v)}{E}U_{ij} = \frac{(1+v)}{E}\sigma_{ij}\varepsilon^{ij} \rightarrow f_{(\varepsilon_{ij}{}^2)} = K_{\left(\frac{1+v}{E}\right)}U$$

A mechanical bridge should be inside κ

The linearized form of Einstein's equation in weak gravitational fields is:

$$\partial^\lambda \partial_\lambda \overline{h}_{\mu\nu} = \Box \overline{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu}$$

Reason 2: Unit

Parallelism of the formalism with the Hook's law

$\delta = (1/K) \times$ Force

Flexibility m N^{-1}

$\varepsilon = (1/E) \times \sigma$

The mechanical aspects of the general relativity should be directly introduced in κ

Conclusion of this first part

1) The principles of the equation of General relativity are close to a Hooke's law as applied to the theory of beams or plates

2) The tensorial formalism presents similarities between the stress and strain tensors on the one hand with the stress energy tensor and the tensor linked to the disturbance of the metric on the other hand

3) A parallelism appears between Hooke's law in a homogeneous and isotropic medium and the general relativity equation

3) An original approach to integrate mechanical characteristics of space into General Relativity is to express Einstein's constant κ as a function of these mechanical parameters

First approach: Mechanical torsion

2) Deformations of space in the case of gravitational waves

- Perturbation theory, linearized equation of Einstein, solution of this equation, order of magnitude of deformations measured by Ligo/Virgo
- Analogy between mechanical torsion of space and polarization of gravitational waves: consequences on G
- An equivalent Young's modulus of the elastic medium space
- Possible mechanical characteristics hidden in Einstein's constant κ
- Necessity to have an anisotropic model of space to be in accordance with the Poisson's ratio $\nu=1$

Case of the gravitational waves far from the source

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T^{\mu\nu}$$

General relativity in weak field

With:

Linearized

$$\bar{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\bar{h}$$

$\bar{h} = -h = h^{\lambda}_{\lambda}$ the trace of h

And the metric of the space time is given by:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$$

Small perturbation f the metric measured by Ligo/Virgo $1\mathrm{x}\ 10^{-21}$

; the solution to solve this equation based on the Greens function:

$$\Box f = \delta^4(x^\mu - x_S^\mu)$$

By analogy with $\Box \bar{h}_{\mu\nu}$

f is a function,

δ^4 delta function of Dirac at 4 dimensions,

x^μ the space variable,

x_S^μ a source point that sends something at a given time to a given location (μ associated with t),

The solution is of the following form:

$$f_{(x^\mu)} = \frac{\delta^3(x^0 - x_s^0 - \|\vec{x} - \vec{x}_s\|)}{4\pi\|\vec{x} - \vec{x}_s\|}\Theta(x^0 - x_s^0)$$

The solution by analogy with the above is therefore:

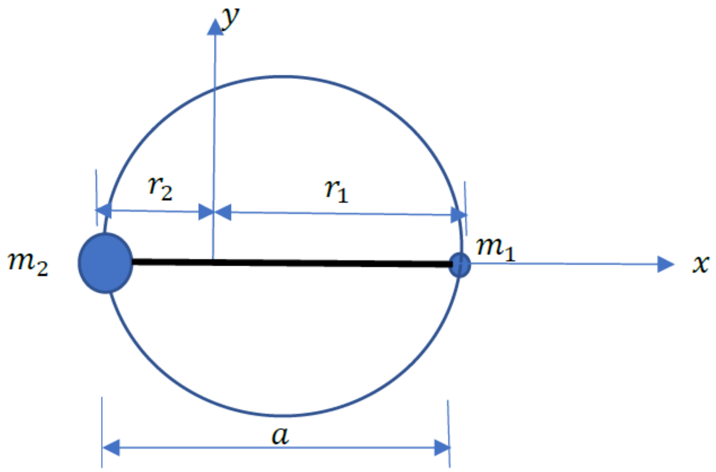
Solution

$$\bar{h}_{\mu\nu} = -\frac{4G}{c^4}\iiint_{Source} \frac{T_{\mu\nu}(x^0 - x_s^0 - \|\vec{x} - \vec{x}_s\|)}{\|\vec{x} - \vec{x}_s\|} d^3\vec{x}_s$$

Case of the gravitational waves far from the source: case of two masses rotating around each other

$$\bar{h}_{ij(t)} = h_{ij(t)} = \frac{2G}{Rc^4}\frac{d^2}{dt^2}I_{ij}\left(t-\frac{R}{c}\right) = \frac{2G}{Rc^4}\ddot{Q}_{ij\left(t-\frac{R}{c}\right)}$$

This expression was demonstrated by Einstein in 1916 **[35]**. We also give this demonstration (see derivation in A.1.3). We check in the equation that $h_{ij(t)}$ has the dimension of a deformation (without unit).



$$1 = \frac{\frac{m^3}{kg s^2}}{\frac{m^4}{m\,[illegible]}}\frac{1}{s^2}kg m^2$$

No unity as a strain

So, we can rewrite the quadrupole formula as:

$$Q = \frac{1}{q''}\iiint \rho\left(z^2 - \frac{r^2}{3}\right)dV$$

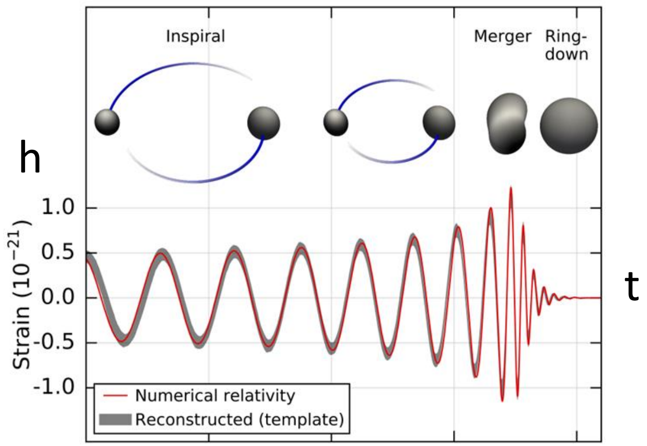
Which is exactly the expression used for gravitational waves:

$$I^{jk} = \iiint_{Source} \rho\left(x_i^j x_i^k - \frac{1}{3}\eta^{jk} r_i^2\right)dV$$

Q= I = Quadrupole

$$h_{ij(t)} = \frac{2G}{Rc^4}\frac{d^2}{dt^2}I_{ij}\left(t-\frac{R}{c}\right) = \frac{2G}{Rc^4}\ddot{Q}_{ij\left(t-\frac{R}{c}\right)} = \frac{4G}{Rc^4}\left(\frac{m_1 m_2}{m_1+m_2}\right)a^2\omega^2\begin{pmatrix} -\cos 2\omega t & -\sin 2\omega t & 0 \\ -\sin 2\omega t & \cos 2\omega t & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gravitational wave: projection of a spheric wave in a plane wave far of the source Via the projection tensor in the plane xy P_{ij}



$$h_{ij}^{TT} = \left(P_{ik}(\vec{n})P_{jl}(\vec{n}) - \frac{1}{2}P_{ij}(\vec{n})P_{kl}(\vec{n})\right)h_{kl}$$

$$P_{ij}(\vec{n}) = \delta_{ij} - n_i n_j \qquad h_{ij}^{TT} = (P_{ijkl})h^{kl}$$

$$h_{ij}^{TT} = \begin{pmatrix} a_+ & a_\times & 0 \\ a_\times & -a_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} e^{i(\vec{k}\vec{x}-\omega t)} + conjugate\ complex$$

$$h_{\mu\nu} = A_{\mu\nu} e^{i\, k_\lambda x^\lambda}$$

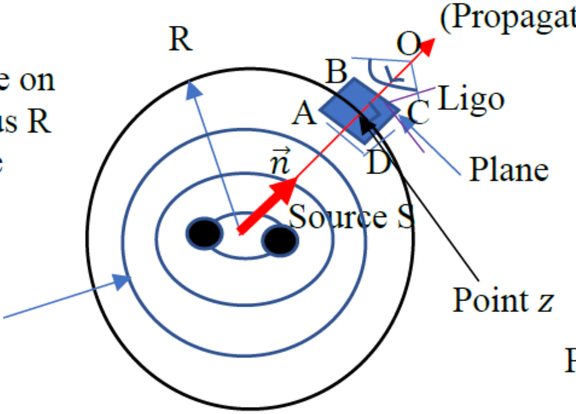
ctor.

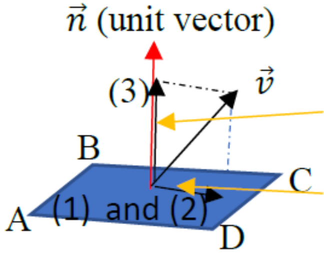
$$A_{\mu\nu} = h_+ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + h_\times \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \cong \eta_{\mu\nu} + 2\varepsilon_{\mu\nu}$$

Locally a spherical wave emission very far from the source becomes a plane wave on the Earth due to the big radius R between the source S and the observer O

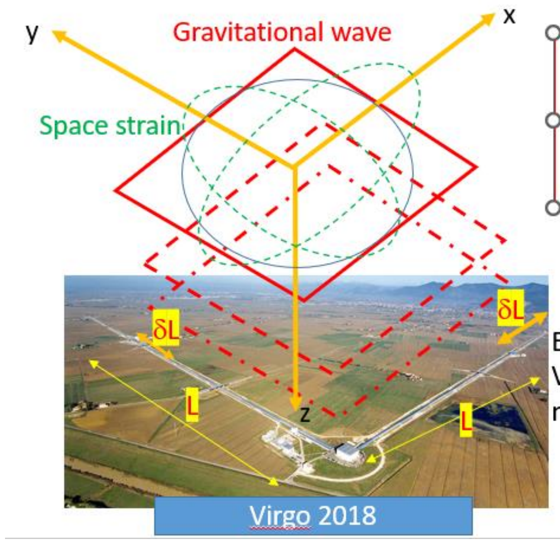
Source emitted Gravitational Waves multipole decomposed.





Projection of the vector $\vec{v}$ along $\vec{n}$ (3)and perpendicular to $\vec{n}$ (1) and (2)

Plane orthogonal to unit vector $\vec{n}$



$$\sigma = \varepsilon E = \frac{\delta L}{L} E$$

Each arms of L = 3km for Virgo and 4km for Ligo measure space strains

GW17

Figure A.6: Projection of a vector $\vec{v}$ – definition of a P_{ij} – (T. Damour source)

Passage of the General relativity linearized Elasticity theory

Study part left of the Einstein Equation

$$g_{ij} = \eta_{ij} + h_{ij} = \eta_{ij} + 2\varepsilon_{ij} \, g_{ij} = \eta_{ij} + h_{ij} = \eta_{ij} + 2\varepsilon_{ij}$$

The linearized form of Einstein's equation in weak gravitational fields is:

$$\partial^\lambda \partial_\lambda \bar{h}_{\mu\nu} = \Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

Study of the correspondence of this part with the elasticity theory $h_{\mu\nu} = 2\varepsilon_{\mu\nu}$

So, the perturbation $h_{\mu\nu}$ and consequently the metric $g_{\mu\nu}$ are close to $2\varepsilon_{\mu\nu}$. So, in weak gravity field we demon-

The linearized form of Einstein's equation in weak gravitational fields is:

$$\partial^{\lambda}\partial_{\lambda}\bar{h}_{\mu\nu} = \Box\bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu}$$

$$\bar{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\bar{h}$$

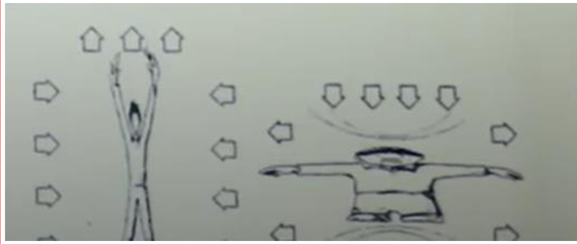
$$\bar{h}_{\mu\nu} = A_{\mu\nu}\cos(k_{\sigma}x^{\sigma})$$

Bridge between mechanic and GR in weak field

2 Polarisations

$$A_{\mu\nu} = A_{+}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + A_{\times}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\eta_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$



Consequence of the bridge $h_{\mu\nu}=2\varepsilon_{\mu\nu}$

h the trace of $h_{\mu\nu}$:

$$\bar{h} = -h$$

$$g_{ij} = \eta_{ij} + h_{ij} = \eta_{ij} + 2\varepsilon_{ij}$$

$$h_{ij} = 2\varepsilon_{ij}$$

$$\frac{\delta L}{L} = \frac{1}{2}h_{ij}n^i n^j$$

Under the weak field condition, the metric tensor can be approximated as,

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\varepsilon_{\mu\nu}, \quad |\varepsilon_{\mu\nu}| \ll 1 \tag{2.9}$$

$$\partial^{\lambda}\partial_{\lambda}\left(h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\bar{h}\right) = \Box\left(h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\bar{h}\right) = -\frac{16\pi G}{c^4}T_{\mu\nu}$$

By replacing $h_{\mu\nu}$ with $2\varepsilon_{\mu\nu}$ we get:

$$\partial^{\lambda}\partial_{\lambda}\left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\bar{\varepsilon}\right) = \Box\left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\bar{\varepsilon}\right) = -\frac{16\pi G}{c^4}T_{\mu\nu}$$

$$\frac{1}{R^2} = \left(\frac{d^2y}{dx^2}\right)^2 = \left(\frac{M}{EI}\right)^2 = \frac{\sigma^2}{E^2\nu^2} = \frac{\varepsilon^2}{\nu^2} = \frac{2}{EI}\frac{U}{L}$$

Mechanical expression of the general relativity linearized

$$\boldsymbol{\partial^{\lambda}\partial_{\lambda}\left(\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\bar{\varepsilon}\right) = \Box\left(\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\bar{\varepsilon}\right) = -\frac{8\pi G}{c^4}T_{\mu\nu}}$$

Passage of the General relativity linearized
Elasticity theory

Study part right of the Einstein Equation

The linearized form of Einstein's equation in weak gravitational fields is:

$$\partial^{\lambda}\partial_{\lambda}\bar{h}_{\mu\nu} = \Box\bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}\boxed{T_{\mu\nu}}$$

Study of the correspondence of
this part with the elasticity theory

Stress energy tensor =stress mechanical tensor

With for the four velocity:

$$u_\mu = \begin{vmatrix} \gamma c \\ \gamma v_x \\ \gamma v_y \\ \gamma v_z \end{vmatrix} \quad (130)$$

We obtain finally the expression of the stress tensor at low speed in function of the energy density ρ and based on the multiplication of the velocity v_i and v_j:

$$\sigma_{ij} = \rho v_i v_j \ (128)$$

The stress energy tensor becomes from the product of the density of energy and the multiplication of the four velocity (4 dimension of the space time) issued from the general relativity.

$$T_{\mu\nu} = \rho u_\mu u_\nu \ (129)$$

Under low speed γ=1 the stress energy tensor becomes:

$$T_{\mu\nu} = \begin{bmatrix} \frac{mc^2}{V} & \rho c v_x & \rho c v_y & \rho c v_z \\ \rho c v_x & \rho v_x v_x & \rho v_x v_y & \rho v_x v_z \\ \rho c v_y & \rho v_y v_x & \rho v_y v_y & \rho v_y v_z \\ \rho c v_z & \rho v_z v_x & \rho v_z v_y & \rho v_z v_z \end{bmatrix} \quad (131)$$

Based on the definition of the stress tensor, (cf. equation 128), the stress energy tensor at low speed can be written as following:

$$T_{\mu\nu} = \begin{bmatrix} \frac{mc^2}{V} & \rho c v_x & \rho c v_y & \rho c v_z \\ \rho c v_x & \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \rho c v_y & \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \rho c v_z & \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \quad (132)$$

The Einstein equation build a link in 4 dimensions (space time) with the curvature tensor $G_{\mu\nu}$(dimension 1/m^2) and the stress energy tensor $T_{\mu\nu}$ (dimension energy/m^3) that is itself a generalization in 4 dimensions of the stress tensor of the continuum mechanics.

$$Pa = N/m^2 = \frac{kg}{m^3} \times \frac{m}{s} \times \frac{m}{s} = \frac{kgm}{s^2} \times \frac{1}{m^2} = pression\ or\ stress = force/Area$$

Geometric Link of GR with the **mechanical** torsion

Question :

- Why there are two polarizations of the gravitational waves except on a mathematical point of view?

For a polarised wave A_+:

$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (30a)$$

For a polarised wave $A_\times$:

$$h_{\mu\nu} = A_\times \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (30b)$$

Possible answer :

$$g_{ij} = \eta_{ij} + h_{ij} = \eta_{ij} + 2\varepsilon_{ij}\, g_{ij} = \eta_{ij} + h_{ij} = \eta_{ij} + 2\varepsilon_{ij}$$

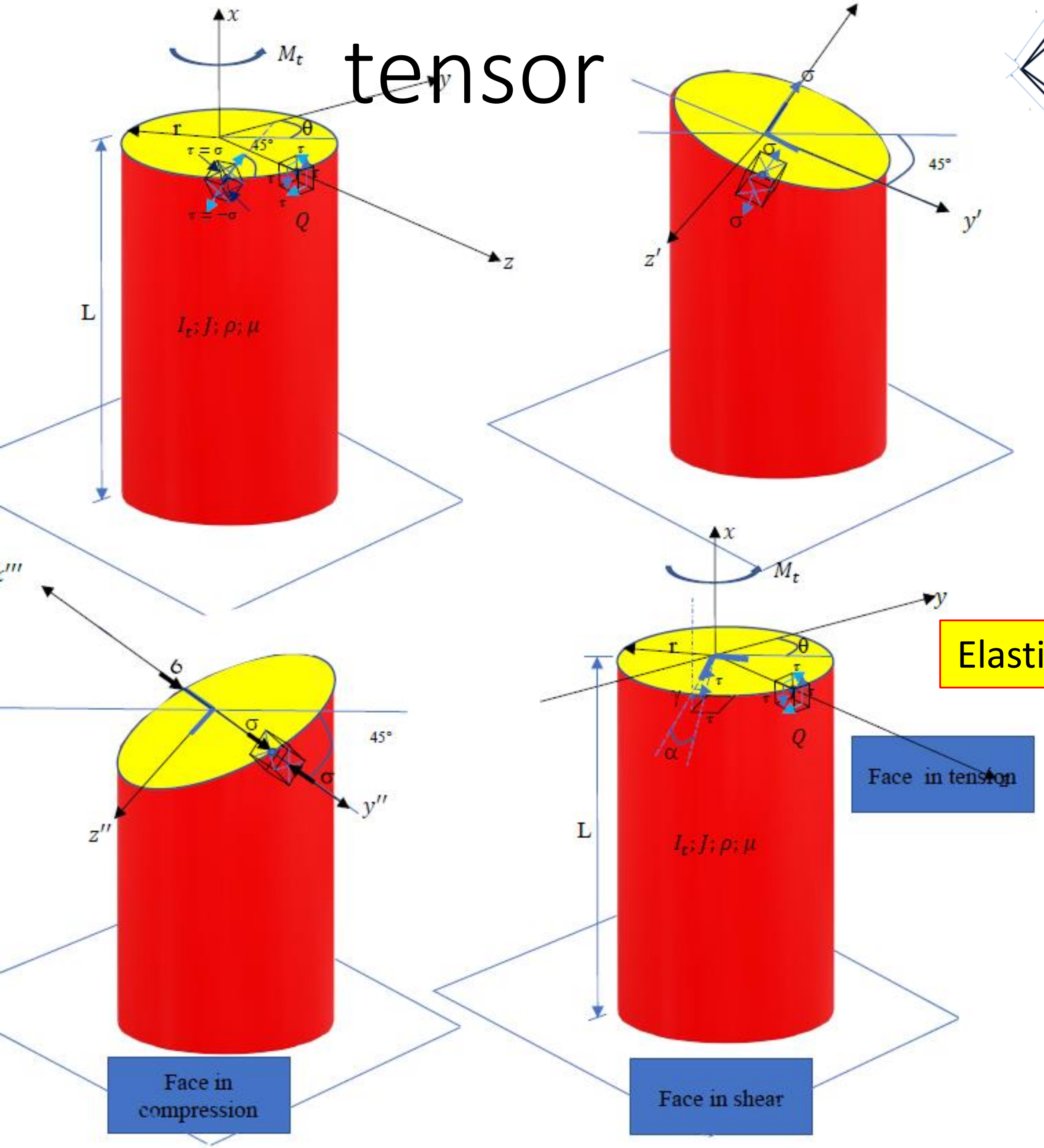
Although Equation (3.8) suggests that there are ostensibly ten strain components, $\varepsilon_{\alpha\beta}$, oscillating independently, in reality only two are independent and the rest are coupled to the two. To show this, consider a traveling wave, which corresponds to a gravity wave, propagating along the x^3 direction. It is necessary that $\varepsilon_{3\alpha} = \varepsilon_{\alpha 3} = 0$ for the wave to be a shear wave. Furthermore, as shown previously, $\varepsilon_{00} = \varepsilon^{3D} = 0$ and $\varepsilon_{j0} = \varepsilon_{0j} = 0$. Finally, we have $\varepsilon^{3D} = \varepsilon_{11} + \varepsilon_{22} = 0$, because $\varepsilon_{33} = 0$ already. Therefore,

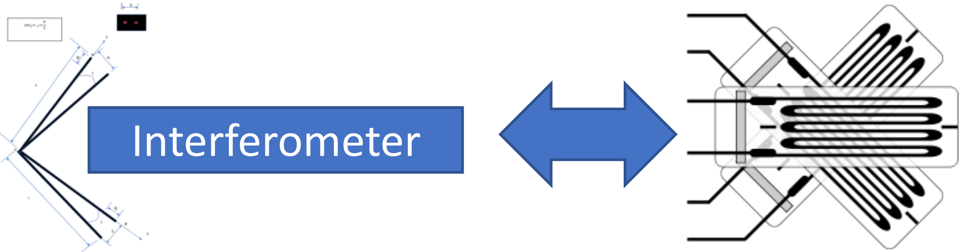
$$\begin{aligned} \varepsilon_{11} &= -\varepsilon_{22} \\ \varepsilon_{12} &= \varepsilon_{21} \end{aligned} \quad (3.12)$$

are the only two independent degrees of freedom left, which implies just two types of wave polarizations. The fact that Equation (3.12) is in terms of the material strain, which has a definite physical meaning, ensures that the waves must also be physical as opposed to being mere coordinate displacements. This result, derived from a Solid Mechanic's perspective, is consistent with the analogous result from General Relativity about the polarization of gravitational waves [26, Ch. 35].

Tenev and Horstemeyer present the results but don't speak of the real geometric consequences

Pure torsion stress tensor





Elastic gauge

GR in weak field Gravitational wave

For a polarised wave A_+:

$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Elasticity

$$[\sigma]^P_{xyz} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \qquad [\sigma]^P_{xyz} = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad [\sigma]^P_{XYZ} = \begin{bmatrix} \tau & 0 & 0 \\ 0 & -\tau & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sigma'_{x,y} = \frac{\sigma}{2} \mp \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$ avec la contrainte normale σ=0

For a polarised wave $A_\times$:

$$\sigma_X = \tau \quad \sigma_Y = -\tau$$

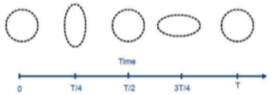
$$h_{\mu\nu} = A_\times \cos\left(\frac{\omega}{c}(ct - z)\right)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \qquad (30b)$$

GR in weak field Gravitational wave

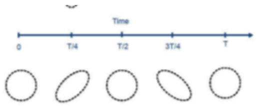
1[rst] consequence: Torsion and Polarisation of gravitational waves (see DI's paper Pramana)

Metric perturbation (2 polarisations)

For a polarised wave A_+:



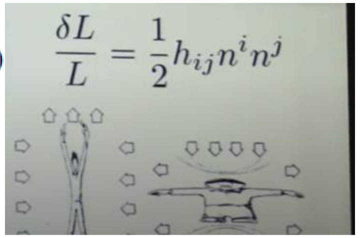
$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (30a)$$



For a polarised wave $A_\times$:

$$h_{\mu\nu} = A_\times \cos\left(\frac{\omega}{c}(ct - z)\right)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (30b)$$

$$g_{\mu\nu} = (\eta_{\mu\nu} + h_{\mu\nu}) = (\eta_{\mu\nu} + 2\varepsilon_{\mu\nu}).$$



$$g_{ij} = \eta_{ij} + h_{ij} = \eta_{ij} + 2\varepsilon_{ij}$$
$$h_{ij} = 2\varepsilon_{ij}$$
$$\frac{\delta L}{L} = \frac{\delta_i}{L} = 10^{-21} = \varepsilon_{ij} = \frac{1}{2}h_{ij}.$$

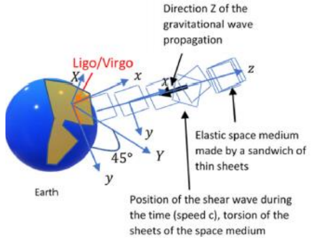


Figure 12. Mohr's circle of the stress state [54].

Figure 13. Morh's circle of the strain state [54].

Strain tensor

$$\varepsilon_{xy(A_+)} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

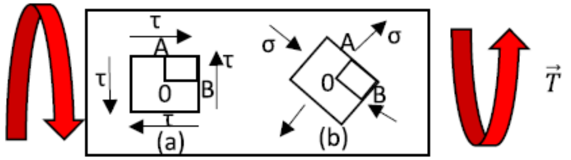
$$\varepsilon_{xy(A_\times)} = \begin{bmatrix} 0 & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

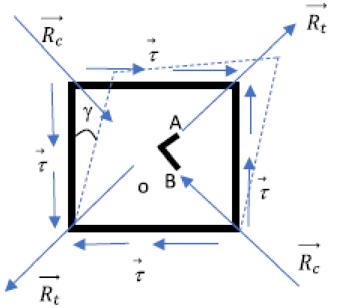
Stress tensor

$$\sigma_{xy(A_+)} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

$$\sigma_{xy(A_\times)} = \begin{bmatrix} 0 & \tau_{xy} = \sigma_{xx} & 0 \\ \tau_{yx} = \sigma_{yy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Associated stress: Torsion state function facet orientation





GR in weak field => gravitational waves with 2 polarisations

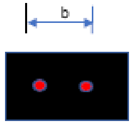
Strain tensor and perturbation of the metric are linked $2\varepsilon_{\mu\nu} = h_{\mu\nu}$

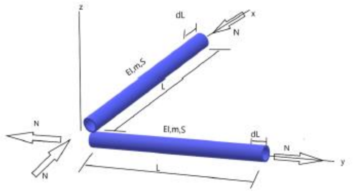
Each polarisation can be associated at two facets of a medium in torsion: black hole generate this torsion (see K. Thorne presentation)

2nd consequence :Ligo/Virgo measures space that is compressed and stretched, but where is the shear strain?

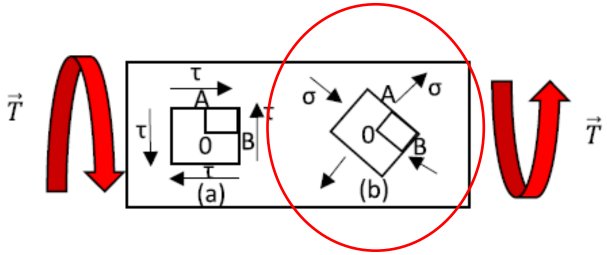
Ligo and Virgo are equivalent stress gauge of the space time structure

$$\tan\gamma \approx \gamma = \frac{b}{L}$$

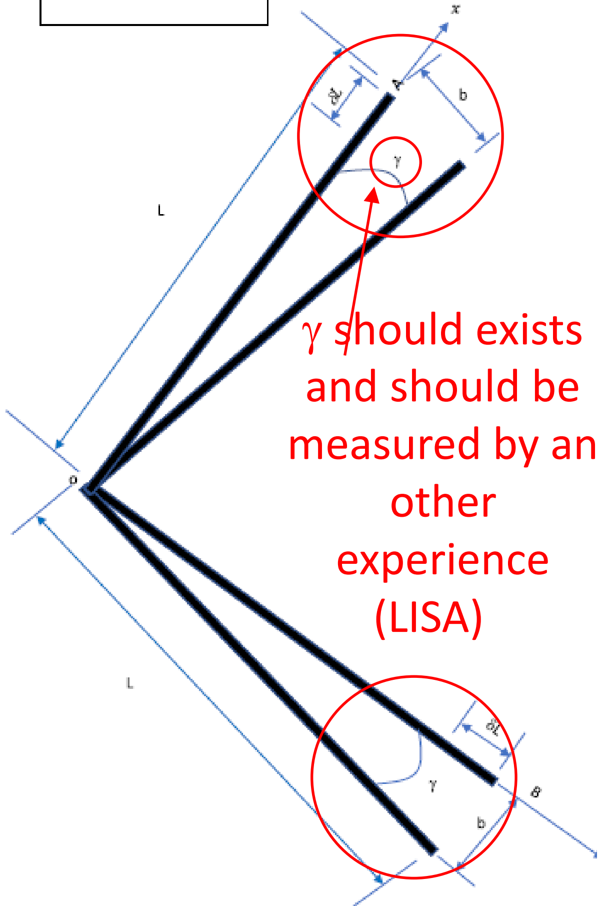




DI: Comment of R. Weiss, impossible to measure the distorsion (shear strain) with Ligo/Virgo, perhaps possible by Lisa (see Nasa)



Only this state is measured by Ligo/Virgo



γ should exists and should be measured by an other experience (LISA)

-----Message d'origine-----
De : Rai Weiss <weiss@ligo.mit.edu>
Envoyé : lundi 24 août 2020 23:32
À : David IZABEL <d.izabel@enveloppe-metallique.fr>
Objet : Re: Possible new measurrments on Ligo.

David I,
I found your paper interesting and a nice addition to looking at the dynamics of GR through the window of elasticity. By the way you will get a quite similar result if you use the weak field energy stored in the gravitational wave and relate it to the strain in space. That also gives the Young's modulus of the new ether. I remember doing this some years ago and getting that at 100 Hz space is 10^20 times stiffer than steel and gets stiffer with (if I remeber right) square of the frequency.

AS to asking LIGO to look for a gravitational wave induced motion of the mirrors transverse to the optical direction, that looks pretty grim in the current geometry. I assume from your calculations that the motions are symmetric, end mirror of the left arm moves outward while the end mirror of the right arm also moves outward - the angle between the arms grows and then in the next cycle of the Gravitational wave the mirrors both move inward . In other words it is not a rotation of the two arms together leaving the angle between the arms constant. In either case, symmetric or antisymmetric the motions we are looking for is h X L. The small misalignemnt of the interferometer would be unmeasurable given the field cnange one would measure on a pair of adjacent photodetectors, The only hope would be to set up an equally sensitive pair of interferometrs to look at this sideways motion, Then it does matter if the motions are symmetric or antisymmetric. I cannot see how to do this without a major reconstruction of the interferometer.

Rainer Weiss

De : Thorpe, Ira (GSFC-6630) <james.i.thorpe@nasa.gov>
Envoyé : mardi 22 septembre 2020 21:50
À : David IZABEL <d.izabel@enveloppe-metallique.fr>
Cc : Kazanas, Demos (GSFC-6630) <demos.kazanas-1@nasa.gov>
Objet : Re: [EXTERNAL] Possible new measurements on Lisa

Dear Pr. Izabel,

Thank you for your message and your article. My personal focus is more on the instrumentation and measurement aspects of LISA so I'm not sure how well I followed the details of your argument but I think I understand some of the basics. As Rai pointed out in the discussion you shared, LISA's triangular configuration provides an opportunity to measure multiple components of the strain tensor. We typically describe this as a capability to simultaneously measure the two polarization modes of standard GR but there has also been some work to extend this to measurements (or constraints) of the four additional basis tensors possible in non-GR metric theories. I suspect a similar framework could be applied to consider LISA measurements in your "elastic gravity" approach. There is also the possibility that LISA will operate simultaneously with a planned Chinese observatory called Taiji, which has similar geometry and performance characteristics. Others have pointed out that a LISA-Taiji network could be particularly powerful for constraining modified gravity as well as for precision astrometry of GW signals. I'm afraid that is about the limit of my capabilities on the subject, but fortunately I am surrounded by colleagues with more knowledge and experience in gravity theory. One of these is Dr. Demos Kazanas, with whom I shared your note and who expressed some interest in further discussions. I've copied him here in case either of you wish to initiate a conversation. Please let me know if I can provide any additional information regarding LISA.
Kind regards,

-Ira Thorpe, NASA/GSFC

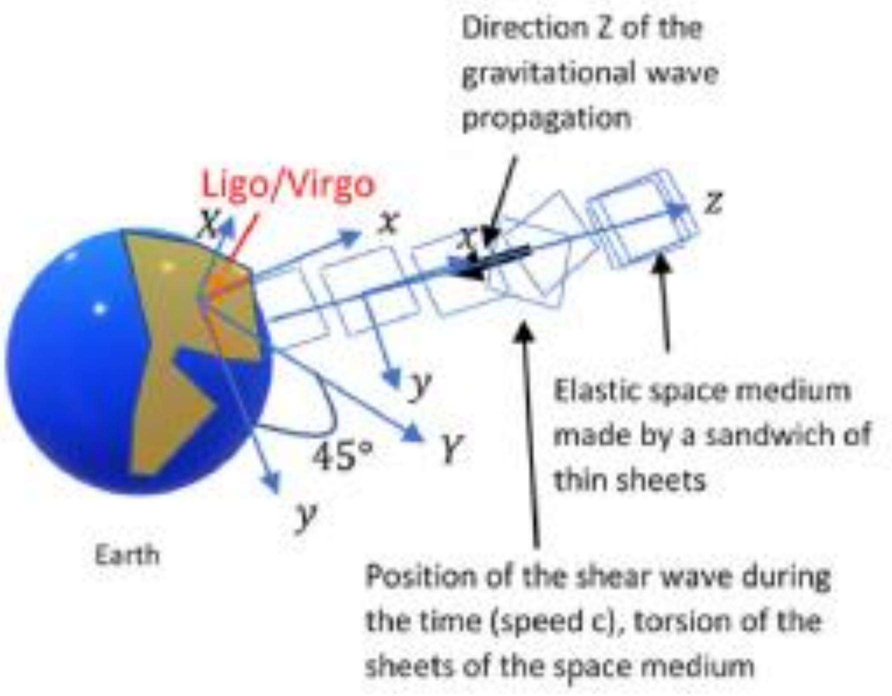
Model 1 : Isotropic medium in the plane of the interferometer : Hooke's law σ = ε Y

For a polarised wave A_+:

$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$[\sigma]^{\gamma}_{XYZ} = \begin{bmatrix} \tau & 0 & 0 \\ 0 & -\tau & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sigma_X = \tau \quad \sigma_Y = -\tau$$



Physic behaviour

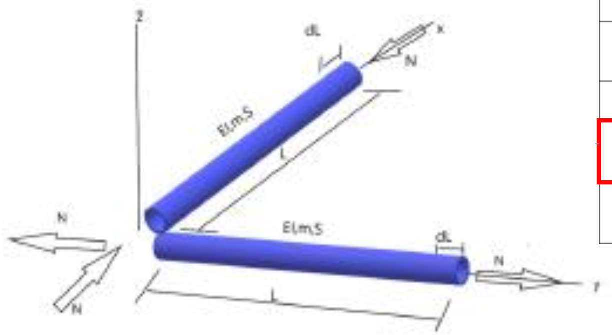


Figure 10. Double perpendicular tube loaded by a normal force.

Sollicitation	Relation between displacement, rotation and strain	Displacement, rotation	Expression of the energy
Bending moment M(x)	$\frac{d^2y}{dx^2} = -\frac{M}{EI} = \frac{1}{R}$	$y_{v(x)}$ Vertical deflection due to bending	$\frac{1}{R^2} = \frac{2}{EI} \times \frac{U}{L}$
Torque of twisting T(x)	$T = GI_t \frac{d\theta}{dx}$	$\theta_{(x)}$ Twisting rotation	$\frac{1}{R_T^2} = \frac{2}{GI_t} \times \frac{U}{L}$
Normal effort N(x)	$N = ES\frac{du}{dx} = ES\varepsilon = \sigma S$	$u_{(x)}$ Lengthening shortening (strain)	$\left(\frac{du}{dx}\right)^2 = \frac{2}{ES}\frac{U}{L}$
Shear force V(x)	$V = GS_r\frac{dy}{dx} = GS_r\gamma_{(x)}$	$\gamma_{(x)}$ Shear force distortion	$\left(\frac{dy}{dx}\right)^2 = \frac{2}{GS_r} \times \frac{U}{L}$

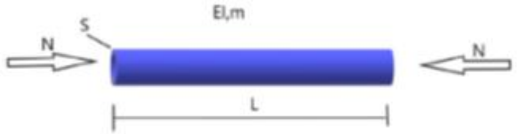


Figure 9. Tube loaded by a normal force N.

The Hooke's law (13) can be written as a function of the displacement $u_{(x)}$:

$$\sigma_{xx} = \varepsilon E = \frac{N}{S} = \left(\frac{u_{(x+dx)} - u_{(x)}}{dx}\right)E = \frac{\delta L}{L}E,$$

Model 1 :Arms in compression/traction via Hooke's law

Reconstruct Einstein's κ: Ligo/Virgo plane stresses and deformations

Indeed, the Laplacian of the gravitational field, $\Delta\phi$, follows the Poisson's equation

$$\Delta\phi = 4\pi G\rho \quad (3a)$$

and the 00 component of the metric $g_{\mu\nu}$ is then

$$g_{00} \approx 1 + \frac{2\phi}{c^2}. \quad (3b)$$

Newton approach component 00 are used => Einstein approach

1

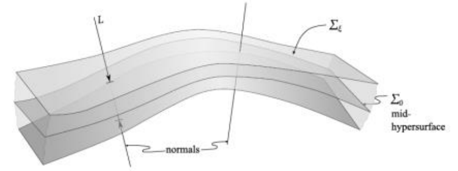
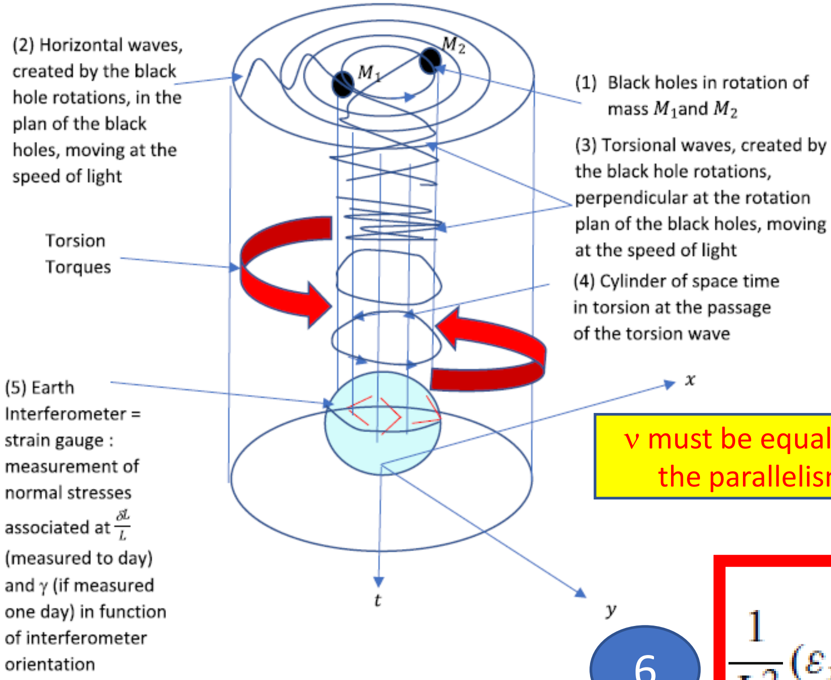


Fig. 4. The cosmic fabric is treated as a stack of three-dimensional hypersurfaces Σ_ξ each parameterized by $\xi \equiv x^4 =$ const, and L is its thickness.

$$\begin{bmatrix} G^{00} & G^{01} & G^{02} & G^{03} \\ G^{10} & G^{11} & G^{12} & G^{13} \\ G^{20} & G^{21} & G^{22} & G^{23} \\ G^{30} & G^{31} & G^{32} & G^{33} \end{bmatrix} = \kappa \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}.$$

DI approach components ij are used => spatial mechanical part of space

2



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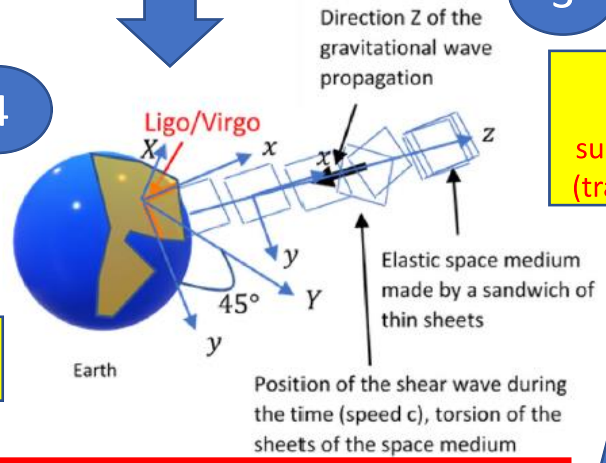
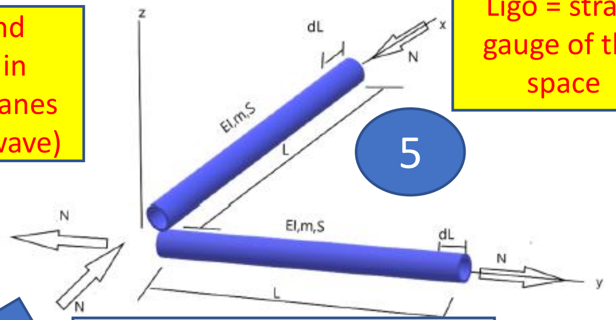


Stresses and strains are in successive planes (transversal wave)

Ligo = strain gauge of the space

5



Demonstration in Pramana paper

ν must be equal to 1 to have the parallelism with GR

6

$$\frac{1}{L^2}(\varepsilon_{xx})^2 = 4(1+\nu) \times \pi \times \frac{\pi f^2}{\rho} \times \frac{1}{c^4} \times \frac{U}{V}$$

$$\frac{1}{L^2}(\varepsilon_{yy})^2 = 4(1+\nu) \times \pi \times \frac{\pi f^2}{\rho} \times \frac{1}{c^4} \times \frac{U}{V}$$

$$\begin{bmatrix} \frac{1}{L^2} & 0 \\ 0 & \frac{1}{L^2} \end{bmatrix} \begin{bmatrix} (\varepsilon_{xx})^2 & 0 \\ 0 & (\varepsilon_{yy})^2 \end{bmatrix} = \frac{8\pi G}{c^4} \begin{bmatrix} T_{xx} & 0 \\ 0 & T_{yy} \end{bmatrix}.$$

See demonstration in DI's Pramana paper, we obtain well G= π f²/ρ and we obtain a parallelism between mechanical and GR equation field

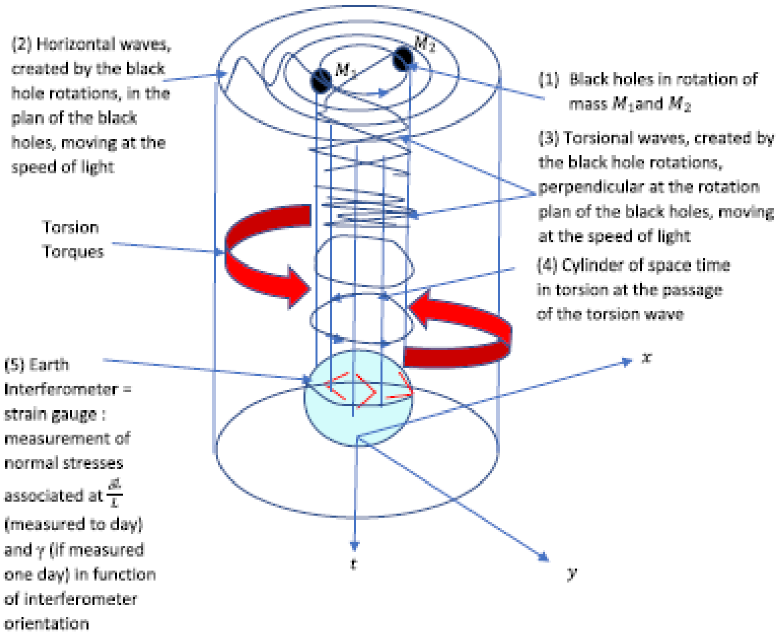
Main consequences

$$G = \frac{\pi f^2}{\rho}$$

7

$$R_{ij} = \frac{8\pi G}{c^4} T_{ij}.$$

Model 2 : Isotropic medium cylinder un torsion : Hooke's law τ = γ G

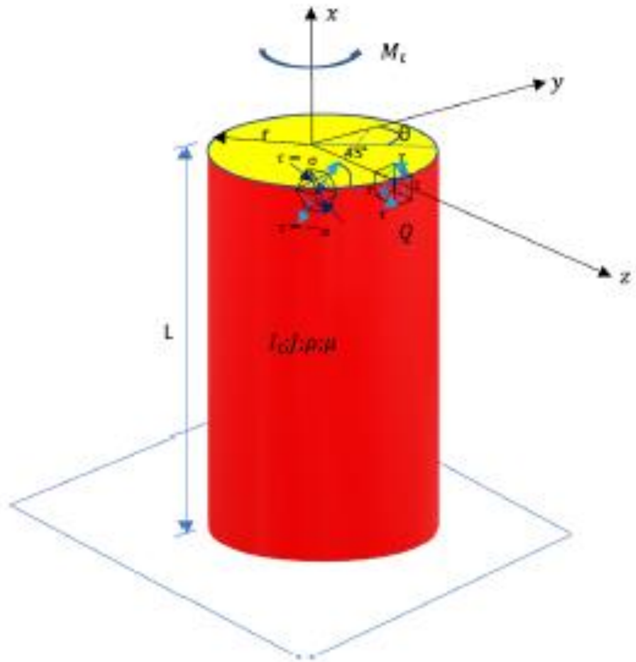


Physic behaviour

Same consequence and conclusion in the case of the cylinder in torsion

For a polarised wave $A_\times$:

$$h_{\mu\nu} = A_\times \cos\left(\frac{\omega}{c}(ct - z)\right)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

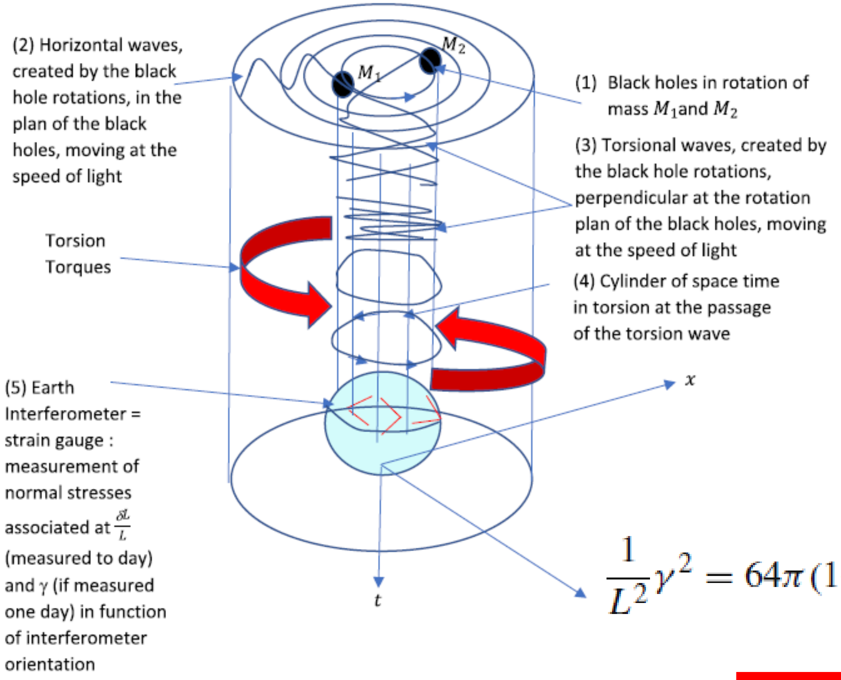


Model 2 :Cylinder in torsion

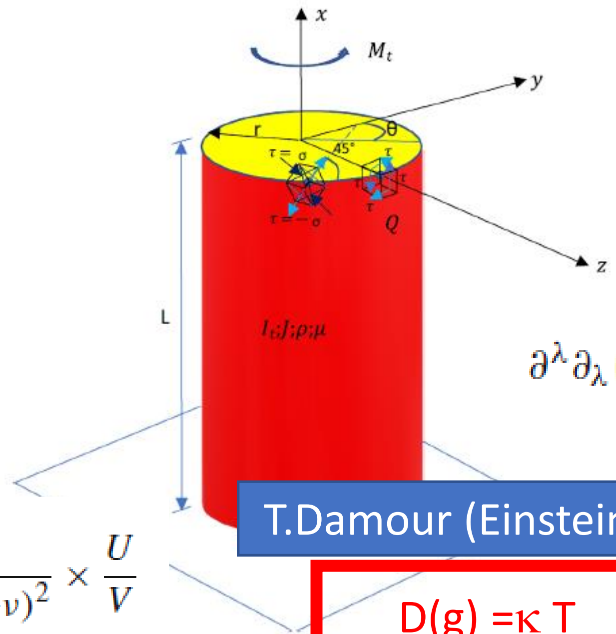
Sollicitation	Relation between displacement, rotation and strain	Displacement, rotation	Expression of the energy
Bending moment M(x)	$\frac{d^2y}{dx^2} = -\frac{M}{EI} = \frac{1}{R}$	$y_{v(x)}$ Vertical deflection due to bending	$\frac{1}{R^2} = \frac{2}{EI} \times \frac{U}{L}$
Torque of twisting T(x)	$T = GI_t \frac{d\theta}{dx}$	$\theta_{(x)}$ Twisting rotation	$\frac{1}{R_T^2} = \frac{2}{GI_t} \times \frac{U}{L}$
Normal effort N(x)	$N = ES\frac{du}{dx} = ES\varepsilon = \sigma S$	$u_{(x)}$ Lengthening shortening (strain)	$\left(\frac{du}{dx}\right)^2 = \frac{2}{ES}\frac{U}{L}$
Shear force V(x)	$V = GS_r\frac{dy}{dx} = GS_r\gamma_{(x)}$	$\gamma_{(x)}$ Shear force distortion	$\left(\frac{dy}{dx}\right)^2 = \frac{2}{GS_r} \times \frac{U}{L}$

$$[\sigma]^{p}_{xyz} = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Reconstruct Einstein's κ by mechanical component of the tensor: Torsion of a space cylinder



$$\frac{1}{L^2}\gamma^2 = 64\pi(1+\nu)^2\frac{\pi f^2}{\rho}\frac{1}{4c^4(1+\nu)^2}\times\frac{U}{V}$$



$$\partial^\lambda\partial_\lambda\bar{h}_{\mu\nu} = \Box\bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu}.$$

$$\partial^\lambda\partial_\lambda\bar{h}_{\mu\nu} = \Box\bar{h}_{\mu\nu} = 0.$$

$$\partial^\lambda\partial_\lambda\left(h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\bar{h}\right) = \Box\left(h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\bar{h}\right) = -\frac{16\pi G}{c^4}T_{\mu\nu}.$$

T.Damour (Einstein GR)

D(g) =κ T

By replacing $h_{\mu\nu}$ by $2\varepsilon_{\mu\nu}$

$$\frac{\gamma^2}{L^2} = 16\pi\frac{\pi f^2}{\rho}\frac{1}{c^4}\times\frac{U}{V}.$$

$$\Box\left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\bar{\varepsilon}\right) = -\frac{16\pi G}{c^4}T_{\mu\nu}.$$

$$\frac{1}{L^2}\gamma^2 = 16\pi\frac{G}{c^4}\times T.$$

Main consequences

$$G = \frac{\pi f^2}{\rho}$$

DI: See demonstration in DI's Pramana paper, we obtain well G= π f²/ρ and we obtain a parallelism between mechanical and GR equation field

2 types of waves in elastic medium, but only one type (shear) in general relativity, Why?

Answer of T Tenev and M.F Horstemeyer

Probably a specific nature of the space time: a special fabric

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + \mu)\vec{\nabla}\left(\vec{\nabla} \bullet \vec{u}\right) + \mu \vec{\nabla}^2 \vec{u} + f_{\text{external}}, \tag{38d}$$

where

$$\vec{\nabla}\vec{u} = \overrightarrow{\text{grad}}(\vec{u}) \tag{38e}$$

$$\vec{\nabla}^2 \vec{u} = \vec{\Delta}(\vec{u}). \tag{38f}$$

When $f_{\text{external}} = 0$, the solution of the equation follows the Helmholtz's decomposition that gives two waves that propagate in the elastic medium:

$$\vec{u} = \vec{u}_{\text{pressure,longitudinal}} + \vec{u}_{\text{shear,transversal}}. \tag{39}$$

A pressure with a longitudinal wave of velocity c_{pressure}:

$$c_{\text{pressure}} = \sqrt{\frac{\lambda + 2\mu}{\rho}}. \tag{40}$$

Does not appear in GR

DI: Following T. Tenev and M.F. Horstemeyer, this unique type of gravitational waves is perhaps a signature of a specific structure 'a fabric) of the analogic elastic medium especially with v =1

direction of propagation (z). A shear with a wave of velocity c_{shear}:

$$c_{\text{shear}} = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{2(1+\nu)\rho}}.$$

Gravitational waves

2.4. *Postulates*

We postulate the cosmic fabric to be (1) an elastic thin hyperplate, with (2) matter-energy fields as inclusions, and (3) lapse rate of proper time proportional to the shear wave speed v_s. Each of these postulates is described and motivated in the sections below.

2.4.1. *Elastic Thin Hyperplate*

Cosmic space is identified with the mid-hypersurface of a hyperplate called the Cosmic Fabric that is thin along the fourth spatial dimension. We imagine the fabric as foliated into 3D hypersurfaces each of which is isotropic and elastic, and each is subject to Hooke's Law (See Fig. 4). Thus, Hooke's Law (1.1) together with concepts such as stress, strain and the Poisson effect (see Fig. 3) apply as conventionally understood in Solid Mechanics, because they pertain to individual hypersurfaces, which are 3D bodies.

Because of its correspondence to physical space, the intrinsic curvature, R^{3D}, of the fabric's mid-hypersurface corresponds to that of three-dimensional (3D) space. Likewise, the intrinsic curvature R of the fabric's world volume, corresponds to that of four-dimensional (4D) spacetime. The term "world volume" refers to the four-dimensional shape traced out by an object in spacetime as it advances in time.

The small transverse thickness of the fabric is needed to create resistance to bending, but once such resistance is accounted for, we treat the fabric as essentially a 3D hypersurface that bends within the 4D reference hyperspace. The thickness must be very small so that the fabric can behave as an essentially 3D object at ordinary length scales and be an appropriate analogy of 3D physical space. The thickness itself defines a microscopic length scale at which the behavior of the physical world

3.3. *Fabric Vibrations and Gravitational Waves*

Having Poisson's ratio $\nu = 1$ also implies that there can only be transverse (shear) waves in the fabric but no longitudinal (pressure) waves. The shear modulus μ and the p-wave modulus M are as follows,

$$\mu = \frac{Y}{2(1+\nu)} = \frac{Y}{4}$$
$$M = Y\frac{1-\nu}{(1-2\nu)(1+\nu)} = 0 \tag{3.4}$$

implying that the transverse (shear) wave velocity $v_s = \sqrt{\mu/\rho} \neq 0$, while the longitudinal (pressure) wave velocity $v_p = \sqrt{M/\rho} = 0$. This result shows why the speed of light is the fastest entity of the universe, given that a longitudinal wave is typically faster than a shear wave. For a shear wave to be the fastest, the Poisson's Ratio must be 1. This conclusion is consistent with observations, because all known waves that propagate in free space, such as gravity or electromagnetic waves, are transverse.

Synthesis of this part=> mechanical conversion of κ

From linearized general relativity in weak field, the link between curvature of space and stresses in this space is the Einstein's constant 2 times:

$$h_{\mu\nu} = 2\varepsilon_{\mu\nu} \Rightarrow \Box\left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\bar{\varepsilon}\right) = -\frac{16\pi G}{c^4}T_{\mu\nu}.$$

$$2\kappa = 16\frac{\pi G}{c^4}$$

From (GW170817) In space gravitational waves are transverse wave and move at c in an equivalent elastic medium constituting space:

$$c = \sqrt{\frac{\mu}{\rho}}$$

From the analogy of the Hooke's law strains in the interferometer's arms or of a cylinder in torsion (rotation two black holes) we obtain (see Pramana Paper):

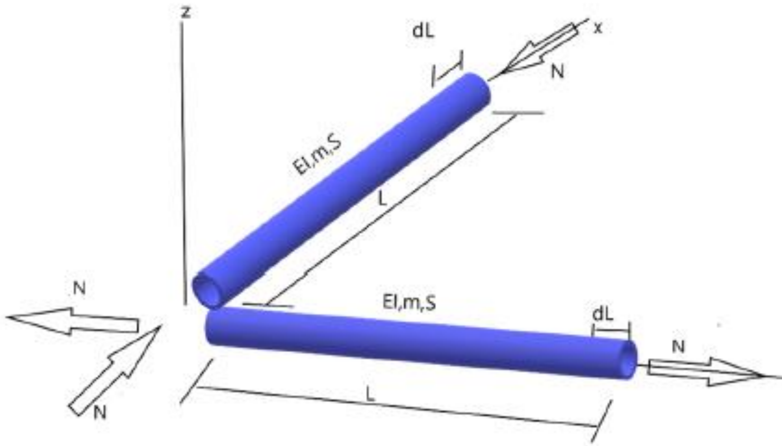
$$G = \frac{\pi f^2}{\rho}$$

So κ can be rewritten in function of mechanical characteristics in the analogy of the space elastic medium:

$$\kappa = \frac{8\pi\left(\frac{\pi f^2}{\rho}\right)}{\left(\sqrt{\frac{\mu}{\rho}}\right)^4} = \frac{8\pi^2 f^2}{\rho\frac{\mu^2}{\rho^2}} = \frac{8\rho\pi^2 f^2}{\mu^2} = \frac{8\rho\pi^2\left(\frac{\omega}{2\pi}\right)^2}{\rho\mu^2} = 2\frac{\rho\omega^2}{\mu^2}$$

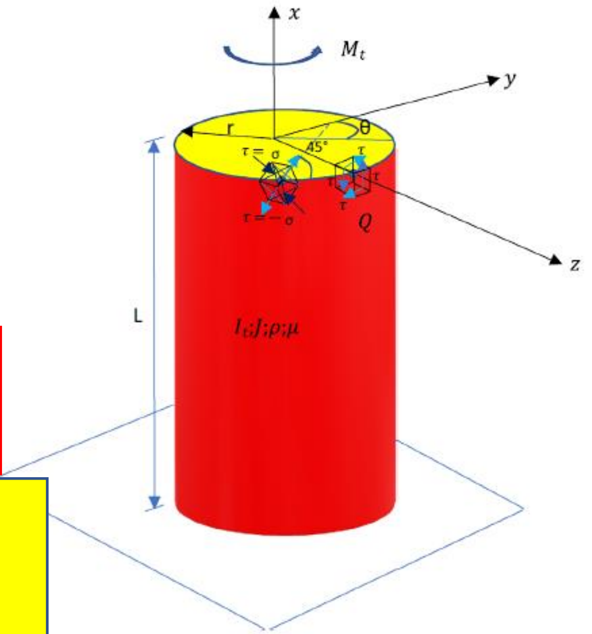
$$\mu = \frac{E}{2(1+\nu)}$$

The consequence about G and κ: DI's paper Pramana and Books



$$G = \frac{\pi f^2}{\rho}$$

$$c_{\text{shear}} = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{2(1+\nu)\rho}}.$$



We obtain well the new expression of κ compatible with states of the art (K. McDonald)

$$\kappa = \frac{8\pi G}{c^4} = (1+\nu)\rho\left(\frac{\omega}{E}\right)^2.$$

$$\kappa = \frac{8\pi G}{c^4} = 2\rho\left(\frac{\omega}{\mu}\right)^2.$$

Results compatible with the expression estimated by the parallelism between GR and elasticity theory

$$\left[\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right)\right]\varepsilon^{ij} = \frac{2(1+\nu)}{E}U_{ij} = \frac{(1+\nu)}{E}\sigma_{ij}\varepsilon^{ij} \rightarrow f_{(\varepsilon_{ij}{}^2)} = K_{\left(\frac{1+\nu}{E}\right)}U$$

BUT we have a price to pay: necessity following this analogy to leave isotropic medium.

Necessity to have an anisotropic model of space to be in accordance with the Poisson's ratio v=1

Gravitational wave = transverse wave

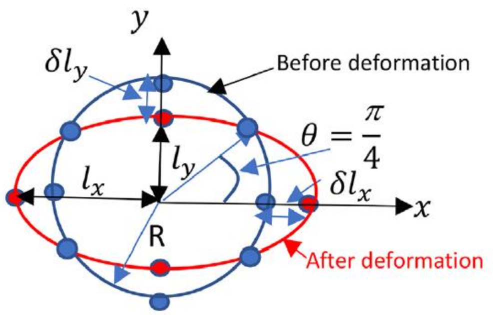
Horstemeyer

Poisson's ratio ν =1

- We demonstrate based on 3 different approaches:

First Approach: Analysis of the Movements of Particles Positioned in Space on a Circle Undergoing the Passage of a Gravitational Wave $\varepsilon_{xx} = -v\varepsilon_{yy}$

Strains measured in the arms of Virgo



Checked on the Ligo/Virgo measurements

Second Approach: In the z Direction, the Gravitational Wave is a Transverse Wave and is Not a Compression Wave

$$c_{\text{pressure}} = \sqrt{\frac{\lambda + 2\mu}{\rho}} = 0$$

$$\lambda = \frac{Ev}{(1+v)(1-2v)}$$

Consequence 1

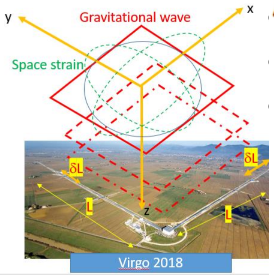
Transverse shear wave implies ν = 1 (outside the range -1, 0,5) so we have to consider an anisotropic medium

Consequence 2

Third Approach: Based on Available Datas

$$\rho = \frac{\mu}{c^2} \rightarrow v = \frac{E}{2c^2\rho} - 1$$

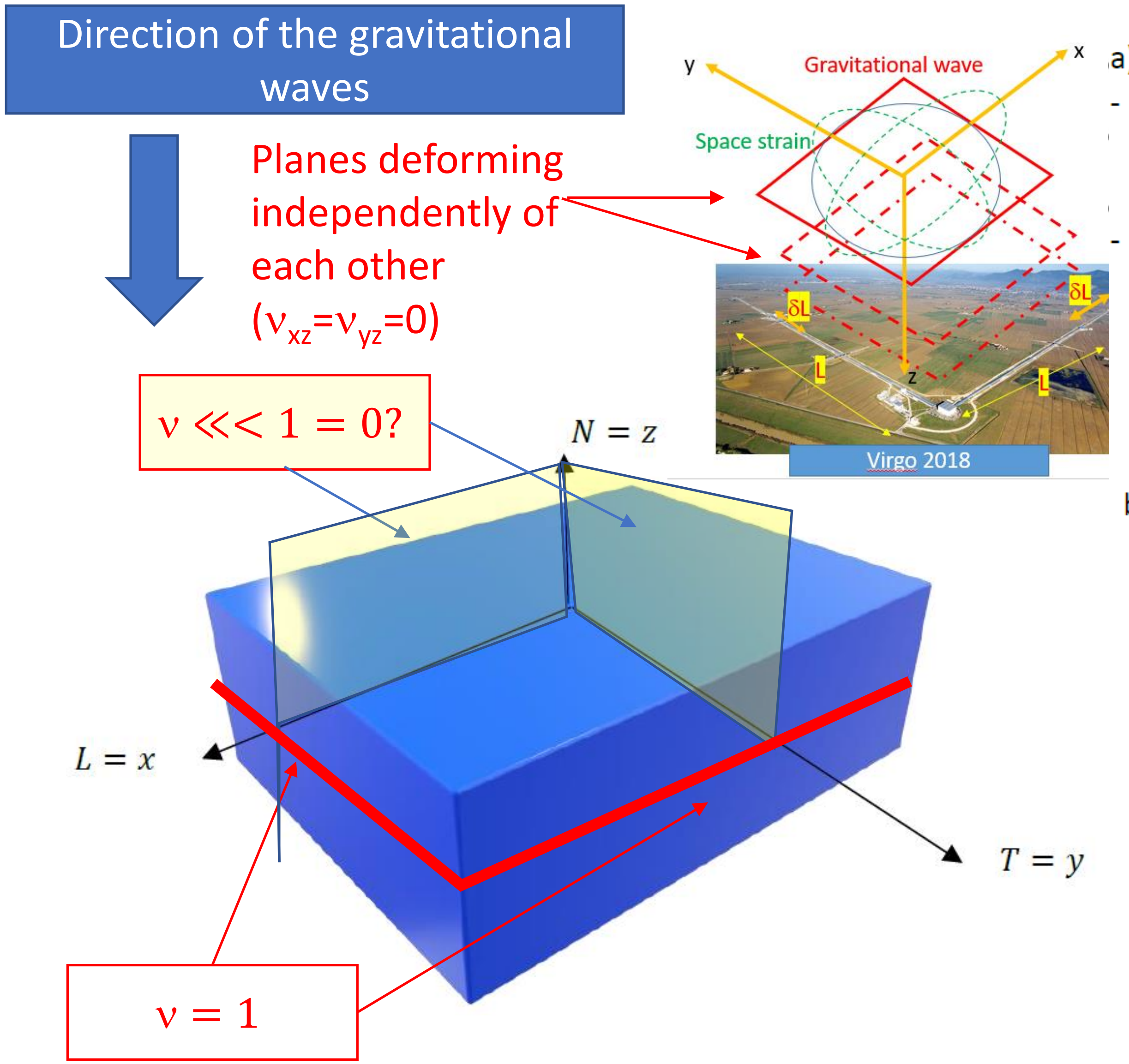
Imply also a behaviour of space as a some of plans deformed independently during the passage of the gravitational wave



$$\mu = \frac{E}{2(1+v)}$$

Continuous medium?

Consequences => the isotropy =>non isotropy of the medium? Limit validity analogy?



a) In the LT plan (x, y)

- For Young's modulus Y=E:

$$Y_L = E_L = Y_T = E_T$$

- For the Poisson's ratio:

$$\nu_{TL} = \nu_{LT}$$

$$G_{LT} = \frac{E_L}{2(1 + \nu_{LT})}$$

b) In LN (xz) and TN plans (yz)

- For the Poisson's ratio:

$$\nu_{NT} = \nu_{NL}$$

$$\nu_{LN} = \nu_{TN}$$

- For shear modulus:

$$G_{TN} = G_{LN}$$

- For the following key relationships between Poisson's ratio and Young's modulus:

$$\frac{\nu_{NT}}{E_N} = \frac{\nu_{LN}}{E_L}$$

Case of medium with transversal isotropy

IV.5.3 Case of transverse isotropy - the case of gravitational waves

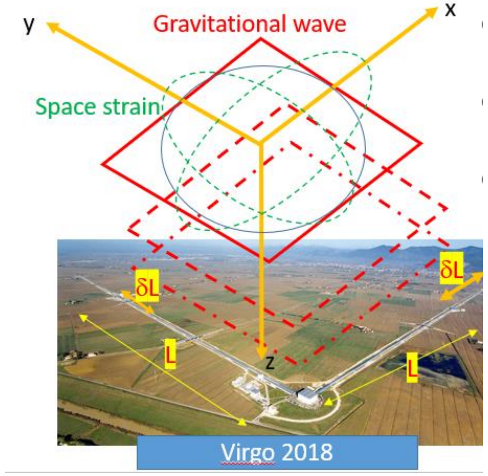
In this case we have:

$\nu_{LT} = \nu_{TL} = 1$

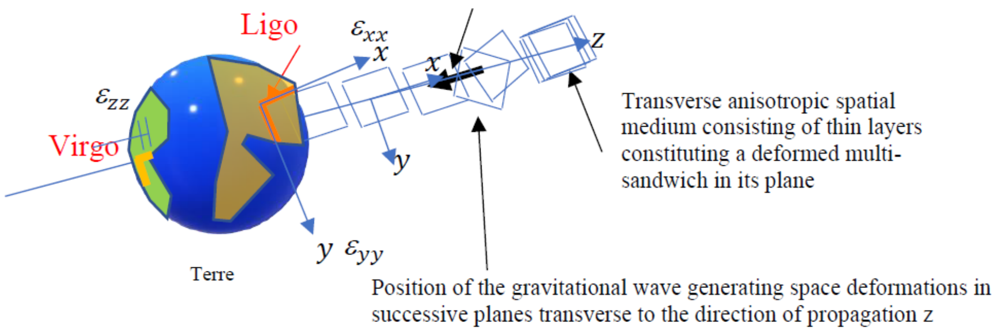
$\nu_{NT}/E_N = \nu_{NL}/E_N = 0$ This implies that E_n is very large so either ν_{NL} and ν_{NT} are very small (no deformation in compression / tension following z in classical RG for polarizations + and x and very small but not zero if Einstein-Cartan torsion (see chapter IV.9 below)

$\nu_{LN}/E_L = \nu_{TN}/E_L = 0$ This implies that E_n is very large so either ν_{NL} and ν_{NT} are very small (no deformation in compression / tension following z in classical RG for polarizations + and x and very small but not zero if Einstein-Cartan torsion (see chapter IV.9 below)

The matrix connecting the deformations to the stresses thus becomes in a transverse anisotropic medium:



$$\begin{Bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{TN} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_L} & \frac{-1}{E_L} & \frac{-\nu_{NL}}{E_N} & 0 & 0 & 0 \\ \frac{-1}{E_L} & \frac{1}{E_L} & \frac{-\nu_{NT}}{E_N} & 0 & 0 & 0 \\ \frac{-\nu_{LN}}{E_L} & \frac{\nu_{TN}}{E_L} & \frac{1}{E_N} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+1)}{E_L} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{LN}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{TN}} \end{bmatrix} \begin{Bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \\ \sigma_{LN} \\ \sigma_{TN} \end{Bmatrix}$$



The following xx and yy do not change parts $((E_L = Y = E)$, so all the developments made within the framework of this thesis that led to remains true $G = \frac{\pi f^2}{\rho}$ see publication of Pramana D. Izabel).

Conclusion of second part

1) The parallelism between the deformation tensor in the case of a mechanical torsion of a cylinder in space by the rotation of very massive objects and the polarizations of gravitational waves emerges from the analogy between elasticity and General relativity in weak field.

2) The writing of the deformations of the arms of the interferometers in terms of dynamic energy variation of the deformations of space during the passage of a gravitational wave leads to express κ and G according to mechanical parameters of elasticity (Y, ν).

3) The same conclusion appears when considering a cylinder of space put in pure torsion.

4) The analogy reaches these limits because it leads by the very nature of gravitational waves which are transverse waves, to a Poisson's ratio = 1, that is to say to an anisotropic medium which is contrary to the hypothesis taken at the start (Hooke's law in a homogeneous and isotropic medium).

5) The multiple observations of gravitational waves since 2016 seems show an identical behaviour in all the directions of space; so, if anisotropy there is, it il homogeneous in all the space: we have a medium with an "isotropic" anisotropy!!!

→ The elastic approach of space conduct at a very special electic medium

Second approach: geometrical torsion

- 3) Limits of the analogy between MMC /RG - Questioning of continuum mechanics in relation to these deformations of space
- Space as an anisotropic medium on a small scale?
- How are the deformations of space transmitted from one plane to another during the propagation of gravitational waves? Local plasticization of an equivalent crystalline medium?
- If the RG has to be modified, the modification must be very small: is geometric torsion a good candidate?
- The contribution of defect theory and its analogy with Einstein Cartan's general relativity, a way to explain the propagation of gravitational waves in space? a local dislocation of the medium?
- Are the polarizations of gravitational waves in the case of GR with torsion a means of supplementing the tensor of plane strains of the space medium observed in the case of GR without geometrical torsion?

Gravitational waves propagations following z = Successive xy planes deformations

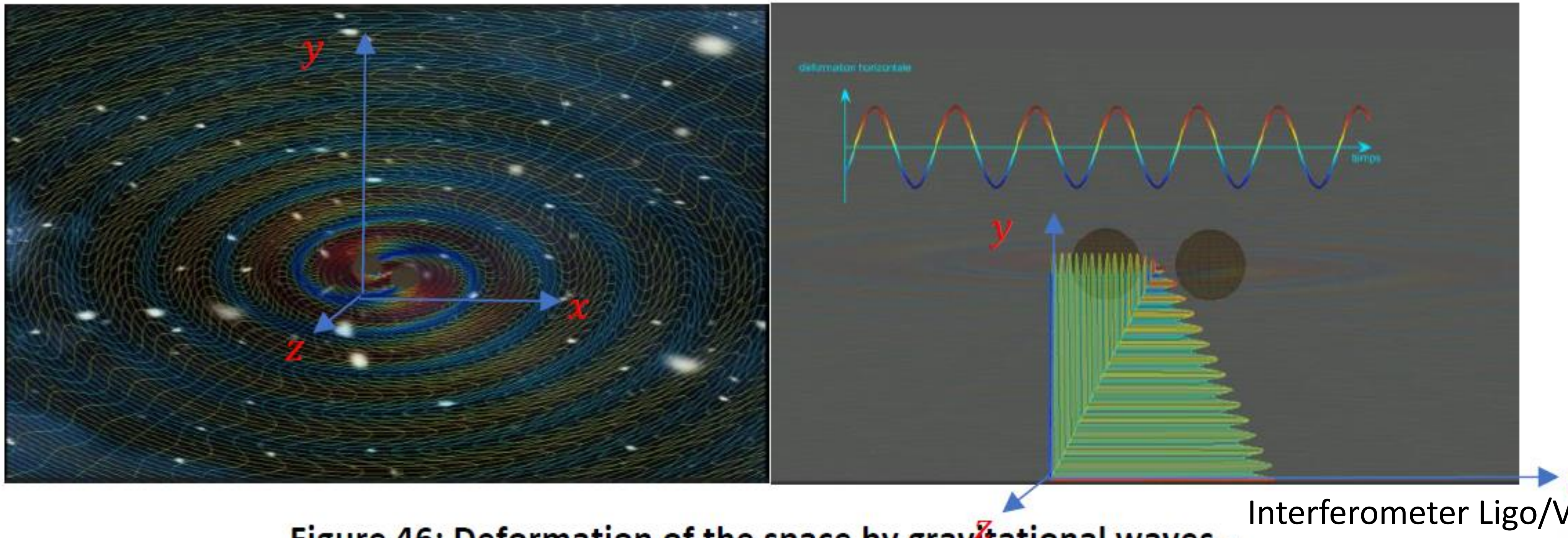


Figure 46: Deformation of the space by gravitational waves -

Interferometer Ligo/Virgo measurments

Source:**[199]** F Bondu Visualization of blue and red tensile and compression deformations in the planes xy following the rotation of the two massive and compact celestial bodies. The space deformations in each plane are contrary signs

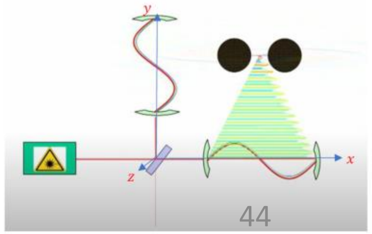
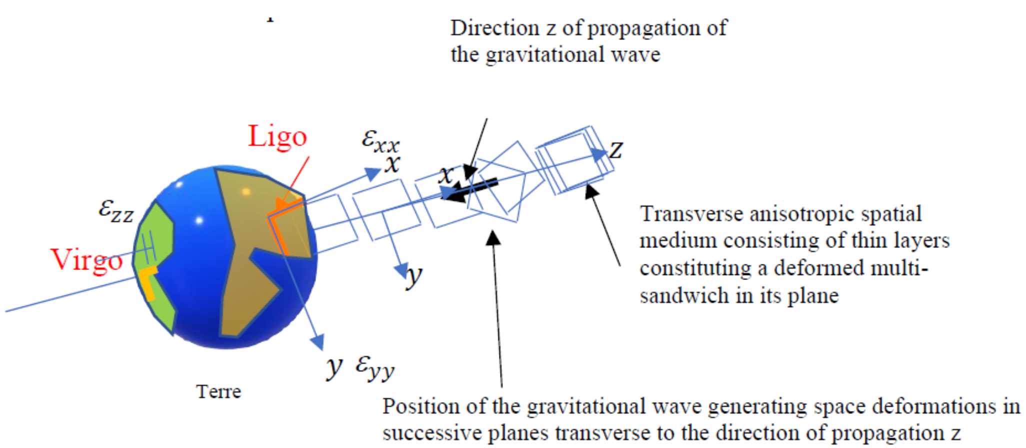


Figure 48: Source: [199] measurement of space deformations by Ligo/Virgo interferometers arms presentation F Bondu -

Limit of the homogenous elastic model

At this stage:

- Our isotropic model of space fails due to Poisson's ratio = 1 (or is valid in plane xy only but not in 3D)
- The shape of the successive deformations of a circle during the passage of a gravitational wave supposes several planes of deformations independent of each other without any link between these successive planes $\nu_{xy} = \nu_{yx} = 1$ $\nu_{xz} = \nu_{yz} = 0$!stange and not in adequacy with the general assumption made in physics of space as isotropic and homogeneous



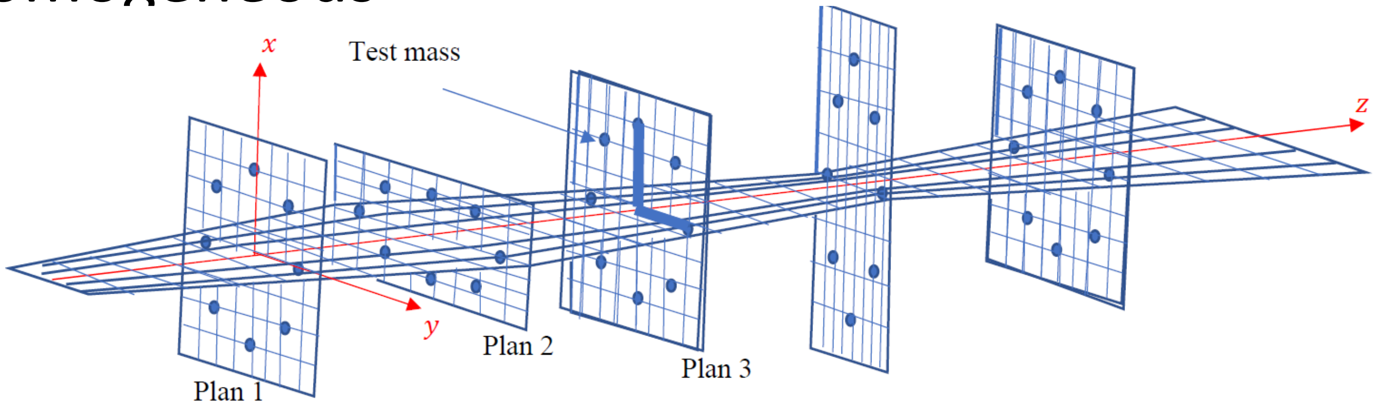


Figure 47: Source: [200] Visualization of deformations in the plans xy due at the rotation of the two massive bodies perpendicular to this plane (z) direction -

If the RG has to be modified, the modification must be very small

Difference between the calculated and measured curves = some %

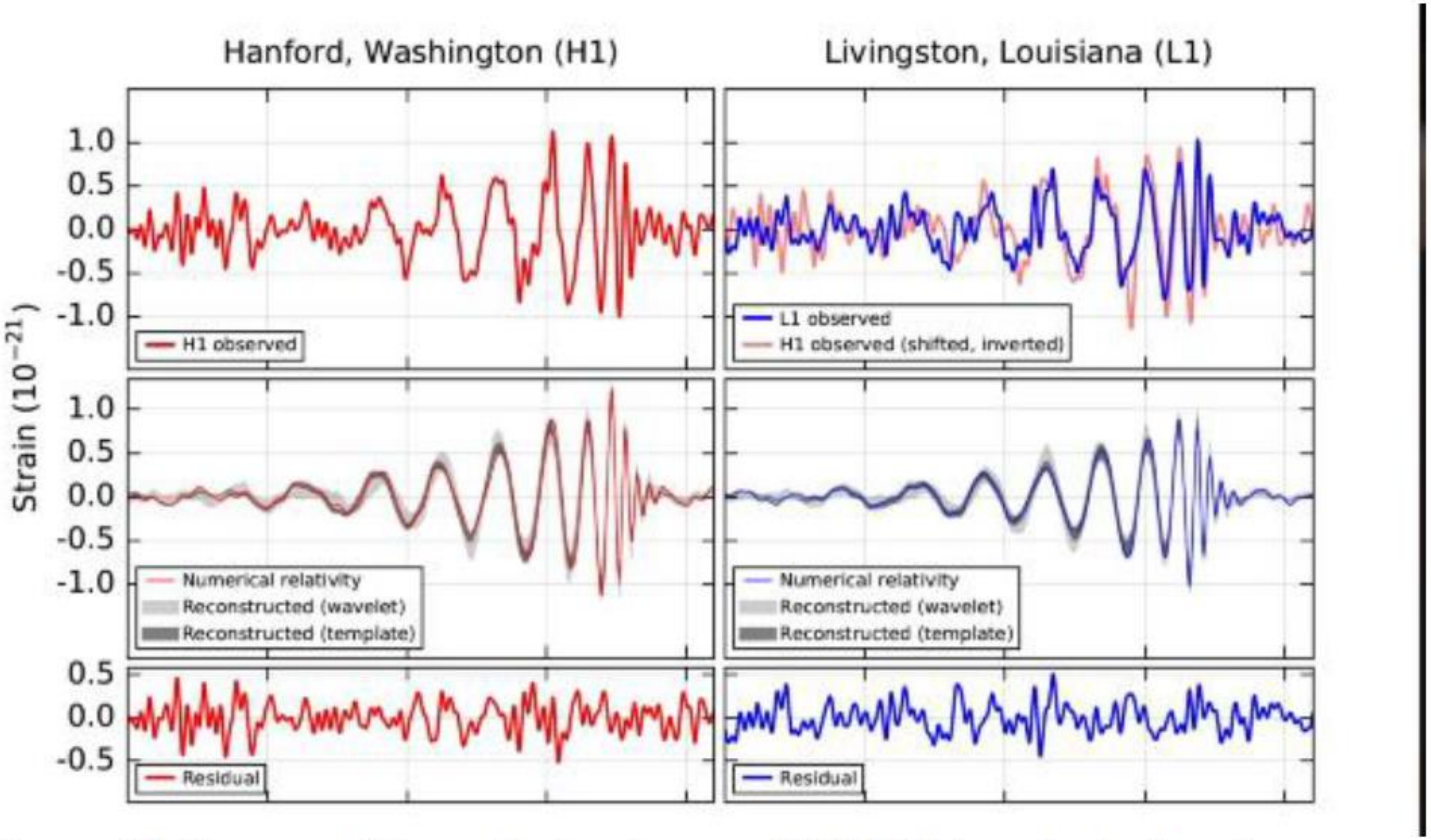


Figure 45: **Source: [43] gravitational wave GW150914 – physical review – measurement of h-deformations of space function of the time in the plane of Ligo interferometers -**

[43] Collective (2016) LIGO Scientific Collaboration and Virgo Collaboration, Physical. Review. Letter. 116, 061102 GW140915

Space as an anisotropic medium on a small scale? **first approach** to solve the problem

Tenev and Horstemeyer propose that the anisotropy is at very small scale (Planck limit) and so, the space stay homogenous at more larger scale:

The Mechanics of Spacetime – A Solid Mechanics Perspective on the Theory of General Relativity

T G Tenev · M F Horstemeyer

Received: date / Accepted: date

Abstract We present an elastic constitutive model of gravity where we identify physical space with the mid-hypersurface of an elastic hyperplate called the "cosmic fabric" and spacetime with the fabric's world volume. Using a Lagrangian formulation, we show that the fabric's behavior as derived from Hooke's Law is analogous to that of spacetime per the Field Equations of General Relativity. The study is conducted in the limit of small strains, or analogously, in the limit of weak and nearly static gravitational fields. The Fabric's Lagrangian outside of inclusions is shown to have the same form as the Einstein-Hilbert Lagrangian for free space. Properties of the fabric such as strain, stress, vibrations, and elastic moduli are related to properties of gravity and space, such as the gravitational potential, gravitational acceleration, gravitational waves, and the energy density of free space. By introducing a mechanical analogy of General Relativity, we enable the application of Solid Mechanics tools to address problems in Cosmology.

Keywords modified gravity · constitutive model · spacetime · cosmic fabric

PACS 04.50.Kd · 46.90.+s

Mathematics Subject Classification (2000) 83D05 · 74L99

IV.8.2 Fabric of equivalent space approach

T. Tenev and M.F. Horstemeyer in the article **[47]** propose on the basis of publications **[134],[135]:** that space would be a kind of superposition of fabrics, a hyper-resistant sandwich. I quote **[47]**:

"Known materials with a Poisson's ratio of = 1 have a fibrous substructure, which suggests that cosmic tissue is, in fact, fabric! For ν= 1, the isostatic modulus of elasticity is $K = Y/[3(1-2\nu)] < 0$. A negative isostatic modulus of elasticity means that the compression of the fabric results in an overall increase in the volume of material and vice versa. Although such behavior is unusual for most conventional materials, stretched compressive [29] and densifying [5] dilating materials have recently been discovered, for which ν = 1 in compression or tensile "

In his thesis T. Tenev considers a scale factor to explain anisotropy at a very small scale and isotropy at a larger scale. We take below Figure 5.4 of it:

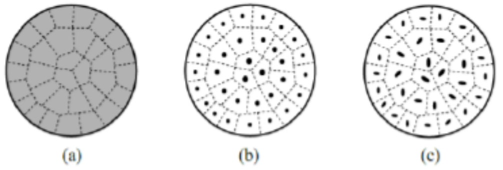


Figure 53: Figure 5.4 Different types of microscopic anisotropy Source [47]

Substructure of a spherically symmetric body

"The substructure of an ostensibly continuous and isotropic body is gradually revealed from (a) to (c), where (a) shows the idealized body as perfectly continuous and spherically symmetric. The dotted lines represent the imaginary subdivision of the body into cells. In (b), the mass of each cell is found to be concentrated into a small particle, while the overall density of the body remains unchanged. In (c), each particle turns out to be locally anisotropic, while the body remains globally ostensibly isotropic" source thesis **[207]** Tenev p108.

Space as an anisotropic medium on a small scale? **second approach** to solve the problem

Tenev and Horstemeyer propose that the fabric of the space time is effectively structured as a fabric that can have this Poisson's ratio of 1

3.2. *Poisson's Ratio and the Substructure of Space*

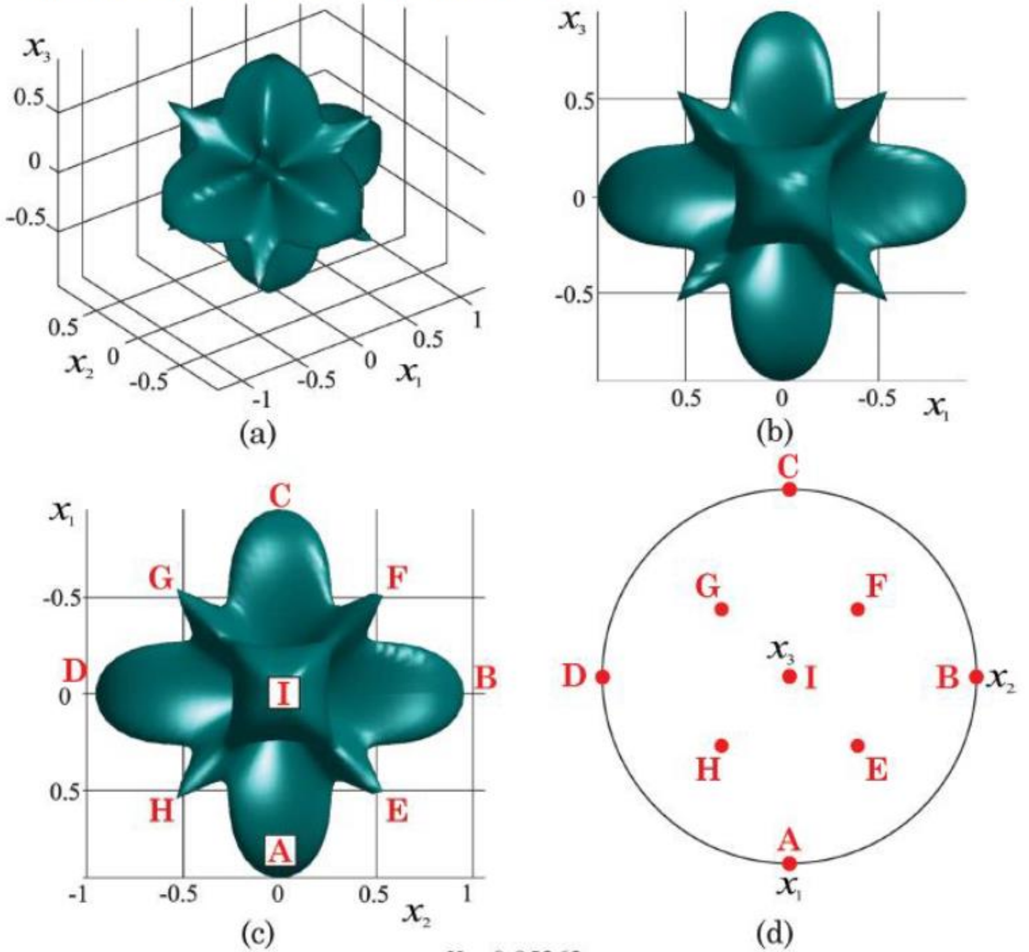
Known materials with a Poisson's ratio of $\nu = 1$ have a fibrous substructure, which suggests that the cosmic fabric is, in fact, a fabric! For $\nu = 1$, the bulk modulus is $K = Y/[3(1-2\nu)] < 0$. A negative bulk modulus means that compressing the fabric results in an overall increase of the material volume and vice versa. Although such behavior is unusual for most conventional materials, there are recently discovered *compressive dilatant*[37] and *stretch densifying*[38] materials, for which $\nu = 1$ in either compression or tension, respectively. Compressive dilatant materials are artificially manufactured and their substructure consists of entangled stiff wires. Stretch densifying materials, have textile-like substructure comprised of woven threads each consisting of twisted fibers.

T. Tenev and M.F. Horstemeyer Publication

37. D. Rodney, B. Gadot, O. R. Martinez, S. R. du Roscoat and L. Org´eas, Nat. Mater. 15 (2016) 72.
38. R. H. Baughman and A. F. Fonseca, Nat. Mater. 15 (2016) 7.

Space as an anisotropic medium on a small scale? **Third approach** to solve the problem

As an example the authors of **[210]** obtain I quote " The maximum Poisson's ratio is 0.820 along [001] V and the minimum is −0.133 along [1 0] V so that$\bar{1}$ K = 0.820 −(−0.133) = .0.953. For [112] V loading (Fig. 3), the maximum Poisson's ratio in the transverse plane is 0.502 along [1 0] V and the minimum value is 0.184 along [111] $\bar{1}$ V. Therefore, r ([112] V) = 0.953 −(0.502 −0.184) = 0.635. » In addition, it is also interesting to note that the directions in the transverse plane at the direction of the effort for which copper has an Isotropic Poisson's ratio form as for gravitational waves two modes, + mode and x mode! (See Figure 52 below from **[210]**).



$K = 0.95363$

Fig. 4. Representation of: (a) the anisotropy surface for copper (point group m$\bar{3}$m); (b) the surface viewed along the $-x_2$ direction; (c) the surface viewed along the $-x_3$ direction; (d) the loading directions in copper for which Poisson's ratio is isotropic in the transverse plane, represented on a stereogram centred at $[001]_V$. The directions have been labelled as follows: A is $[100]_V$, B is $[010]_V$, C is $[\bar{1}00]_V$, D is $[0\bar{1}0]_V$, E is $[111]_V$, F is $[\bar{1}11]_V$, G is $[\bar{1}\bar{1}1]_V$, H is $[1\bar{1}1]_V$ and I is $[001]_V$. For clarity, only loading directions in the northern hemisphere and on the equatorial plane have been specified in the stereogram in (d) and other stereograms in this paper.

Figure 52 Source [210] Figure 4

- At small scale the medium as a **crystallin structure with some particular direction the Poisson's ratio is 1** and other directions is in the curent range -1 < v < 0,5
- Observe in this example the direction that are as the wave polarisation + and x

[210] S. Shrikanth , Kevin M. Knowles , Suresh Neelakantan , Rajesh Prasad dans leur publication de (2020) « Planes of isotropic Poisson's ratio in anisotropic crystalline solids »

Space as an anisotropic medium on a small scale? **fourth approach** to solve the problem

One way to satisfy the criteria of small modification of the classical RG is to consider in addition of the classical curvature a geometric torsion.

M.L Ruggiero and A, Tartaglia in « Einstein-Cartan theory of defect of space time » precise that effectively the effect of the torsion is very small:

Einstein-Cartan theory as a theory of defects in space-time

M. L. Ruggiero* and A. Tartaglia†
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The Einstein-Cartan theory of gravitation and the classical theory of defects in an elastic medium are presented and compared. The former is an extension of general relativity and refers to four-dimensional space-time, while we introduce the latter as a description of the equilibrium state of a three-dimensional continuum. Despite these important differences, an analogy is built on their common geometrical foundations, and it is shown that a space-time with curvature and torsion can be considered as a state of a four-dimensional continuum containing defects. This formal analogy is useful for illustrating the geometrical concept of torsion by applying it to concrete physical problems. Moreover, the presentation of these theories using a common geometrical basis allows a deeper understanding of their foundations.

Trautman[28] introduced a characteristic length to estimate the effects of torsion, the "Cartan" radius. To achieve the condition $\rho \simeq \kappa s^2$, we can imagine that a nucleon of mass m should be squeezed so that its radius coincides with the Cartan radius r_{Cart}:[28]

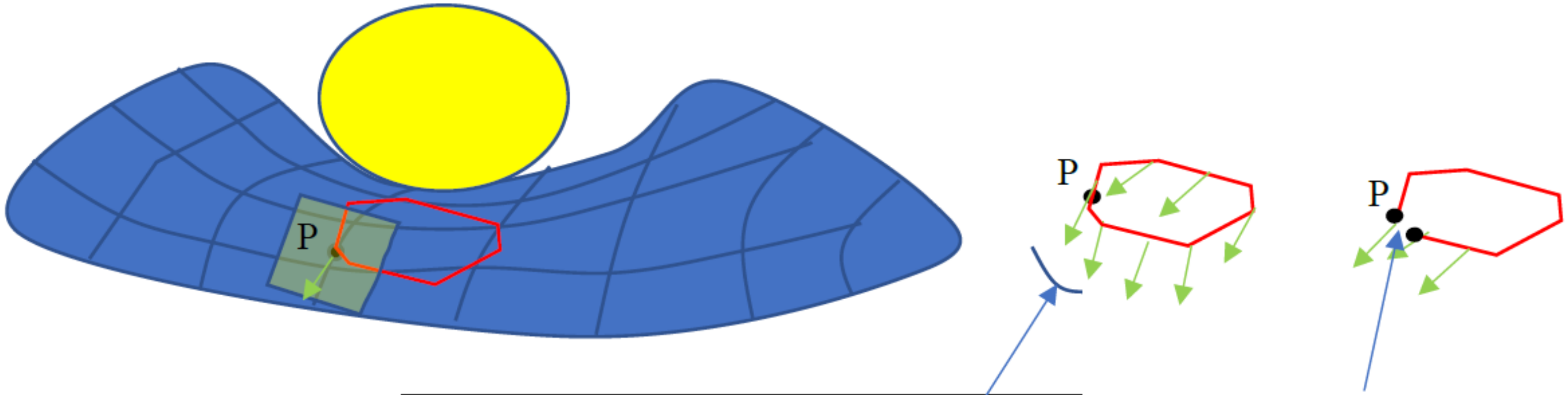
$$\frac{m}{r^3_{\mathrm{Cart}}} \simeq \kappa\left(\frac{\hbar}{r^3_{\mathrm{Cart}}}\right)^2, \tag{38}$$

whence

$$r_{\mathrm{Cart}} \simeq (l^*)^{\frac{2}{3}}(r_{\mathrm{Compt}})^{\frac{1}{3}}, \tag{39}$$

where $l^* \simeq 1.6 \times 10^{-33}$ cm is the Planck length, and r_{Compt} is the Compton length. For a nucleon we obtain $r_{\mathrm{Cart}} \simeq 10^{-26}$ cm, which is very small when compared with macroscopical scales, but it is larger than the Planck length. Hence, torsion must be taken into account to achieve a quantum theory of gravity.

The Einstein-Cartan theory in some words and equations



The vectors do not point in the same direction so there is curvature

If the curve does not close, then there is geometric torsion (modified general

Link with quantum field theory:
Curvature $G_{\mu\nu}$ => mass energy $T_{\mu\nu}$
Twisting by torsion => spin of

Figure 23: Curvature of space-time parallel transport - twisting of space-time [162] -

[162] V. Nikiforova (2020) Einstein généralisé - Théorie Cartanienne de la gravité Institut des hautes études Scientifiques (IHES) https://av.tib.eu/media/50915

Mathematically the Riemann tensor with geometric torsion change

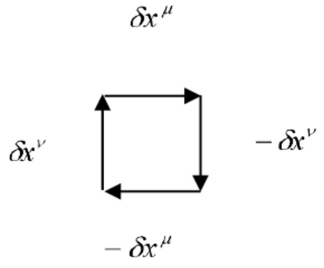
- Classical general relativity **(without geometric torsion)**

$$R_{\alpha\beta\mu\nu} = \frac{1}{2}\{g_{\beta\mu,\nu\alpha} - g_{\beta\nu,\mu\alpha} + g_{\alpha\nu,\mu\beta} - g_{\alpha\mu,\nu\beta}\} - g_{\sigma\rho}\{\Gamma^{\sigma}{}_{\alpha\mu}\Gamma^{\rho}{}_{\beta\nu} - \Gamma^{\sigma}{}_{\alpha\nu}\Gamma^{\rho}{}_{\beta\mu}\}$$

With: $R_{\alpha\beta\mu\nu} + R_{\alpha\nu\beta\mu} + R_{\alpha\mu\nu\beta} = 0$

$$g_{\beta\mu,\nu\alpha} = \frac{\partial^2 g_{\beta\mu}}{\partial x^{\nu}\partial x^{\alpha}};\ g_{\beta\nu,\mu\alpha} = \frac{\partial^2 g_{\beta\nu}}{\partial x^{\mu}\partial x^{\alpha}};\ g_{\alpha\nu,\mu\beta} = \frac{\partial^2 g_{\alpha\nu}}{\partial x^{\mu}\partial x^{\beta}};\ g_{\alpha\mu,\nu\beta} = \frac{\partial^2 g_{\alpha\mu}}{\partial x^{\nu}\partial x^{\beta}}$$

The curvature tensor therefore tells us how a vector changes when transported parallel to itself along an infinitesimal closed curve:



The curvature is determined using the parallel transport tool associated at the covariant derivative

So, without torsion we have for the covariant derivative of the vector V^{μ} transported parallel along a closed curve (vector at a point P located in the plane tangent to the surface along a curve, closed See Figure 24) equal to 0:

The covariant derivative = 0

$$D_{\nu}V^{\mu} = 0 \rightarrow D_{\nu}V^{\mu} = \partial_{\nu}V^{\mu} + \Gamma^{\mu}{}_{\nu\alpha}V^{\alpha} = 0$$

$$\partial_{\nu}V^{\mu} = \frac{\partial}{\partial x^{\nu}}V^{\mu} = -\Gamma^{\mu}{}_{\nu\alpha}V^{\alpha} = 0$$

Mathematically the Riemann tensor with geometric torsion change

=> Modified General relativity **(with geometric torsion):**

The parallel transport does not close

From a mathematical point of view the variation of the component μ of the contravariant vector V in

general relativity without torsion during parallel transport $\delta V^{//\mu}$:

$$\delta V^{//\mu} = -(\Gamma^{\mu}_{\ \nu\alpha} V^{\alpha})\delta x^{\nu}$$

Implies in the case of a geometric torsion of the Einstein-Cartan type:

Main difference

$$T^{\mu}_{\ \nu\alpha} = \Gamma^{\mu}_{\ \nu\alpha} - \Gamma^{\mu}_{\ \alpha\nu} = 2\Gamma^{\mu}_{\ [\nu\alpha]} \neq 0$$

The geometric torsion destroys the parallelogram in Figure 25.

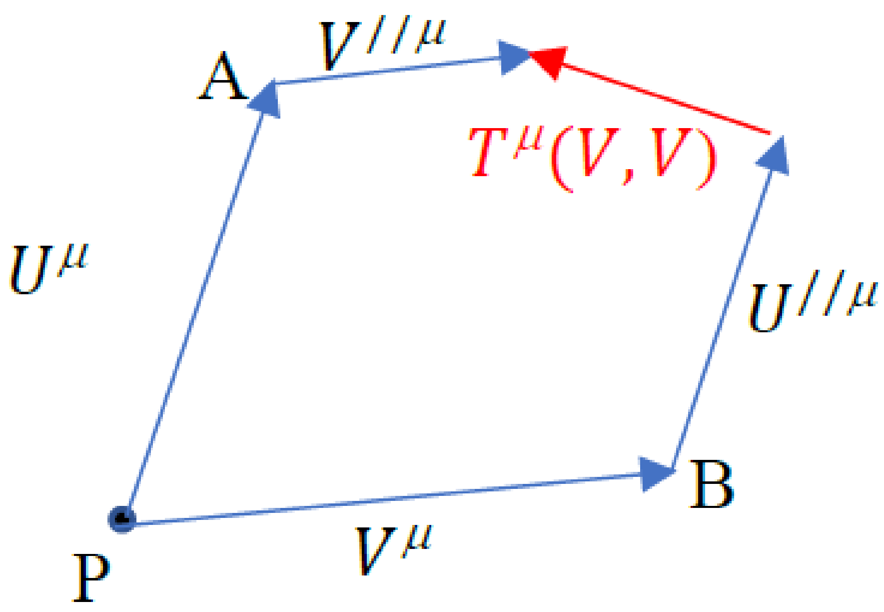
For symmetric components:

$$[\nu\alpha] = \frac{1}{2}(\nu\alpha + \alpha\nu)$$

For anti-symmetrical components:

$$[\nu\alpha] = \frac{1}{2}(\nu\alpha - \alpha\nu)$$

In classical Einstein general relativity $\Gamma^{\mu}_{\ \nu\alpha} = \Gamma^{\mu}_{\ \alpha\nu}$ and therefore the geometric torsion disappears and its remains the well-known the Riemann tensor.



The Riemann Tensor with geometric Torsion

$$[D_\mu, D_\nu]V^\rho = D_\mu D_\nu V^\rho - D_\nu D_\mu V^\rho = (\partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma})V^\sigma - 2\Gamma^\lambda_{[\mu\nu]}D_\lambda V^\rho$$

We define the Riemann tensor:

$$R^\rho_{\sigma\mu\nu} = (\partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma})$$

Complementary term du to the geometric torsion

We therefore obtain at the end the following expressions of the Riemann tensor and the torsion tensor obtained from the second derivatives covariants of the contravariant tensor V^ρ:

$$[D_\mu, D_\nu]V^\rho = D_\mu D_\nu V^\rho - D_\nu D_\mu V^\rho = R^\rho_{\sigma\mu\nu}V^\sigma - T^\lambda_{\mu\nu}D_\lambda V^\rho$$

$$[D_\mu, D_\nu]V^\rho = D_\mu D_\nu V^\rho - D_\nu D_\mu V^\rho = R^\rho_{\sigma\mu\nu}V^\sigma - 2\Gamma^\lambda_{[\mu\nu]}D_\lambda V^\rho$$

$$[D_\mu, D_\nu]V^\rho = D_\mu D_\nu V^\rho - D_\nu D_\mu V^\rho = (\partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma})V^\sigma + (\Gamma^\lambda_{\nu\mu} - \Gamma^\lambda_{\mu\nu})D_\lambda V^\rho$$

$$[D_\mu, D_\nu]V^\rho = D_\mu D_\nu V^\rho - D_\nu D_\mu V^\rho = (\partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma})V^\sigma + (\Gamma^\lambda_{\nu\mu} - \Gamma^\lambda_{\mu\nu})(\partial_\lambda V^\rho + \Gamma^\rho_{\lambda\sigma}V^\sigma)$$

Different ways to present mathematically the geometric torsion

Remark

We had obtained:

$$D_\alpha(D_\nu V^\mu) - D_\nu(D_\alpha V^\mu) = (-\partial_\nu \Gamma^\rho_{\alpha\mu} + \partial_\alpha \Gamma^\rho_{\nu\mu} + \Gamma^\rho_{\alpha\sigma}\Gamma^\sigma_{\nu\mu} - \Gamma^\rho_{\nu\sigma}\Gamma^\sigma_{\alpha\mu})V^\mu$$

In the right parenthesis of the above expressions if we make the following index change: ($\mu \rightarrow \sigma$; $\alpha \rightarrow \mu$; $\sigma \rightarrow \lambda$), we get well:

$$D_\mu(D_\nu V^\rho) - D_\nu(D_\mu V^\rho) = (-\partial_\nu \Gamma^\rho_{\mu\sigma} + \partial_\mu \Gamma^\rho_{\nu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma})V^\sigma$$

Einstein-Cartan theory in 3 and 4 dimensions

III.3.15 Einstein-Cartan Linearized 3-dimensional theory [159] and [153 chapters 3.2 and 3.3]

In **[153]** p1395 formula 3.52 and 3.53 and **[159]** formula 40 and 41 it is shown that in 3 dimensions the linearized gravitational field equation with geometric torsion now has 2 equations:

First equation (3.52) of [153]:

Curvature

$$G_{ij} - \frac{1}{2}\partial_k\left(S_{ij,k} - S_{jk,i} - S_{ki,j}\right) = -\frac{8\pi G}{c^4}T_{ij} = -\kappa T_{ij}$$

With:

The torsion tensor (related to the density of dislocations)

$$S^{\mu}{}_{\nu\lambda} = \frac{1}{2}\left(\Gamma^{\mu}{}_{\nu\lambda} - \Gamma^{\mu}{}_{\lambda\nu}\right)$$

$$S_{\mu\nu\lambda} = S_{\mu\nu}{}^{\eta} g_{\eta\lambda}$$

Or depending on the dislocation density:

$$S_{ij,k} = \varepsilon_{ijl}\alpha_{kl}$$

Einstein's tensor:

$$G_{ij} = R_{ji} - \frac{1}{2}g_{ij}R^k_k$$

The classical stress-energy tensor of General Relativity:

$$T_{ij}$$

$$G_{ij} - \frac{1}{2}\partial_k\left(P^k_{ij} - P^k_{ji} - P^k_{ij}\right) = \kappa T_{ij}$$

$$P_{ijk} = 2S_{ijk} + 2\delta_{ik}S_j - 2\delta_{jk}S_i = \kappa\sum\nolimits_{ijk}$$

With in 3 dimensions:

T_{ij} the stress field (force)

$\sum_{ijk}$ constraint field (Moment)

$P_{ijk} = \varepsilon_{ijl}\alpha^l_k$

α^l_k dislocation density s

Einstein-Cartan theory =2 equations

Second equation (3.53) of [153]:

And for spin density, a complementary equation:

spin

$$-\frac{1}{2}\kappa\sum\nolimits_{ij,k} = \frac{1}{2}S_{ij,k} = S_{ijk} + \delta_{ik}S_j - \delta_{jk}S_i$$

$$\kappa\sum\nolimits_{ij,k} = 2\,S_{ijk} + 2\delta_{ik}S_j - 2\delta_{jk}S_i$$

Or depending on the dislocation density:

$$\kappa\sum\nolimits_{ij,k} = \varepsilon_{ijl}\,\alpha_{kl}$$

It can also be shown that the stress energy tensor is equivalent to the default density at factor 1/κ ready:

$$T_{ij} = -\frac{1}{\kappa}\eta_{ij}$$

According to M Ruggiero **[159]** formula (40) and (41) the Einstein-Cartan theory linearized theory in 3D is written:

III.3.16 Einstein-Cartan 4-dimensional theory [159] and [153 chapters s 3.2 and 3.3]

Einstein's theory of relativity is written without twisting:

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

With:

$$\kappa = \frac{8\pi G}{c^4}$$

When we include geometric torsion, we introduce a new degree of freedom, and the starting equation is now broken down into 2 equations described in the Einstein-Cartan formalism (See **[159]** formula (34) and (35)):

$$\begin{cases} G^{\mu\nu} - \frac{1}{2}D^*_{\gamma}\left(P^{\mu\nu\gamma} - P^{\nu\gamma\mu} + P^{\gamma\mu\nu}\right) = \kappa T^{\mu\nu} \\ P^{\gamma}_{\mu\nu} = \kappa\sum\nolimits^{\gamma}_{\mu\nu} \end{cases}$$

With:

D_{γ} the covariant derivative

$D^*_{\gamma} = D_{\gamma} + 2S_{\gamma} = D_{\gamma} + 2S^{\lambda}{}_{\gamma\lambda}$

$P^{\gamma}_{\mu\nu}$ is the Palatini tensor: $(\frac{1}{2}P^{\gamma}{}_{\mu\nu} = S^{\gamma}_{\mu\nu} + \delta^{\gamma}_{\mu}S_{\nu} - \delta^{\gamma}_{\nu}S_{\mu}$

$S^h_{ml} = \alpha^h_{ml}$ for the torsion tensor

[159] M. L. Ruggiero et A. Tartaglia (2003) «Einstein-Cartan theory as a theory of defects in spacetime» American Journal of Physics 71, 1303

[153] H. Kleinert (1989) «Gauge Fields in Condensed Matter, II. Stresses an Defects»WorldScientific,Singapore,

Einstein Cartan Theory in 3D

III.3.15 Einstein-Cartan Linearized 3-dimensional theory [159] and [153 chapters 3.2 and 3.3]

In **[153]** p1395 formula 3.52 and 3.53 and **[159]** formula 40 and 41 it is shown that in 3 dimensions the linearized gravitational field equation with geometric torsion now has 2 equations:

First equation (3.52) of [153]:

Curvature

$$G_{ij} - \frac{1}{2}\partial_k(S_{ij,k} - S_{jk,i} - S_{ki,j}) = -\frac{8\pi G}{c^4}T_{ij} = -\kappa T_{ij}$$

With:

The torsion tensor (related to the density of dislocations)

$$S^{\mu}{}_{\nu\lambda} = \frac{1}{2}(\Gamma^{\mu}{}_{\nu\lambda} - \Gamma^{\mu}{}_{\lambda\nu})$$

$$S_{\mu\nu\lambda} = S_{\mu\nu}{}^{\eta} g_{\eta\lambda}$$

Or depending on the dislocation density:

$$S_{ij,k} = \varepsilon_{ijl}\alpha_{kl}$$

Einstein's tensor:

$$G_{ij} = R_{jl} - \frac{1}{2}g_{ij}R^k_k$$

The classical stress-energy tensor of General Relativity:

$$T_{ij}$$

$$G_{ij} - \frac{1}{2}\partial_k(P^k_{ij} - P^k_{ji} - P^k_{ij}) = \kappa T_{ij}$$

$$P_{ijk} = 2S_{ijk} + 2\delta_{ik}S_j - 2\delta_{jk}S_i = \kappa\sum_{ijk}$$

With in 3 dimensions:

T_{ij} the stress field (force)

$\sum_{ijk}$ constraint field (Moment)

$P_{ijk} = \varepsilon_{ijl}\alpha^l_k$

α^l_k dislocation density s

Second equation (3.53) of [153]:

And for spin density, a complementary equation:

spin

$$-\frac{1}{2}\kappa\sum_{ij,k} = \frac{1}{2}S_{ij,k} = S_{ijk} + \delta_{ik}S_j - \delta_{jk}S_i$$

$$\kappa\sum_{ij,k} = 2S_{ijk} + 2\delta_{ik}S_j - 2\delta_{jk}S_i$$

Or depending on the dislocation density:

$$\kappa\sum_{ij,k} = \varepsilon_{ijl}\alpha_{kl}$$

It can also be shown that the stress energy tensor is equivalent to the default density at factor 1/κ ready:

$$T_{ij} = -\frac{1}{\kappa}\eta_{ij}$$

According to M Ruggiero **[159]** formula (40) and (41) the Einstein-Cartan theory linearized theory in 3D is written:

Einstein-Cartan theory =2 equations

The density of defects χ^{ij} is expressed in terms of the density of dislocations α_{ij} and the disclinations Θ_{ij} by the following relation (see, for example, Ref. 7):

$$\chi^{ij} \equiv \Theta^{ji} - \epsilon^{jmn}\partial_m(-\alpha^i_n + \tfrac{1}{2}\delta^i_n\alpha^k_k). \qquad (15)$$

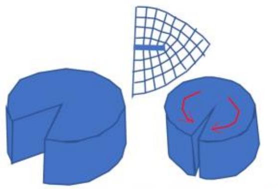


Figure 33 : Disclination [153 fig 2.10] -

The disclination density corresponding to each line is given by

$$\Theta_{ij}(\mathbf{x}) = \delta_i(L)\Omega_j, \qquad (12)$$

where $\delta_i(L)$ is a Dirac delta function which is nonzero only along the disclination line L.[7]

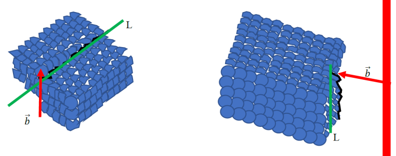


Figure 32a: Screw dislocation- [153 fig2.9] Figure 32b: Edge Dislocation –

The meaning of Eq. (25) is that dislocations, through their density α^h_{ml} [defined in Eq. (23)], constitute the sources for torsion. This result is very general, and does not depend on the coordinates we used, because torsion is a tensor.

Defect theory in 3D

…e curvature of the medium also is influenced by the presence of defects.

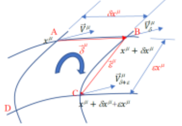
Defect = Disclination =curvature

$$\Theta_{ij} = \varepsilon_{ikl}\partial_k\partial_l\omega_j$$

$$V^{\mu}_{\varepsilon+\delta} - V^{\mu}_{\delta+\varepsilon} = -\frac{1}{2}R^{\mu}_{\lambda\kappa\nu}(\delta x^{\kappa}\varepsilon x^{\nu} - \delta x^{\nu}\varepsilon x^{\kappa})V^{\lambda} = -\frac{1}{2}R^{\mu}_{\lambda\kappa\nu}V^{\lambda}ds^{\kappa\nu}$$

$$\Delta v^h = -\tfrac{1}{2}R^h_{nml}\Delta A^{nm}v^l$$

$$R^h_{nml} = \Theta^h_{nml}$$



$$G^{ij} \equiv \tfrac{1}{4}e^{inm}e^{jlq}R_{nmlq}$$

$$G^{ij} = \sigma^{ij},$$

$$\sigma^{ij} = \chi^{ij} + \partial_k\epsilon^{jkl}(-\alpha^i_l + \tfrac{1}{2}\delta^i_l\alpha^n_n)$$

$$\chi^{ij} = G^{ij} - \partial_k\epsilon^{jkl}k^i_l$$

$$k^i_l = (-\alpha^i_l + \tfrac{1}{2}\delta^i_l\alpha^n_n)$$

Defect = Dislocation = torsion via the Burgers vector

$$\alpha_{ij} = \varepsilon_{ikl}\partial_k\partial_l u_j$$

$S^h_{ml} = \Gamma^h_{[ml]}$ is called the torsion tensor.

$$dA^{ml} = d_1x^m d_2x^l - d_1x^l d_2x^m$$

$$db^h = S^h_{ml}dA^{ml}.$$

$$db^h = \tfrac{1}{2}\alpha^{nh}e_{nml}dA^{ml} \doteq \alpha^h_{ml}dA^{ml}$$

$$S^h_{ml} = \alpha^h_{ml}.$$

$P^{\gamma}_{\alpha\beta}$ is the Palatini tensor $(\frac{1}{2}P^{\gamma}_{\alpha\beta} = S^{\gamma}_{\alpha\beta} + \delta^{\gamma}_{\alpha}S_{\beta} - \delta^{\gamma}_{\beta}S_{\alpha})$ and $S^{\gamma}_{\alpha\beta} = \alpha^{\gamma}_{\alpha\beta}$

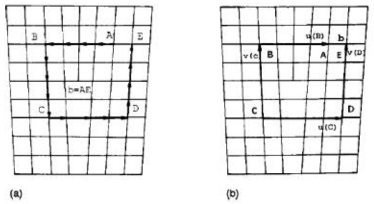


Fig. 7. Dislocations and geometry. (a) on the left the usual Burgers circuit around an edge dislocation, and (b), on the right, the differential geometric view in terms of parallel transport.

where e_{nml} and e^{nml} are defined in terms of the completely antisymmetric tensors ϵ_{nml}, ϵ^{nml}, and the determinant of the metric g:

$$e_{nml} = \sqrt{g}\,\epsilon_{nml} \quad e^{nml} = \frac{1}{\sqrt{g}}\epsilon^{nml}.$$

Einstein Cartan Theory

That we can with the Einstein-Cartan theory linearized in 4 Dimensions (See **[159]** formula (46) at (48))

$$\begin{cases} G^{\mu\nu} = \kappa \Psi^{\mu\nu} \\ P^{\gamma}_{\mu\nu} = \kappa \Sigma^{\gamma}_{\mu\nu} \end{cases}$$

With:

$$\Psi^{\mu\nu} = T^{\mu\nu} + \frac{1}{2} D^{*}_{\gamma} (\Sigma^{\mu\nu\gamma} - \Sigma^{\nu\gamma\mu} + \Sigma^{\gamma\mu\nu})$$

It can therefore be seen that the formalism of the theory of defects is identical to that of the Einstein-Cartan theory.

$P^{\gamma}_{\alpha\beta}$ is the Palatini tensor ($\frac{1}{2} P^{\gamma}_{\alpha\beta} \equiv S^{\gamma}_{\alpha\beta} + \delta^{\gamma}_{\alpha} S_{\beta} - \delta^{\gamma}_{\beta} S_{\alpha}$)

and $S^{\gamma}_{\alpha\beta} = \alpha^{\gamma}_{\alpha\beta}$

$$\sigma^{ij} = G^{ij} - \partial_k \varepsilon^{jkl} k^{i}_{l} + \partial_k \varepsilon^{jkl} \left(-\alpha^{i}_{l} + \frac{1}{2} \delta^{i}_{l} \alpha^{n}_{n} \right)$$

$$\sigma^{ij} = G^{ij} - \partial_k \varepsilon^{jkl} \left(-\alpha^{i}_{l} + \frac{1}{2} \delta^{i}_{l} \alpha^{n}_{n} \right) + \partial_k \varepsilon^{jkl} \left(-\alpha^{i}_{l} + \frac{1}{2} \delta^{i}_{l} \alpha^{n}_{n} \right)$$

Consequently, there is the following analogy between the defect theory and the general relativity modified (Einstein Cartan with geometric torsion):

A link between the curvature tensor of Riemann and the **disclination density** (see **[159]** formula (27)):

$$R^{h}_{nml} = \Theta^{h}_{nml}$$

A link between the torsion tensor of Einstein-cartan and the **dislocation density** (see **[159]** formula

Defect theory

The extension of the 4-dimensional defect theory is therefore written (see **[159]** formula (44) and (45)):

$$\begin{cases} G^{\mu\nu} = \sigma^{\mu\nu} \\ S^{\gamma}_{\mu\nu} = \alpha^{\gamma}_{\mu\nu} \end{cases}$$

With (see **[159]** formula 28 to 31):

$$\sigma^{ij} = \chi^{ij} + \partial_k \varepsilon^{jkl} \left(-\alpha^{i}_{l} + \frac{1}{2} \delta^{i}_{l} \alpha^{n}_{n} \right)$$

$$\chi^{ij} = G^{ij} - \partial_k \varepsilon^{jkl} k^{i}_{l}$$

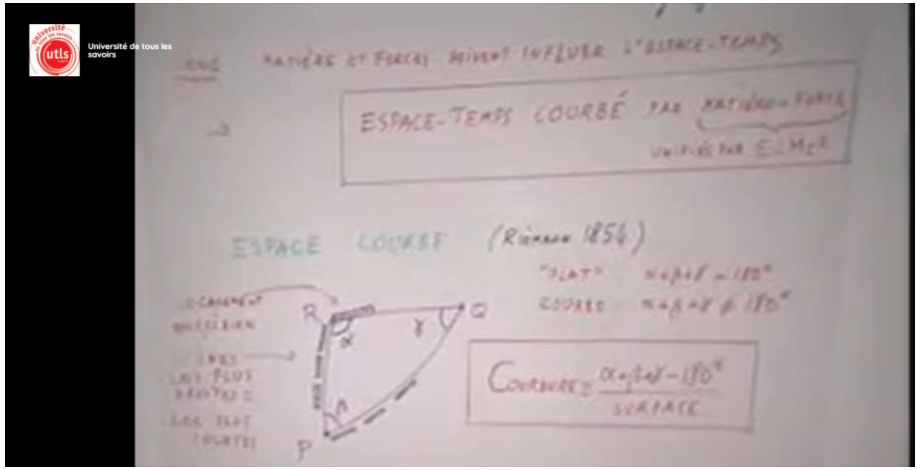
$$G^{ij} = \frac{1}{4} e^{inm} e^{jlq} R_{mnlq}$$

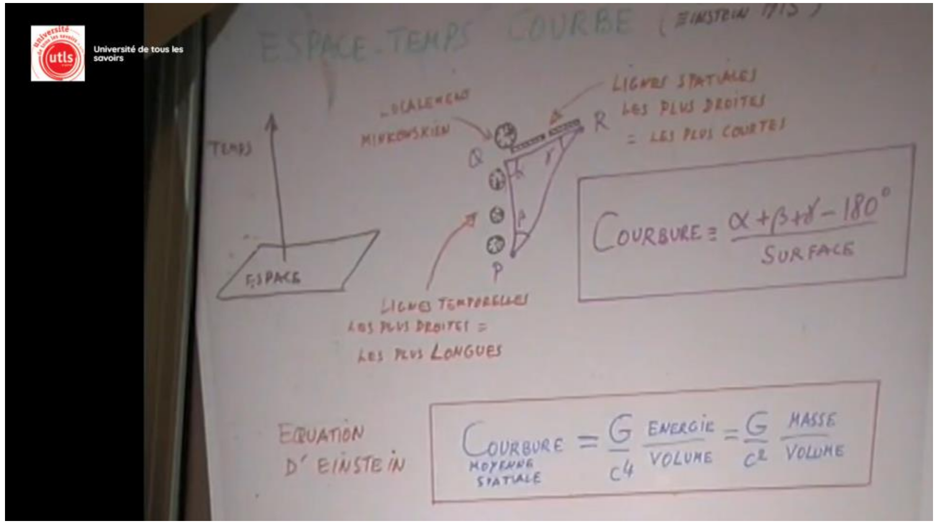
$$e_{nml} = \sqrt{g}\, \varepsilon_{nml} e^{nml} = \frac{1}{\sqrt{g}} \varepsilon^{nml}$$

$$k^{i}_{l} = \left(-\alpha^{i}_{l} + \frac{1}{2} \delta^{i}_{l} \alpha^{n}_{n} \right)$$

Curvature

Source université de tous les savoirs
T.Damour 2000



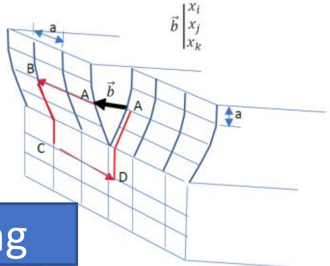


Torsion

$db^{\mu} = -I_{[\nu\lambda]}^{\ \ \mu} dA^{\nu\lambda}$

$S_{\nu\lambda}^{\ \ \mu} = I_{[\nu\lambda]}^{\ \ \mu} = (1/2) T_{\nu\lambda}^{\ \ \mu} = 0$

$-I_{[\nu\lambda]}^{\ \ \mu} \rightarrow \lim \dfrac{db^{\mu}}{dA^{\nu\lambda}}$



Systèmes en involution
et parallélisme absolu

Paris, IHP
11 mai 2007

Joël MERKER
(CNRS & DMA)

Theorie der Transformationsgruppen

I. Le problème de Riemann–Lie–Helmholtz

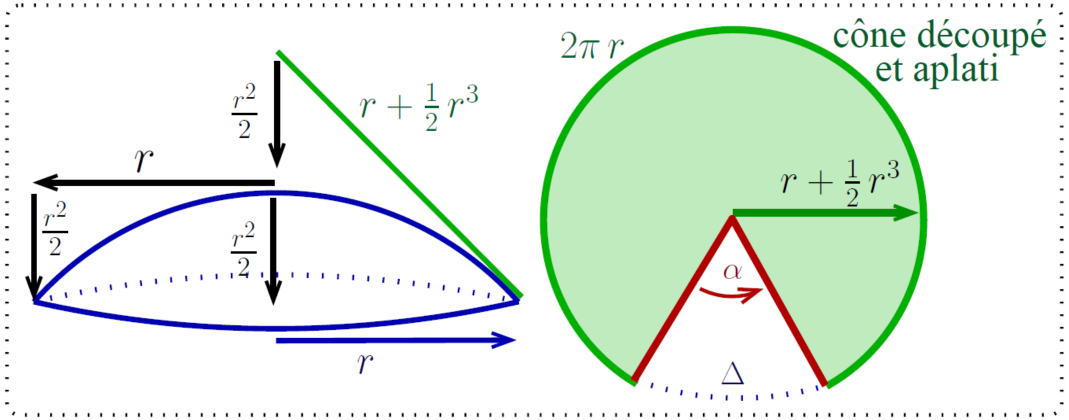
www.dma.ens.fr/~merker/

spacing

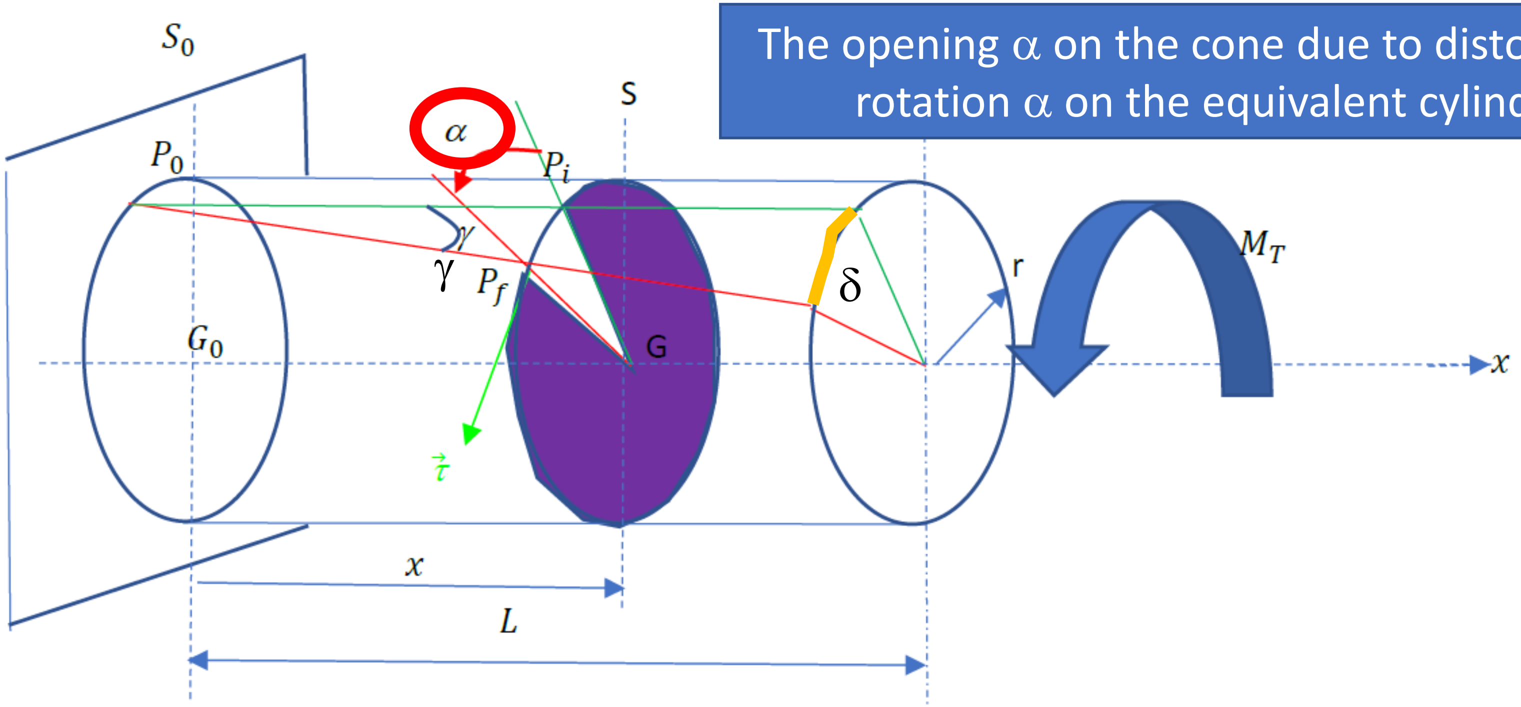
Area

$$\textbf{Torsion} := \lim \frac{\textbf{Écartement}}{\textbf{aire}} = \lim \frac{\Delta}{\pi r^2} = \lim \frac{\pi r^3}{\pi r^2} = 0.$$

$$\textbf{courbure} := \lim \frac{\textbf{angle}}{\textbf{aire}} = \lim \frac{\alpha}{\pi r^2} = \lim \frac{\pi r^2}{\pi r^2} = 1.$$



Proposal conclusion about the interaction of the different views about torsion



The opening α on the cone due to distorsion =equivalent rotation α on the equivalent cylinder in torsion

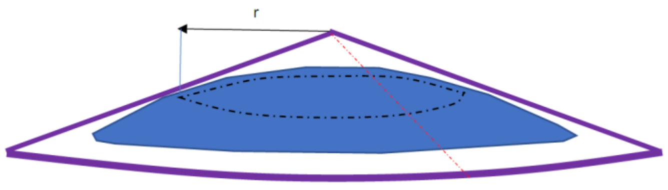


Figure F.1: Cone tangent to a spherical cap of unit radius Source **[168]** and **[169]** -

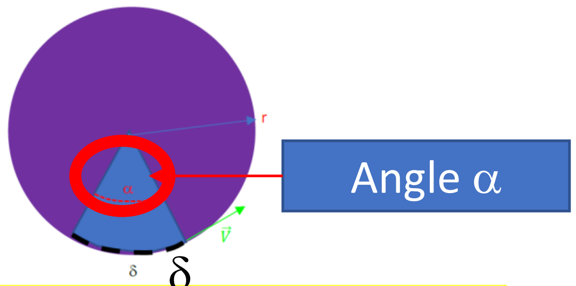


Figure F.2: Flat cut cone. Visualization of the cut-off δ and visualization of the parallel transport of the vector $\vec{V}$ **[168]** and **[169]** -

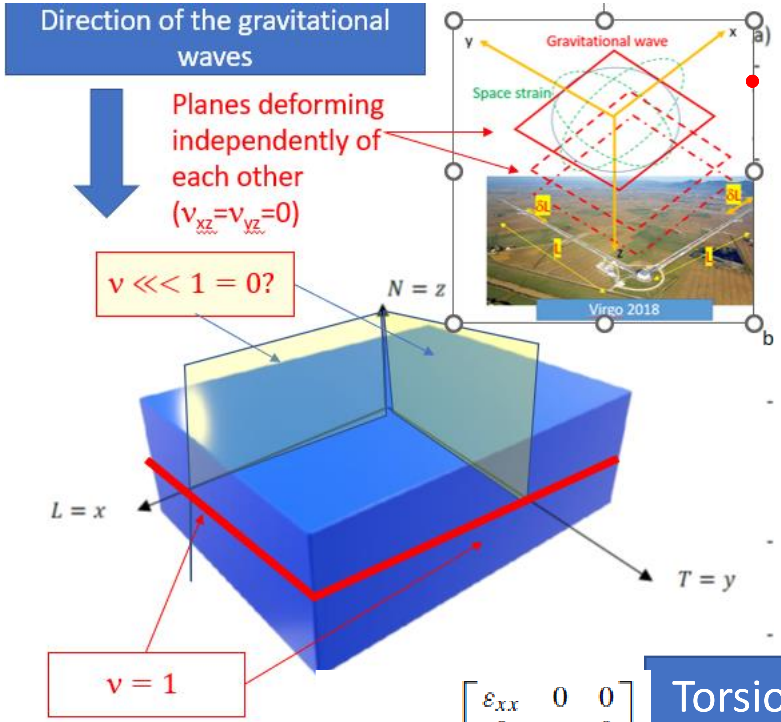
$$\gamma = \frac{\alpha}{x} r$$

$$\delta = \alpha r = \left(\frac{\gamma x}{r}\right) r = \gamma x$$

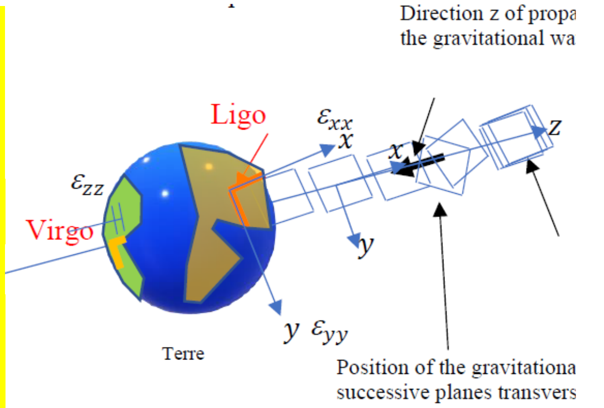
$$Torsion = \lim \frac{Spacing}{Area} = \lim \frac{\delta}{\pi r^2} = \lim \frac{\alpha r}{\pi r^2} = \lim \frac{\gamma x}{\pi r^2}$$

$$\delta = \alpha r$$

First interest of the torsion: The geometric torsion by this analogy with the defects theory allow to propose a way to explain how the gravitational wave deformation pass to a plan to an other successively by local dislocation in plasticity



- The propagation of the deformation of the space put in torsion by rotation for example of two black holes is equivalent at a propagation of a dislocation of the equivalent elastic material of the space time. Thus, the Einstein-Cartan and defect theory (α angle opening of dislocation associated with the burgers Vector) are correlated with the mechanical torsion (γ for the shear strain due to the torsion torque T and α the rotation of the section in pure torsion) must be considered in the Einstein-Field equation.



$$\varepsilon_{xy(A_+)} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Torsion : a solution to link the several plans distorted by GW

$$\varepsilon_{xy(A_\times)} = \begin{bmatrix} 0 & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

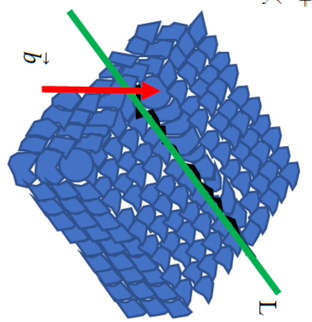


Figure 32a: Screw dislocation- [153 fig2.9]

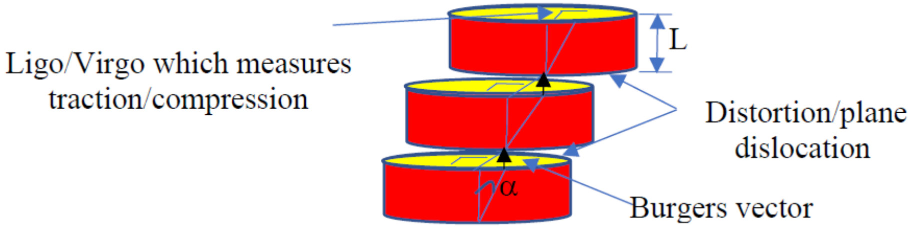


Figure 55. Visualization of the dislocations between the different layers of the structure of the fabric of space-time during the passage of a gravitational wave -

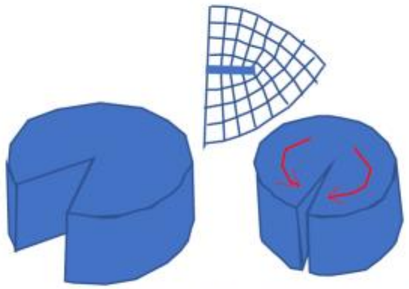


Figure 33 : Disclination [153 fig 2.10] -

Second interest of the torsion: the Shapes of the polarizations of the gravitational waves in the case of GR with geometric torsion correspond by analogy at the component of the stain tensor that are missing to have a spatial deformation and not only a plane deformation as in classical general relativity

Einstein-Cartan-Sciamma-Kibble

Gravitational Waves in ECSK theory: Robustness of mergers as standard sirens and nonvanishing torsion

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April 4, 2022

arXiv:2204.00090v1 [gr-qc] 31 Mar 2022

Abstract

The amplitude propagation of gravitational waves in an Einstein-Cartan-Sciamma-Kibble (ECSK) theory is studied by assuming a dark matter spin tensor sourcing for spacetime torsion at cosmological scales. The analysis focuses on a "weak-torsion regime," such that gravitational wave emission, at leading and subleading orders, does not deviate from standard General Relativity. We show that, in principle, the background torsion induced by an eventual dark matter spin component could lead to an anomalous dampening or amplification of the gravitational wave amplitude, after going across a long cosmological distance. The importance of this torsion-induced anomalous propagation of amplitude for binary black hole mergers is assessed. For realistic late-universe astrophysical scenarios, the effect is tiny and falls below detection thresholds, even for near-future interferometers such as LISA. To detect this effect may not be impossible, but it is still beyond our technological capabilities.

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of H_{ab}. For instance, it is possible to write the polarization of a gravitational wave propagating in the third direction of the orthonormal frame as

$$P_{ab} = p_{(+)}\mathbb{P}^{(+)}_{ab} + p_{(\times)}\mathbb{P}^{(\times)}_{ab} + p_{(b)}\mathbb{P}^{(b)}_{ab} + p_{(l)}\mathbb{P}^{(l)}_{ab} + p_{(x)}\mathbb{P}^{(x)}_{ab} + p_{(y)}\mathbb{P}^{(y)}_{ab}\,,$$

with the orthonormal polarization basis

$$\mathbb{P}^{(+)}_{ab} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbb{P}^{(\times)}_{ab} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

classical

$$\mathbb{P}^{(b)}_{ab} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbb{P}^{(l)}_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\mathbb{P}^{(x)}_{ab} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \mathbb{P}^{(y)}_{ab} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

New

and where

$$\bar{p}_{(+)}p_{(+)} + \bar{p}_{(\times)}p_{(\times)} + \bar{p}_{(b)}p_{(b)} + \bar{p}_{(l)}p_{(l)} + \bar{p}_{(x)}p_{(x)} + \bar{p}_{(y)}p_{(y)} = 1\,.$$

It is essential to remember that, even in the case of standard torsionless GR, further gauge fixing (as the transverse traceless gauge) is only approximate on a generic background geometry, and it is only valid at leading and subleading orders in the eikonal expansion. For a generic geometry background, this means that in standard torsionless GR, some polarization components dominate over others,

$$\bar{p}_{(+)}p_{(+)} + \bar{p}_{(\times)}p_{(\times)} \approx 1\,, \tag{2.20}$$

$$\bar{p}_{(b)}p_{(b)} + \bar{p}_{(l)}p_{(l)} + \bar{p}_{(x)}p_{(x)} + \bar{p}_{(y)}p_{(y)} \leq \epsilon^2\,. \tag{2.21}$$

This hierarchy of polarization modes can break down in a generic torsional background, and in principle, the modes beyond $(+)$ and $(\times)$ could become significant.

Use of the analogy

Time

$$g_{ij} = \eta_{ij} + h_{ij} = \eta_{ij} + 2\varepsilon_{ij}\, g_{ij} = \eta_{ij} + h_{ij} = \eta_{ij} + 2\varepsilon_{ij}$$

$$\mathbb{P}^{(b)}_{ab} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \mathbb{P}^{(l)}_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\mathbb{P}^{(x)}_{ab} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \qquad \mathbb{P}^{(y)}_{ab} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

$$T_{\mu\nu} = \begin{bmatrix} \frac{mc^2}{V} & \rho c v_x & \rho c v_y & \rho c v_z \\ \rho c v_x & \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \rho c v_y & \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \rho c v_z & \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

The analogy of the polarization with the deformation tensor makes it possible to immediately check the plausibility of the polarizations obtained. Here deformations appear in the direction of propagation of the gravitational wave suppressing the apparent discontinuity between deformation planes obtained in GR without torsion

Tensile Strain in the propagation direction of the gravitational wave

Shear Strain in the propagation direction z of the gravitational wave

Shape of the potential additional polarisation in case of geometric torsion

Strains in the plane xy due to the classical polarization in general relativity

Complementary strains in zx and zy due to polarizations of the gravitational wave in direction (z)

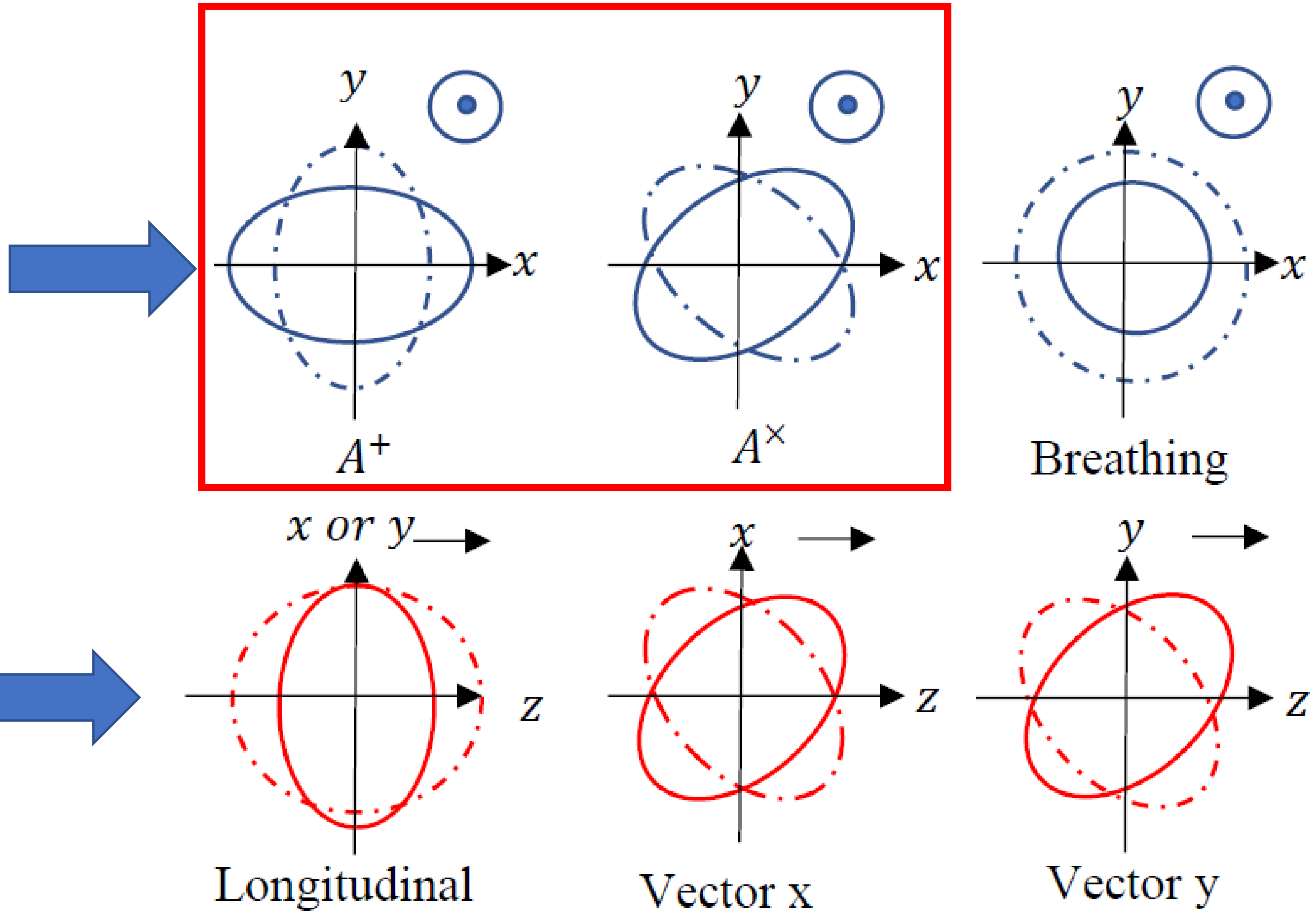

Figure 56: Different polarizations -Source Teukolsky

Third interest of the torsion: the Einstein Cartan theory and by analogy the defect theory introduce a second spin equation in general relativity that is a step in direction of the quantum gravity where spin is necessary

Einstein-Cartan theory as a theory of defects in space-time

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Dip. Fisica, Politecnico and INFN, Torino, Italy, I-10129

VI. SUMMARY

The Einstein–Cartan theory of gravitation has been introduced starting from an analogy with the static theory of defects, which describes the equilibrium state of a three-dimensional continuum; ECT agrees with the known tests of general relativity. Moreover, we stressed that the Einstein–Cartan theory has a richer geometric and physical structure. In particular, a nonsymmetric connection is used, and torsion is linked to the density of spin. In ECT, both the mass and spin determine the geometric properties of space–time and shape its structure.

We also showed that in the classical theory of defects, a geometric approach is possible and leads to the description of a continuous medium by means of geometric entities that are determined by the presence of defects, such as disclinations and dislocations which we related to curvature and torsion. Then we outlined a comparison between these two theories, which share a similar underlying geometric structure, even though they apply to very different physical phenomena.

We showed that the equations that describe the state of the medium and its structure in the presence of defects may be interpreted as the Einstein–Cartan field equations for a three-dimensional continuum, at least in the linear approximation. On the other hand, the incompatibility equation of the theory of defects, which is usually obtained in the linear approximation, can be extended to a more general situation, where defects are not necessarily assumed to be small.

By pursuing this formal analogy, space–time, as described by ECT, can be interpreted as a defect state of a four-dimensional continuum. We suggest that this analogy, although formal, might be useful in modern astrophysics, because cosmic strings may be interpreted as extensions of three-dimensional defects.

It is fascinating that the theory of defects, whose origin dates back to the beginning of the 20th century, can have such a strong and fruitful analogy with recent developments in theoretical physics. We believe that the analogy we have outlined can be useful for understanding the key concepts of differential geometry and the geometric theories of gravitation as well as helping to stimulate interest in these fields.

For me defect theory is near plastic deformation, in the thesis we stay in elastic

4 A rule in three dimensions: Dislocation density equals torsion

- In the 1930s, **the concept of a crystal dislocation was introduced in order to understand the plastic deformation of crystalline solids**, as, for instance, of iron. Dislocations are one-dimensional lattice defects. Basically, there exist two types of dislocations, edge and screw dislocation, see Weertman & Weertman

J. and J.A. Weertman, *Elementary Dislocation Theory*, MacMillan, London (1969).

Conclusion of Third part

1) Several solutions are possible to take into account the potential anisotropy of space and to complete the GR with small value of the deformations of the space (anisotropy at plank scale, direction of anisotropy due at crystallography structure, geometric torsion via Einstein-Cartan Theory),

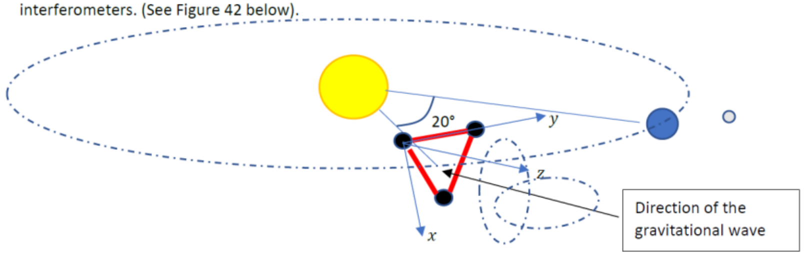
2) Einstein-Cartan theory and defect theory present a similar formalism. This analogy allow to have an image in elasticity of the geometrical torsion (local dislocation and yielding of the medium). This behaviour could a way to transmit the deformation plane to plane during the propagation of a gravitational wave,

3) The potential complementary polarizations associated with the geometric torsion complete the plane strain tensor associated at the classical general relativity,

4) The geometric torsion added at the general relativity is a step-in direction of the quantum gravity via the additional equation in link with spins and quantum gravity,

5) Complementary measurements via for example LISA are necessary to confirm or not these complementary deformations in the wave propagation direction.

The CONSTELLATION of LISA Satellites will therefore be sent into space in the coming years. The 3 satellites 2.5 million km apart will "exchange" their positions via lasers as in the LIGO/VIRGO interferometers. (See Figure 42 below).



How the distorsion (shear strain) γ can be measured by Lisa ?

DI: The Mohr 's circle explains how to check if there are or not distorsion (shear strain) of space when a gravitational waves passes

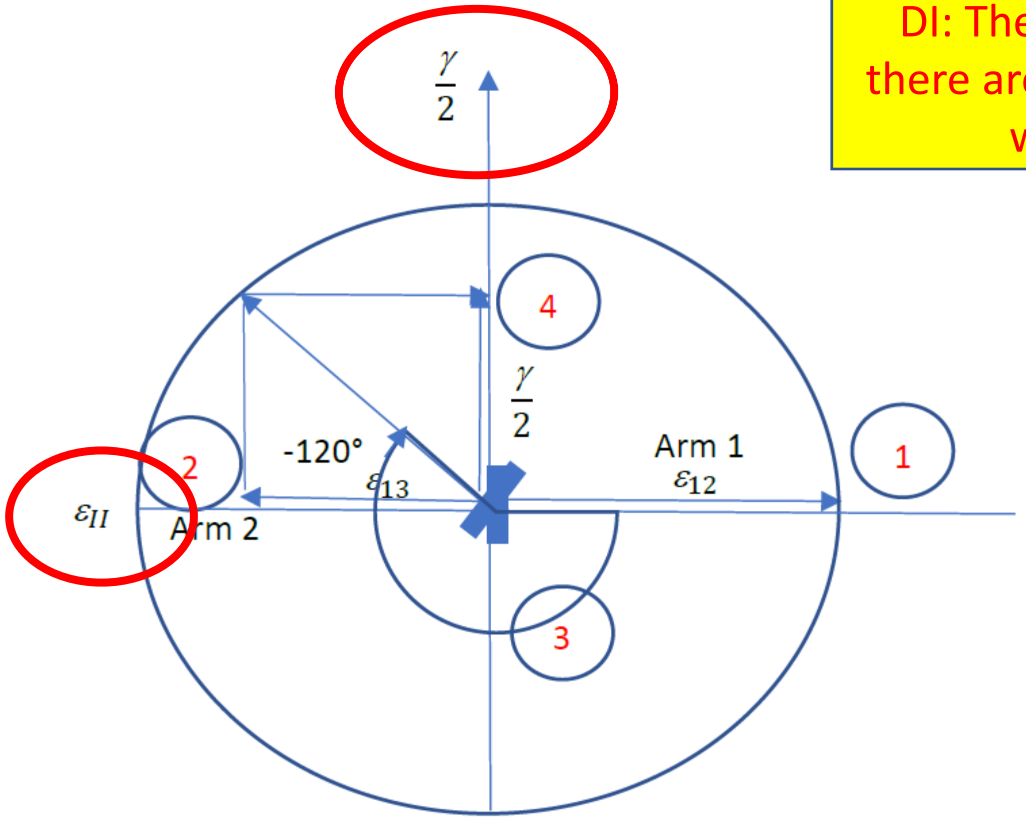


Figure 46: Mohr's circle of the strains plane $\vec{x}, \vec{y}$

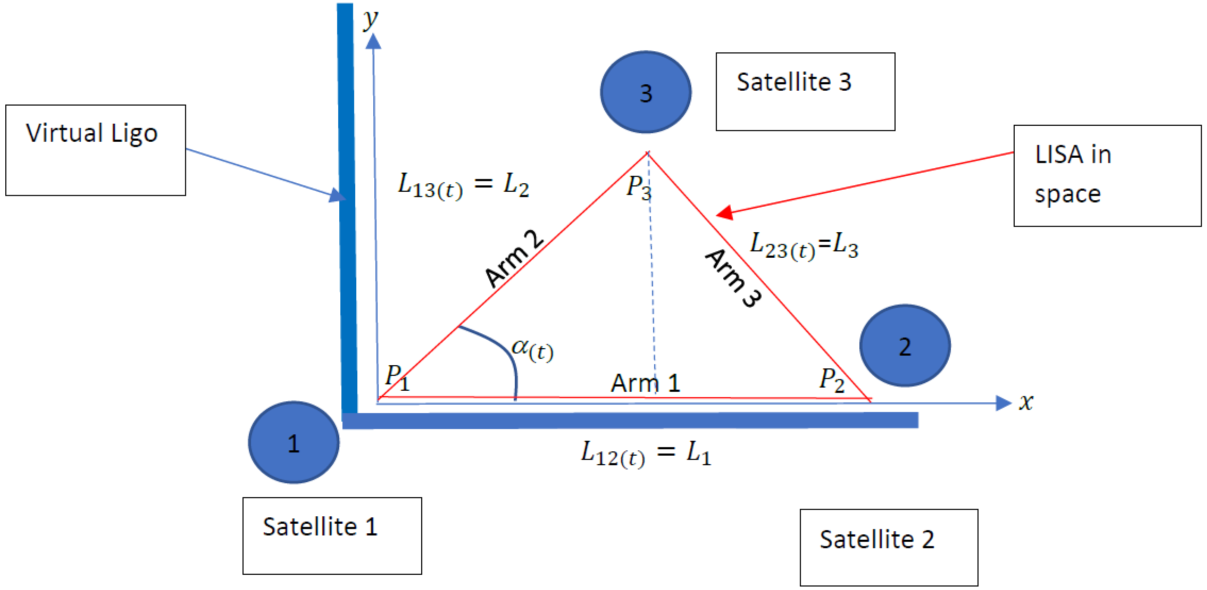


Figure 44: Locating LISA versus A Virtual Positioning of LIGO -

4) Conclusion

-

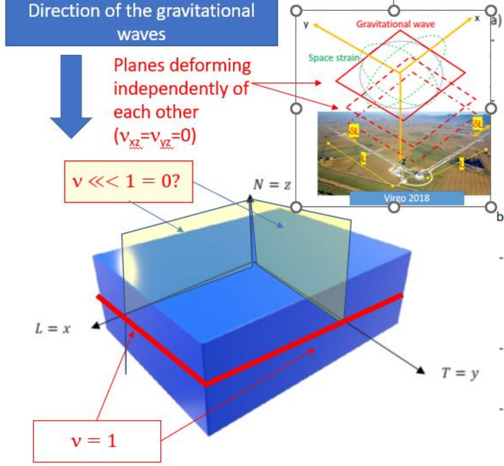
The interests and limits of this analogy : convergence points

Source T.Damour presentation

- At this stage :
- The analogy of the elastic medium with general relativity has many points of convergence:
  - The measurements of the deformations of space are compatible with an elastic medium responding to the stresses resulting from moving energy masses
  - The general form of Einstein's equation is compatible with Hooke's law transcribed in 1D in beam theory, in 2D in plate theory and in 3D in the stress-strain tensor relations in isotropic elasticity and in particular under mechanical torsion stresses
  - The shape of the deformation tensors and the polarizations of gravitational waves with and without geometric torsion (Einstein-Cartan) is compatible

the interest and limits of this analogy : Divergence points

Source wikipedia



- But the analogy reaches its limits:
- The correspondence between the polarizations of the gravitational waves and the deformation tensor, in line with the measurements made, lead to a Poisson's ratio of 1 in the plane of the interferometers, which is contrary to an isotropic medium, the basic hypothesis of relativity general when considering space and in cosmology (Friedmann Lemaitre equations)
- Geometric torsion partly repairs the behavior of space solicited by gravitational waves in independent planes deforming successively one after the other and independently of each other in the case of General Relativity without geometric torsion, but the deformations outside of these planes remain extremely small (scale) of Planck and therefore do not resolve this functioning in therefore discontinuous and non-homogeneous planes of space
- These plane distortions of space manifest themselves in all directions of space regardless of the direction of the gravitational waves! We thus arrive at a homogeneity of the anisotropy! which is paradoxical
- The deformation of time remains unexplained by an elastic medium even if the tensorial expressions make it possible to integrate it into a 4-dimensional elasticity
- How can one have a hyper-rigid elastic medium and celestial bodies which move freely within it? This rigidity only manifests itself at the speeds of light. Would space therefore be a low-speed fluid and a hyper-rigid medium at the speed of light rather than a crystalline medium made up of space atoms the size of the Planck length?

Next steps…

- Test the analogy (elasticity strain deformation/ G W polarisations) on the differents ways to introduce geometric torsion in GR,
- Test the analogy (elasticity strain deformation/ G W polarisations) on the classical general relativity but with linearisation based on quadratic form of assembly A+ and Ax at second order,
- See potential interactions between the analogy and the elastic space medium concerning the dark matter and dark energy,
- Verifiy the order of magnitude of the equivalent Young’s modulus of space time base on a model of the Prob B angular distorsion of the space,
- Mind about the possible structure of the space (interaction between the deformation in the plane of the interferometer and out of this plane with the different Poisson’s ratio, possible anisotropy etc),
- Identify when the analogy work and where are the limits,

Other possible displacements perpendicular at the plane of the test masses (linear general relativity at second order)

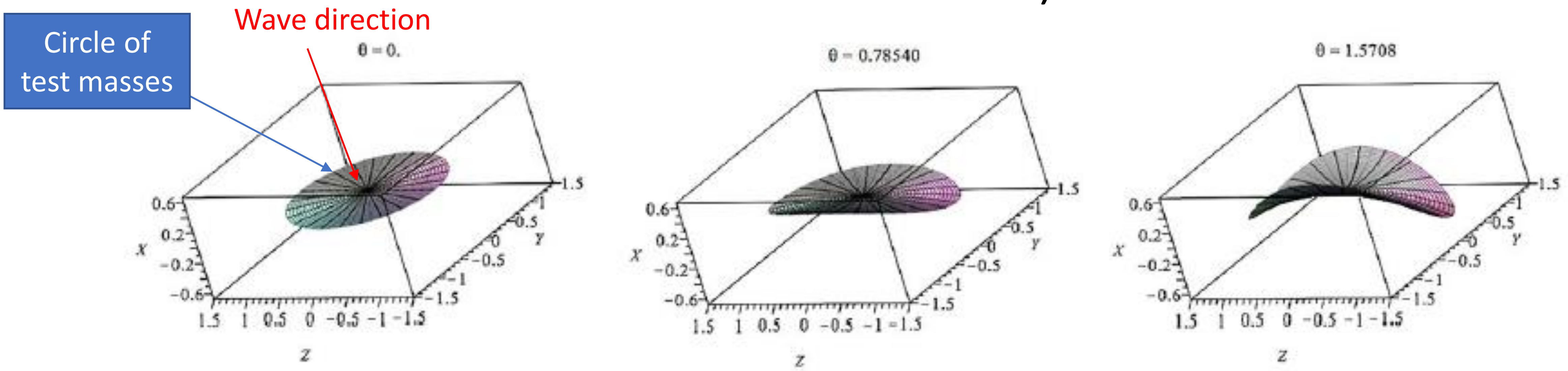


Fig. 1 Time evolution due to the gravitomagnetic effects of the + polarization; $\theta = \omega T$

Gravitomagnetic induction in the field of a gravitational wave

Matteo Luca Ruggiero[1,2]

Received: 20 April 2022 / Accepted: 25 August 2022

Abstract
The interaction of a plane gravitational wave with test masses can be described in the proper detector frame, using Fermi coordinates, in terms of a gravitoelectric and a gravitomagnetic field. We use this approach to calculate the displacements produced by gravitational waves up to second order in the distance parameter and, in doing so, we emphasize the relevance of the gravitomagnetic contribution related to gravitational induction. In addition, we show how this approach can be generalized to calculate displacements up to arbitrary order.

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